Combined Adaptive and Variable Structure Control for Constrained Robots*

CHUN-YI SU,† YURY STEPANENKO† and TIN-PUI LEUNG‡

Key Words—Robot; constraints; variable structure control; adaptive control; efficiency.

Abstract—This paper addresses the problem of controller design for constrained robots. A control algorithm, described as a combination of the direct adaptive and the variable structure method, is presented for the trajectory tracking of an end-effector on a constrained surface with specified constraint forces. This scheme has certain advantages, both with respect to computational efficiency and with respect to design. With respect to computational efficiency, the calculation of the regressor is avoided. With respect to design, since the number of parameter update laws required in the adaptation is independent of the number of links of the robot, the difficulty of tuning numerous adaptation gains is avoided.

1. Introduction

To achieve a wider class of tasks that involve contact with a manipulator environment, issues of appropriate modeling and of effective new control strategies arise, since such contact usually results in the generation of external forces acting on the end-effector of a manipulator, and modifies the dynamic behavior of a manipulator. The mathematical model of a robot system in contact with its environment, when it is described by a holonomic smooth manifold, gives rise to a mathematical system composed of differential and algebraic equations (McClamroch and Huang, 1985). The control of such systems is called constrained robot control (see e.g. McClamroch and Wang, 1988; Mills and Goldenberg, 1989). The objective of control is to determine the input torques to achieve tracking for a desired trajectory on a constrained surface with specified constraint forces.

The initial study on controlling a constrained robot was done by Hemami and Wyman (1979). Later, a number of strong results were obtained, including those of Yoshikawa (1987), McClamroch and Wang (1988), Mills and Goldenberg (1989), Yoon and Salam (1988), Yun (1988), Young (1988) and Lozano and Brogliato (1990). A general theoretical framework of constrained motion control was proposed by McClamroch and Wang (1988), who developed a rigorous mathematical model for constrained robots explicitly incorporating the constraint description. This model was then used to develop a modified computed torque controller guaranteeing global asymptotic stability for position and force tracking. However, this modified computed torque controller required exact knowledge of robot dynamics. To deal with the uncertainty in the constrained robot model, adaptive controls (Carelli and Kelly, 1991; Jean and Fu, 1991; Han et al., 1992) and variable structure control (Huang and Lin, 1992) were developed based on the model given by McClamroch and Wang (1988).

In Su et al. (1990, 1992), an alternative mathematical model for constrained robots was developed that embedded the constraint equations into the dynamic equation, resulting in an affine nonlinear system without constraints. Then adaptive control (Su et al., 1990) and variable structure control (Su et al., 1992) were proposed. Although these control laws can be shown to achieve asymptotic tracking, owing to the use of a regressor matrix, the computational complexity required for their implementation may be considerable. Therefore, a modified scheme (Jean and Fu 1991) to allow off-line computation of the regressors using the desired values instead of actual measurements was proposed. However, it should also be emphasized that use of the regressor is not the only method for controller design. In fact, robust controllers for robot motion control without the use of regressors are quite successful (see e.g. Stepanenko and Yuan 1992).

In this paper, based on the model of constrained robots established in Su et al., (1990, 1992), a combined adaptive and variable structure control strategy is proposed. The justification for combining adaptive and variable structure methods was given by Narendra and Boskovic (1992), who demonstrated through a simple first-order linear plant that such a combination can overcome the principle drawbacks of a variable structure method, and is superior to any of the methods currently available in adaptive control. Therefore, a new class of robust adaptive control laws for nonlinear robotic systems has been developed. Compared with other robust methods (Carelli and Kelly, 1991; Su et al., 1990, 1992; Jean and Fu, 1991; Han et al., 1992), this scheme has certain advantages, both with respect to computational efficiency and with respect to design. With respect to computational efficiency, the calculation of the regressor is avoided. With respect to design, our algorithm requires the tuning of three updated parameters, independent of the number of links of the robot, whereas typical parameter adaptive algorithms can require updating of as many as 10 parameters for each link of the robot (Spong, 1993). Stability analysis shows that the controller guarantees the boundedness of the tracking error. Moreover, the tracking error can be made arbitrarily small.

