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SBox: A Qualitative Spatial Reasoner –Progress Report–

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Abstract

This paper presents a method for reasoning about spatial objects and their qualitative spatial relationships (e.g. touches, overlaps etc.) on the basis of a description logic framework. We apply this method to the domain of deductive geographic information systems. In contrast to existing work, which mainly focuses on reasoning about qualitative spatial relations alone, we integrate quantitative and qualitative information with terminological reasoning by extending description logics with a space box reasoner which is inspired by an extension to description logics called "concrete domains." With the space box reasoner presented in this paper it is possible to combine qualitative spatial reasoning and description logic classification processes.

Keywords— Qualitative spatial reasoning, description logics, deductive geographical information systems.

1 Introduction

Qualitative relations play an important role in formal reasoning systems. We emphasize that inferences about spatial relations should not be considered in isolation but should be integrated with formal inferences about concepts (e.g. automatic consistency checking and classification). The semantics of qualitative relations should be grounded in a quantitative representation of spatial data. In our opinion, the abstractions provided by qualitative spatial relations can be interpreted as an interface from a conceptual model about the world to quantitative spatial data representing spatial information about domain objects.

The combination of conceptual and spatial inference services can be used to solve important application problems. Continuing our work presented in [10] we show how terminological inferences with spatial relations can be used for image interpretation. The characteristic of these problems is that it is often very difficult to describe a fixed algorithm that defines an exact sequence of "interpretation steps" because several different "cues" have to be integrated. In other words: the solution must be computed by adequately integrating partial information about domain objects. The information about objects is given by conceptual background knowledge, the image itself and different kinds of intermediate interpretation results. According to the work of Schröder and Neumann [19] who are inspired by the MAPSEE approach [17], image interpretation can be defined as a (re-)construction process of a specific possible world that is consistent with the given knowledge (see also Section 5).

In this paper, we consider a map interpretation problem and demonstrate how conceptual background knowledge can be exploited for image interpretation tasks. As an example, a subsection of a map from the city of Hamburg is shown in Figure 1. In a geographical information system, queries like "search for a living area in a border district with recreation areas" might be defined. We assume that the necessary data are automatically gathered using image interpretation techniques. Note that in our setting image interpretation starts with vector data, i.e. areas are defined by polygons (see the emphasized polygon in the center of Figure 1). Polygons from the image data are already annotated with labels like "living-area", "ordinary-road" etc. In order to interpret the image, different kinds of world knowledge are required. For instance, with background knowledge one can infer that the large number 7434 in the upper right corner of Figure 1 cannot be a zip code nor can it describe the height of a mountain (not in Northern Germany). The required inference steps can be formalized by combining spatial and terminological reasoning.

The spatial part of the theory is based on Egenhofer's set of topological relations. In contrast to [10] and [9] where topological relations are used as primitives in the sense of logic (i.e. they are semantically uninterpreted), we extend the treatment of topological relations by interpreting their semantic definition with respect to concept entailment (cf. the notion of subsumption: one concept is more general than another) and demonstrate their influence on automatic concept classification.

Thus, the theory presented in this paper allows to detect both inconsistencies and implicit information in formal conceptual models for spatial domain objects. On the one hand, it can be shown that concept definitions and subsumption (or inheritance) relations restrict the set of possible relations between domain objects. On the other hand, definitions about topolog-

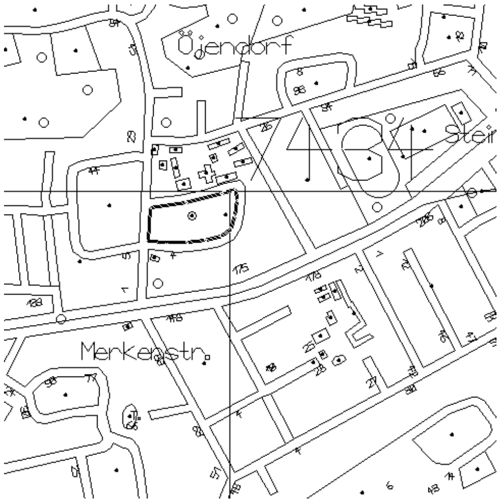

Figure 1: Subsection of Öjendorf, a district of the city of Hamburg.

ical relations might define implicit subsumption relationships which have to be automatically detected to capture all kinds of possible inferences that are sanctioned by the semantics of the representation formalism.

