CONCORDIA UNIVERSITY

Department of Computer Science and Software Engineering

COMP361/5611 - Numerical Methods

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Course Outline - Fall 2021

Objective: Advanced numerical algorithms are widely used in virtually all areas of Science and Engineering. Computer Scientists are expected to possess some knowledge of the most common numerical algorithms. The main purpose of this course is to introduce some basic concepts and elementary algorithms that are fundamental to almost all scientific and engineering computations, with emphasis on their mathematical understanding.

Topics: These topics are included: Vector norms and matrix norms, Gauss elimination and LU-decomposition, row-pivoting, condition number, residual correction, nonlinear equations, iterative methods, Newton's method, convergence analysis, polynomial interpolation, numerical differentiation, best approximation, orthogonal polynomials, numerical integration, and Gauss quadrature.

Prerequisites: COMP232 and COMP249. Basic knowledge of Linear Algebra (vectors and matrices) and Calculus (derivatives and integrals) is essential.

Course Material: The Lecture Notes are posted on the COMP361 Web Page

`https://users.encs.concordia.ca/~doedel/courses/comp-361/`

(Be sure to enter the "widetilde" (" ~ ") manually if you screen copy the above address.)

The Lecture Notes include all necessary course material in a concise form. Students are expected to thoroughly understand the material covered, and to be able to apply it to relatively simple problems. In this respect the Lecture Notes contain various representative problems, with solutions provided for many of these.

A useful reference may be "Numerical Computing with MATLAB", by Cleve Moler, which can be downloaded from `http://www.mathworks.com/moler/`. Note, however, that there is no formal text book in this course. Students wishing to learn additional material, beyond the requirements of the course, are encouraged to search for books on "Numerical Methods" or "Numerical Analysis". However, such supplementary reading is not necessary to prepare for the examinations.

Lectures: Prerecorded lecture segments can be viewed on the COMP361 Moodle page, where they are posted under "YuJa Lecture Capture". Note that Moodle now requires a more secure login procedure, called "MFA" (multi-factor authentication), as described here:

`https://www.concordia.ca/it/security/mfa.html`

The lecture segments are arranged according to the corresponding pages of the Lecture Notes. Click on "SORT", and then "Alphabetical", to list the videos in proper order. For each week there are several videos, each focusing on a specific topic, as shown in the Table on the next page. The videos show pages from the Lecture Notes with accompanying narrative, much like what will be done in the actual lecture room H535, but without an opportunity for questions and discussions. An effective approach is to preview pages in the Lecture Notes, followed by viewing the corresponding video, and then study the pages more carefully. Thereafter attend the actual class in H535 for a more lively presentation, with opportunities for questions and discussions.

WEEK	LECTURE NOTES	TOPICS	ASSIGNMENT DUE
Week 01 Sep 10	Pages-001-005 Pages-006-019 Pages-020-024	Vector Norms Matrix Norms Banach Lemma	
Week 02 Sep 17	Pages-025-030 Pages-031-036 Pages-037-039 Pages-040-046	Gauss Elimination LU-Decomposition Tridiagonal Systems Inverses	
Week 03 Sep 24	Pages-047-055 Pages-056-072	Pivoting Condition Number	Assignment 1 Problems 1-6
Week 04 Oct 01	Pages-073-091 Pages-092-098 Pages-099-101	Nonlinear Equations Newton for Systems Residual Correction	
Week 05 Oct 08	Pages-102-110 Pages-110-116 Pages-117-125 Pages-126-130	Fixed Point Iteration Fixed Point Theorem Logistic Equation Rate of Convergence	Assignment 2 Problems 7-16
Week 06 Oct 15	Pages-131-144 Pages-145-157	More Newton's Method Convergence for Systems	
Week 07 Oct 22	TERM TEST 1		
Week 08 Oct 29	Pages-158-165 Pages-166-175 Pages-176-184	Function Norms Lagrange Interpolation Lagrange Theorem	Assignment 3 Problems 17-26
Week 09 Nov 05	Pages-185-190 Pages-191-200 Pages-201-206	Chebyshev Polynomials Chebyshev Theorem Chebyshev Examples	
Week 10 Nov 12	Pages-207-215 Pages-216-230 Pages-231-239	Taylor Polynomial Local Interpolation Numerical Derivatives	
Week 11 Nov 19	Pages 268-277 Pages-278-286	Numerical Integration Integration Theorem	Assignment 4 Problems 27-33
Week 12 Nov 26	TERM TEST 2		
Week 13 Dec 03	Pages-240-246 Pages-247-255 Pages-256-267 Pages-287-295	Best Approx. in R^n General Best Approx. Gram-Schmidt Gauss Quadrature	Assignment 5 Problems 34-41
TBA	FINAL EXAM		

Assignments: The assignment problems are posted on the COMP361 Web Page by the beginning of the course. Your solutions to these problems are due as indicated in the Table on the preceding page. Most assignment problems are of a theoretical nature, but some also require programming. Programming can be done in any appropriate language, such as Python, while keeping in mind that most large-scale scientific computations on supercomputers rely heavily on Fortran and C.