2. Constrained robot dynamics

In the Euler–Lagrange formulation, the motion equation of an n-link rigid constrained robot can be expressed in joint space as

\[
D(q)\ddot{q} + B(q, \dot{q})\dot{q} + G(q) = u + J'(q)A, \\
\phi(q) = 0,
\]

where \( q \in \mathbb{R}^n \) represents the generalized coordinates (joint positions), \( u \in \mathbb{R}^n \) is the vector of applied joint torques, \( J'(q) \in \mathbb{R}^{n \times n} \) is the vector of constraint forces in joint space, \( A \in \mathbb{R}^n \) is the associated Lagrange multiplier, \( D(q) \in \mathbb{R}^{n \times n} \) is the symmetric, bounded, positive-definite inertia matrix (Ghorbel et al., 1993), the vector \( B(q, \dot{q}) \in \mathbb{R}^n \) represents...
the centripetal and Coriolis torques, $G(q)$ ∈ $\mathbb{R}^n$ is the vector of gravitational torques, which is a bounded $C^1$ function, $\dot{\psi} = 0$ is the constraint equation in joint space (the mapping $\psi: \mathbb{R}^n \rightarrow \mathbb{R}^m$ is twice continuously differentiable), and $J(q) = \partial \psi / \partial q$ is the Jacobian matrix of the constraint equation (2). Two simplifying properties should be noted about this dynamic structure.

**Property 1.** A suitable definition of $B(q, \dot{q})$ makes the matrix $D - 2B$ skew symmetric.

**Property 2.** There exist positive constants $\rho_i$, $i = 1, 2, 3$, such that
\[
\|D(q)\| \leq \rho_1, \\
\|B(q, \dot{q})\| \leq \rho_2, \\
\|G(q)\| \leq \rho_3.
\]

Since $\psi(q) = 0$ is identically satisfied, it is evident that $J(q) = 0$. Thus, the effect of the constraints on the end-effector is linear and positive definite.

**Property 3.** The matrix $A(q') = L^T(q')B(q', \dot{q}')$ is skew symmetric.

**Property 4.** The matrix $\hat{A}(q') = L^T(q')B(q', \dot{q}')$, is skew symmetric.

**Property 5.**
\[
J(q')L(q') = L^T(q')J(q') = 0.
\]

The above properties are fundamental for designing the force/motion control law. Throughout this paper, the norm of a vector $x$ is defined as
\[
\|x\| = \left( \sum_{i=1}^{n} x_i^2 \right)^{1/2}
\]
and that of a matrix $A$ as the corresponding induced norm
\[
\|A\| = \max_i \left\{ \text{eigenvalue} \ A^T A \right\}^{1/2}.
\]

3. Reduced-order control

3.1. Robust adaptive controller design. The considered adaptive controller design problem is stated as follows. Given a desired joint trajectory $q_d$ and desired constraints force $f_2$, in fact, equivalently, a desired multiplier $\lambda \phi$, that satisfy the imposed constraints, i.e. $\psi(q) = 0$ and $f_2 = J(q')\lambda$, determine a control law such that for all $(q(0), \dot{q}(0)) \in \Omega, q(\cdot) = \dot{q}(\cdot)$, and $f \rightarrow f_2$ as $t \rightarrow \infty$.

It should be noted that, since $q' = \sigma(q')$, it is only required to find a control law to satisfy $q' \rightarrow q_m'$, as $t \rightarrow \infty$.

In order to derive the controller, the following assumptions are required.

**Assumption A.1.** The desired trajectory $q_d(t)$ is chosen such that $q_{d1}, \dot{q}_{d2}$ and $\dot{q}_{d3}$ are all bounded signals.

**Assumption A.2.** The minimum and maximum eigenvalues of the matrix $L^T(q')L(q')$ satisfy
\[
\lambda_{\text{min}}(L^T L) \geq \beta_1, \quad \lambda_{\text{max}}(L^T L) \leq \beta_2(q')
\]
where $\beta_1$ is a strictly positive constant and $\beta_2(q')$ is a known positive function, bounded for bounded $q'$.