The major contribution of this paper is the incorporation of characteristics of space into the semantics of the inference system. The main idea is to treat a region as a set of points and to extend the subsumption relationship between concepts to subsumption between spatial regions. A region R_1 can be defined to subsume another region R_2 when R_1 "contains" R_2 (see Section 2 for a formal definition of spatial relations). Basically, for spatial subsumption, the same set-inclusion semantics as for concept languages is used. For our application we consider spatial point sets defined by polygons. Qualitative relations between two dimensional areas are defined by topological relations between polygons.

2 Qualitative Modeling

The previous section motivated the formalization (qualitative modeling) of space with the help of conceptual and spatial inference services. This section introduces the formal tools used for qualitative modeling. We define spatial regions and their qualitative relationships and combine them with a description logic framework extended by a space box reasoner.

2.1 Objects and their Spatial Relationships

The definition of basic geometric objects usually relies on topology which is itself a basis for defining relationships between objects. In the following we assume the usual concepts of point-set topology with open and closed sets [20]. The *interior* of a set λ_i (denoted by λ_i°) is the union of all open sets in λ_i. The *closure*

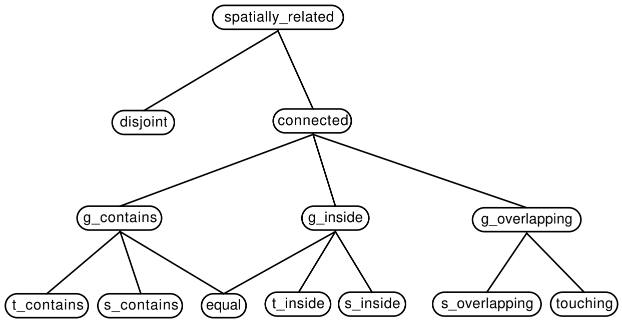

Figure 2: Hierarchy of spatial relations

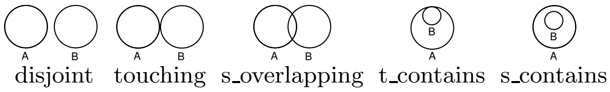

Figure 3: Spatial relations between A and B

of λ_i (denoted by $\overline{\lambda_i}$) is the intersection of all closed sets containing λ_i. The *complement* of λ_i (denoted by λ_i^{-1}) with respect to the embedding space $\Re^n$ is the set of all points of $\Re^n$ not contained in λ_i. The *boundary* of λ_i (denoted by $\partial\lambda_i$) is the intersection of the closure of λ_i and the closure of the complement of λ_i.

The following restrictions apply to every pair of sets. (1) λ_i, λ_j be n-dimensional and $\lambda_i, \lambda_j \subset \Re^n$, (2) $\lambda_i, \lambda_j \neq \emptyset$, (3) all boundaries, interiors, and complements are connected, and (4) $\lambda_i = \overline{\lambda_i^\circ}$ and $\lambda_j = \overline{\lambda_j^\circ}$.