The way to submit assignments will be announced before the first assignment is due. Solutions must be handwritten, and be presented in the order the problems appear in the posted file with assignment problems. Discussing the assignments with other students and with the tutors is encouraged, but it is very important that you do the actual work yourself. The assignments constitute an essential learning experience without which it may be difficult to pass the examinations.

Tutorials: Depending on the availability of tutors there will be scheduled tutorials, starting in the second week of classes, where problems related to the lectures and assignments can be discussed. In particular, the tutorials provide an opportunity to discuss the exercises in the Lecture Notes, many but not all of which are accompanied by solutions.

Questions related to the lectures and the assignments can also be discussed with the instructor during his office hours, which will be posted on the COMP361 Web page.

Assessment: There will be 5 assignments, two term tests, and a final examination. Term Test 2, which will be near the end of the term, will examine a student's knowledge of all lecture and assignment material covered so far during the term. Based on the student's performance on both term tests and the assignments, a provisional course letter grade will be assigned, according to the following weighting scheme: Assignments: 20%, Term Test 1: 20%, Term Test 2: 60%. However, if the result of the Term Test 2 is better than the result of Term Test 1 (as a percentage out of 100) then Term Test 1 will not count, and the weight of Term Test 2 will become 80%.

Students are then given the option of accepting the provisional course letter grade, in which case they will not write the final exam, or not accepting the provisional course letter grade, in which case they must write the final exam. There will be a strict deadline for acceptance of the provisional course letter grade. If no acceptance is received by the deadline then the student must write the final exam.

If a student chooses to write the final exam then the course letter grade will be based on the following weighting scheme: Assignments: 20%, Term Test 1: 10%, Term Test 2: 20%, Final Exam: 50%. However, if the result of the Final Exam is better than the result of Term Test 1 (as a percentage out of 100) then the weight of Term Test 1 will be transferred to the weight of the Final Exam. Moreover, if the result of the Final Exam is better than the result of Term Test 2 (as a percentage out of 100) then the weight of Term Test 2 will be transferred to the Final Exam.

Note that there is no standard relationship between numerical percentages and letter grades.

Important Additional Information: Students and faculty members must adhere strictly to the safety rules set by the government and the university with respect to the covid 19 pandemic. This includes rules on the wearing of masks and distancing rules, which can be found here:

`https://www.concordia.ca/coronavirus/return-to-campus.html`

In the eventuality that the evolving circumstances require a return to on-line teaching then there will be important changes to the information that is currently in this outline. This may include complete reliance on the recorded lectures, with more email communication with the instuctor and with the tutors. It can also result in the cancellation of the final exam, in which case the course grade will be based only on the assignments and the two Terms Tests, precisely as already presented as an option in the assessment section above.

CEAB Graduate Attributes:

As part of either the Computer Science or Software Engineering program curriculum, the content of this course includes material and exercises related to the teaching and evaluation of graduate attributes. Graduate attributes are skills that have been identified by the Canadian Engineering Accreditation Board (CEAB) and the Canadian Information Processing Society (CIPS) as being central to the formation of Engineers, Computer Scientists and Information Technology professionals. As such, the accreditation criteria for the Software Engineering and Computer Science programmes dictate that graduate attributes are taught and evaluated as part of the courses. The following is the list of graduate attributes covered in this course, along with a description of how these attributes are incorporated in the course.

Graduate attributes for COMP361 are:

Attribute 1: Knowledge-base: Knowledge of a wide array of fundamental numerical methods used in Science and Engineering, as stated in the course description.

Indicator 1.1: Knowledge base of mathematics

Attribute 2: Problem analysis: Use a wide array of basic numerical methods to model and analyze complex problems in order to establish the requirements and constraints on their design, implementation and deployment solutions.

Indicator 2.1: Problem identification and formulation

Course Learning Objectives:

Introduce the students to basic numerical techniques that are fundamental to Scientific and Engineering computing.

Make the students aware of the complexity and the limitations of numerical algorithms.

Teach the students basic concepts and analytical techniques that allow the determination of key properties of numerical algorithms.

Prepare the students for more advanced courses in Scientific and Engineering computing.