**Assumption A.3.** There exists a known positive function $\beta_3(q')$, bounded for bounded $q'$, such that
\[
\|L(q')\| \leq \beta_3(q') \quad \text{for} \quad \frac{\partial L(q')}{\partial q'} \neq 0.
\]

**Remark.** Assumptions A.2 and A.3 can always be satisfied, and need not be stated as assumptions, since $L(q')$ is assumed to be known. One can always find, if it exists, a known constant $\beta_1$ and known functions $\beta_2(q')$ and $\beta_3(q')$ so that Assumptions A.2 and A.3 are satisfied. The purpose of using $\beta_1$, $\beta_2$ and $\beta_3$ is to try to make full use of the available knowledge so as to reduce the control gains.

With Property 2 and Assumptions A.2 and A.3, and noting that $\beta_2(q') \equiv 1$ owing to the structure property of $L$, the following property holds.

**Property 6.** For all $(q', \dot{q}') \in \mathbb{R}^{2(n-m)}$, the matrices $L^T D(q')L, L^T B(q', \dot{q}')$ and $L^T G(q')$ satisfy
\[
\|L^T D(q')L\| \leq \rho_1 \beta_2, \\
\|L^T B(q', \dot{q}')\| \leq \beta_2(\rho_1 + \rho_2)\|q'\|, \\
\|L^T G(q')\| \leq \rho_2 \beta_3.
\]

where $\rho_i$, $i = 1, 2, 3$, are positive constants.

A sliding variable $s_i = [s_1 \cdots s_{n-m}] \in \mathbb{R}^{n-m}$ is defined as
\[
s_i = \tilde{e}_n + \Lambda e_{m},
\]
where $e_m = q' - q_{d5}$ denotes the tracking error and $\Lambda$ is a
positive-definite matrix whose eigenvalues are strictly in the right half complex plane. The robust adaptive control law is then synthesized as

\[ u = -K_d L \delta - \beta_1 J_T(D\delta + \delta) \beta_1 \]

where \( K_d \in \mathbb{R}^{m \times m} \) is a positive-definite matrix, \( \beta_i \), \( i = 1, 2, 3 \), are the adaptive control gains, \( \delta \) is a strictly positive constant and \( D \in \mathbb{R}^{n \times m} \) is a vector of auxiliary signals defined by

\[ D = \sum_{i=1}^{3} \beta_i \varphi_i \]

The force term \( \lambda_k \) in (11) is defined as

\[ \lambda_k = \lambda_d - K_d e_k, \]

where \( K_d \in \mathbb{R}^{m \times m} \) is a constant diagonal matrix of force feedback control gains, \( e_k = \lambda - \lambda_d \).

The updated laws \( \beta_i \), \( i = 1, 2, 3 \), are defined as follows:

\[ \dot{\beta}_1 = \eta_1 \beta_1 m(||x||) \left[ ||x||^2 |||\|\phi||\|\right], \]

\[ \dot{\beta}_2 = \eta_2 \beta_2 (1 + \beta_3) m(||x||) \left[ ||x||^2 ||\phi|| + ||\phi||^2 \right], \]

\[ \dot{\beta}_3 = \eta_3 \beta_3 m(||x||) \left[ ||x||^2 + ||\phi||^2 + \beta_1 \right], \]

where \( \eta_i > 0 \), \( i = 1, 2, 3 \), are constants, determining the rates of adaptation, \( m(||x||) \) is a modulation function, which allows the controller transition between a nonadaptive and adaptive model and will be defined in the next section, and \( x^T = [\phi^T, \chi^T] \).

Remark. The controllers given by Carelli and Kelly (1991), Su et al. (1992) and Han et al. (1992) actually belong to the linear parameterization approach, where on-line calculation of regressors is required. For a typical six-joint robot this involves very intensive computations (Stepanenko and Yuan, 1992). However, first, the control law (11) gives an ahema-
Therefore, the force tracking error \( f - f_0 \) is bounded and can be adjusted by changing the feedback gain \( K_x \). Thus, the lemma is proved.

3.2.2. Unknown parameter upper bounds. We now return to the main purpose of the paper, i.e. \( \hat{\beta}_i, i = 1, 2, 3 \), in (11) are unknown and adjusted by the adaptive law (14)–(16). Following the method given by Brogliato and Trofino-Neto (1992), let \( m(|x|) \) in (14)–(16) be chosen as depicted in Fig. 1. For the existence of the closed-loop systems, see Brogliato and Trofino-Neto (1992). Now, we can present the following stability theorem for the control law (11) and (14)–(16).

**Theorem 1.** Set \( \epsilon > 0 \) as the admissible magnitude of \( s_i \). Then the control law (11) with the updated laws (14)–(16) in closed loop with the constrained robots modeled in the reduced form (6) guarantees that the following hold:

(i) the system state \( x \) is uniformly ultimately bounded, and the total time spent by \( x \) outside the ball \( \{x: |x| < \gamma_1(1/2R^{-1}(\epsilon)) \} \) is finite;

(ii) the steady-state force \( J_{Te}, \) is bounded and inversely proportional to the norm of the matrix \( K_x + I \).

**Proof.** Let us consider the positive-definition function

\[
V(t) = V_1(t) + \sum_{i=1}^{3} \left( \rho_{2i} - \rho_i \right) \| x_i \|^2 / \eta_i,
\]

where \( V_1(t) \) is defined in (18) and \( \rho_i \) are estimates of \( \rho_{2i}. \)

Since the state \( x \) is outside \( R(e) \), differentiating (25) with respect to time along the trajectories of (17) leads to

\[
\dot{V} = \sum_{i=1}^{3} \left( \rho_{2i} - \rho_i \right) \dot{x}_i \times \left[ (\rho_{2i} - \rho_i) \dot{x}_i - \rho_i x_i \right] / \eta_i.
\]

Since \( m(|x|) \) are chosen as depicted in Fig. 1, following the results and argument given by Brogliato and Trofino-Neto (1992), the state \( x \) is uniformly ultimately bounded in the sense that given any \( \epsilon > 0 \), the total time spent by \( x \) outside the set \( \gamma_1(1/2R^{-1}(\epsilon)) \) is finite, and \( \rho_i, i = 1, \ldots, 4 \) remain bounded. Thus, (i) has been proved.

Since \( \hat{\beta}_i, i = 1, 2, 3 \) and therefore \( \hat{e}_m \) and \( \hat{e}_e \) remain bounded, the proof of (ii) is similar to that of Lemma 1, and is omitted here to save space. □

**Remarks.**

1. For the controller design, the existence of \( \rho_{2i}, i = 1, 2, 3 \), is necessary to guarantee the stability of the closed-loop system. However, these constants are not explicitly involved in the control inputs; the existence of \( \rho_{2i} \) is sufficient for the validity of Theorem 1. The control inputs will rise to whatever level is necessary to ensure the stability of the overall system. The residual set can be made as small as desired by choosing the design parameters. The trade-off between the size of the residual set and the control gain can be determined by the choice of \( \delta \) and \( \epsilon. \)

2. Since the choice of \( y_1(\cdot) \) and \( y_2(\cdot) \) is related to the unknown inertia matrix \( L^TD_L \), one can always choose suitable \( y_1(\cdot) \) and \( y_2(\cdot) \), although such a choice may be conservative. Moreover, even if \( y_1(\cdot) \) and \( y_2(\cdot) \) are unknown, following the results and argument of Brogliato and Trofino-Neto (1992), one can replace the value \( \gamma_1(1/2R^{-1}(\epsilon)) \) in the definition of \( m(|x|) \) by any constant \( \xi > \gamma_1(1/2R^{-1}(\epsilon)) \), and the above results will still hold. We should mention that, even with the very conservative choice of \( y_1(\cdot) \) and \( y_2(\cdot) \), they are only related to the transition between the nonadaptive and adaptive modes, not to the controller gains. This differs from the case where the upper bounds are required to be known.

3. Since the matrix \( I \) is known, \( \beta_1 \) and \( \beta_2 \) given in Assumption A.2 should be chosen in such a way as to reduce the magnitude of \( \beta_2/\beta_1 \). This, in turn, reduces the magnitude of the control law.

4. No perfect force tracking is objective is warranted. Nevertheless, part (ii) of Theorem 1 has a particular value, because improved steady state constraint force accuracy is obtained with sufficiently high force gain. This result is similar to the results presented in Carelli and Kelly (1991).

5. As \( \delta \to 0 \), the function \( s_1/(|s_i| + \delta) \) eventually becomes discontinuous. In such a case, the controller becomes a typically variable structure control scheme, which may cause chattering phenomena. As a matter of fact, the control law (11) is just a smoothing realization of the switching function \( \text{sgn}(s_i) \) to overcome chattering, which is undesirable in practice. It should be noted that if \( \delta \) is chosen too small, such that the linear region of the function \( s_1/(|s_i| + \delta) \) is very thin, there is a risk of exciting high frequency dynamics. This suggests that a trade-off must be made between the value of \( \delta \) and trajectory-following requirements. An effective way to implement such a trade-off is to actually let \( \delta \) vary depending on the state location. For a discussion of the tuning of \( \delta \) in this spirit, the reader is referred to Slotine (1984).

**4. Simulation results**

A two-link robotic manipulator with a circular path constraint, as given by Young (1988) and Su et al. (1990, 1992), is used to verify the validity of the control approach outlined in this paper. The original model, in the form of (1), can be written as

\[
\begin{bmatrix}
D_1(q_1) & D_3(q_2) & q_1 \\
D_2(q_1) & D_3(q_2) & q_2 \\
F_1(q_1) & F_2(q_1) & 0 \\
F_1(q_1) & F_2(q_1) & 0
\end{bmatrix}
\begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2 \\
\dot{q}_3 \\
\dot{q}_4
\end{bmatrix} +
\begin{bmatrix}
\ddot{q}_1(q_1, q_2) \dot{q}_1(q_1, q_2) \\
\ddot{q}_2(q_1, q_2) \dot{q}_2(q_1, q_2) \\
\ddot{q}_3(q_1, q_2) \dot{q}_3(q_1, q_2) \\
\ddot{q}_4(q_1, q_2) \dot{q}_4(q_1, q_2)
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
\psi(q_1, q_2) \psi(q_1, q_2) \\
\psi(q_1, q_2) \psi(q_1, q_2) \\
\psi(q_1, q_2) \psi(q_1, q_2) \\
\psi(q_1, q_2) \psi(q_1, q_2)
\end{bmatrix}
\begin{bmatrix}
\ddot{q}_1(q_1, q_2) \\
\ddot{q}_2(q_1, q_2) \\
\ddot{q}_3(q_1, q_2) \\
\ddot{q}_4(q_1, q_2)
\end{bmatrix},
\]

where

- \( D_1(q_1) = (m_1 + m_2)q_1^2 + m_2l_2^2 + 2m_2l_1^2 \cos q_2 \)
- \( D_2(q_2) = m_2l_2^2 + m_2l_1^2 \cos q_1 \)
- \( D_3(q_2) = m_2l_2^2 \sin q_2 \)
- \( F_1(q_1) = -m_2l_2 \sin q_2 \)
- \( F_2(q_1) = -m_2l_2 \cos q_1 \)
- \( \ddot{q}_1(q_1, q_2) = -m_2l_2 \cos q_1 \)
- \( \ddot{q}_2(q_1, q_2) = m_2l_2 \cos q_1 \)
- \( \ddot{q}_3(q_1, q_2) = -m_2l_2 \cos q_1 \)
- \( \ddot{q}_4(q_1, q_2) = m_2l_2 \cos q_1 \)

The parameter values used are the same as those of Su et al. (1992), namely \( l_1 = 1, l_2 = 0.5 \), \( m_1 = 0.5 \) kg, \( m_2 = 0.5 \) kg, \( J_1 = 0.97 \) kg m^2 and \( J_2 = 0.8 \) kg m^2.

The constraint is a circle in the work space (the \( x, y \) plane) whose center coincides with the axis of rotation of the first link. Figure 2 depicts the two-link manipulator and the constraint. The constraint, when expressed in terms of joint space, is

\[
\psi(q) = l_1^2 + l_2^2 + 2l_1l_2 \cos q_2 - r^2 \leq 0,
\]

which has an unique constant solution for \( q_2 \).

\[
q_2 = \cos^{-1} \left( \frac{r^2 - (l_1^2 + l_2^2)}{2l_1l_2} \right) = q_f. \]
The Jacobian matrix of (28) is
\[ J(q) = \begin{bmatrix} 0 \\ -2l_1 l_2 \sin q_2 \end{bmatrix} \] (30)

Therefore, the matrix \( L \) defined in (5) is
\[ l(q^1) = \begin{bmatrix} 1 & 0 \end{bmatrix}^T. \] (31)

The constrained robot motion equation (6), when restricted to the circle, can be expressed as
\[ \begin{bmatrix} D_{11}(q^1) & D_{12}(q^2) \\ D_{21}(q^1) & D_{22}(q^2) \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} g_1(q_1, q_2) \\ g_2(q_1, q_2) \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -2l_1 l_2 \sin q_2 \end{bmatrix} \lambda. \] (32)

The constraint forces are
\[ f_1 = 0, \]
\[ f_2 = -2l_1 l_2 \lambda \sin q_2. \] (33)

The control objective is to determine a feedback control so that the joint \( q_1 \) tracks the desired trajectory \( q_{1d} \) and maintains the constraint force \( f_2 \) to the desired \( f_{2d} \), where \( q_{1d} \) and \( f_{2d} \) are assumed to be consistent with the imposed constraint.

Since \( \lambda \to \lambda_d \) means \( f_2 \to f_{2d} \), in this simulation \( q_{1d} \) and \( f_{2d} \) are chosen as
\[ q_{1d} = \begin{bmatrix} -90 + 52.5(1 - \cos 1.26t) \\ 15 \end{bmatrix}, \]
\[ \lambda_d = 10. \] (34)

The parameters \( \beta_1, \beta_2, \) and \( \beta_3 \) are chosen as \( \beta_1, \beta_2 = 1 \) and \( \beta_3 = 0 \). The control parameters are chosen as \( K_1 = 5I, K_2 = 10I, \lambda = 4I \) and \( K_5 = 0.8I \). The adaptive gains are chosen as \( \eta_1 = 4.5, \eta_2 = 4.5 \) and \( \eta_3 = 4.5 \). \( \delta \) and \( \epsilon \) are chosen as \( \delta = 0.2 \) and \( \epsilon = 0.2 \).

Since trajectory tracking on the constrained surface with specified constraint force is of interest, the initial position and velocity of the manipulator are chosen on the desired trajectory:
\[ q_1(0) = -90^\circ, \quad q_2(0) = 80^\circ, \quad \dot{q}_1(0) = 0, \quad \dot{q}_2(0) = 0. \]

The initial constraint force is assumed as \( f_2 = 0 \), i.e. \( \lambda = 0 \). The initial control gains \( \beta_i(0), i = 1, 2, 3, \) are arbitrarily taken as \( \beta_1(0) = 2, \beta_2(0) = 1 \) and \( \beta_3(0) = 1 \).

The simulation is conducted for two cases: the fixed-parameters controller (dashed curves) and the adaptive controller (solid curves), where the fixed gains are chosen as \( \rho_{1f} = 3.5, \rho_{2f} = 3.5, \) and \( \rho_{3f} = 20 \). Figure 3 shows the desired joint trajectory and Fig. 4 shows the contact force \( \Lambda \). The final maximum error with \( \Lambda_d \) is 1.5 N. Figures 6 and 7 show the torques exerted at manipulator joints, and the changes in the adapted parameters are shown in Fig. 8. These results show that the control objective is achieved.

5. Conclusions
In this paper, we have derived an adaptive scheme, described as a combination of adaptive and variable structure control techniques, for constrained robots. The developed scheme actually utilizes the tracking error feedback with adaptive gains that grow until they reach whatever levels are necessary to cancel the nonlinearities. Compared with previous designs (parameter adaptive algorithms), clearly, the parameter adaptive algorithms are model-based methods, use more knowledge about the dynamic system and would be expected to work better. However, it is known that computation of the regressor is a time-consuming task and numerous parameters are required to be updated; they may
also exhibit poor robustness to unmodeled dynamics and external disturbances unless the algorithms are modified (Spong, 1993). As an alternative to the parameter adaptive designs, our design has certain advantages, particularly with respect to computation, design and robustness. Therefore, the proposed scheme would be attractive in an environment where the computing capability has some limitations, the knowledge of the mathematical model is somewhat lacking, and unmodeled dynamics is of concern.

Acknowledgements—The authors wish to acknowledge the support of the Natural Science and Engineering Research Council of Canada, the Institute for Robotic and Intelligent Systems (IRIS) and Precarn Associates Inc. The authors also wish to thank anonymous referees for their helpful suggestions.

References