Using these definitions we can define 13 binary topological relations that are organized in a subsumption hierarchy (see Figure 2). The leaves of this graph represent eight mutually exclusive relations that cover all possible cases with respect to the restrictions mentioned above. The eight relations are also referred to as *elementary relations*. The elementary relations are equivalent to the set of eight relations defined by Egenhofer [7] and others [16, 6]. Figure 3 illustrates five of these eight relations (the inverses and the relation equal are omitted). The 13 relations are defined as follows:

- **spatially_related**: Two objects have a *spatial relationship* between each other. This relation is defined as the disjunction of its two mutually exclusive subrelations disjoint and connected.
  spatially_related$(\lambda_1, \lambda_2) \equiv$
  disjoint$(\lambda_1, \lambda_2) \vee$ connected(λ_1, λ_2)
- **disjoint**: Two objects are *disjoint* if their intersection is empty; disjoint is symmetric.
  disjoint$(\lambda_1, \lambda_2) \equiv \lambda_1 \cap \lambda_2 = \emptyset$
- **connected**: Two objects are *connected* if their intersection is non-empty; connected is symmetric.
  connected$(\lambda_1, \lambda_2) \equiv \lambda_1 \cap \lambda_2 \neq \emptyset$

- **g_overlapping**: Two objects are *generally overlapping*. This relation is defined as the disjunction of its two mutually exclusive subrelations touching and s_overlapping; g_overlapping is symmetric.

$$\mathsf{g\_overlapping}(\lambda_1, \lambda_2) \equiv \mathsf{touching}(\lambda_1, \lambda_2) \vee \mathsf{s\_overlapping}(\lambda_1, \lambda_2)$$

- **touching**: Two objects are *touching* if only their boundaries are intersecting; touching is symmetric.

$$\mathsf{touching}(\lambda_1, \lambda_2) \equiv \mathsf{connected}(\lambda_1, \lambda_2) \wedge (\lambda_1^\circ \cap \lambda_2^\circ = \emptyset)$$

- **s_overlapping**: Two objects are *strictly overlapping* if they are connected and their intersection is not equal to either of them; s_overlapping is symmetric.

$$\mathsf{s\_overlapping}(\lambda_1, \lambda_2) \equiv \mathsf{connected}(\lambda_1, \lambda_2) \wedge (\lambda_1 \cap \lambda_2 \neq \lambda_1) \wedge (\lambda_1 \cap \lambda_2 \neq \lambda_2) \wedge (\lambda_1^\circ \cap \lambda_2^\circ \neq \emptyset)$$

- **g_contains/g_inside**: An object λ_1 *generally contains* an object λ_2. This relation is defined as the disjunction of its three mutually exclusive subrelations equal, t_contains, and s_contains; g_inside is the inverse of g_contains; g_contains and g_inside are reflexive, antisymmetric, and transitive.

$$\mathsf{g\_contains}(\lambda_1, \lambda_2) \equiv \mathsf{t\_contains}(\lambda_1, \lambda_2) \vee \mathsf{s\_contains}(\lambda_1, \lambda_2) \vee \mathsf{equal}(\lambda_1, \lambda_2)$$

- **equal**: The relation equal is symmetric and transitive.

$$\mathsf{equal}(\lambda_1, \lambda_2) \equiv \lambda_1 = \lambda_2$$

- **t_contains/t_inside**: An object λ_1 *tangentially contains* an object λ_2 if their intersection is equal to λ_2 and the intersection of their boundaries is non-empty; the inverse of t_contains is t_inside; t_contains and t_inside are asymmetric.

$$\mathsf{t\_contains}(\lambda_1, \lambda_2) \equiv (\lambda_1 \cap \lambda_2 = \lambda_2) \wedge (\lambda_1 \cap \lambda_2^{-1} \neq \emptyset) \wedge (\partial\lambda_1 \cap \partial\lambda_2 \neq \emptyset)$$

- **s_contains/s_inside**: An object λ_1 *strictly contains* an object λ_2 if their intersection is equal to λ_2 and only the interiors of their regions intersect; the inverse of s_contains is s_inside; s_contains and s_inside are asymmetric and transitive.

$$\mathsf{s\_contains}(\lambda_1, \lambda_2) \equiv (\lambda_1 \cap \lambda_2 = \lambda_2) \wedge (\lambda_1 \cap \lambda_2^{-1} \neq \emptyset) \wedge (\partial\lambda_1 \cap \partial\lambda_2 = \emptyset)$$

2.2 Description Logic

We model terminological knowledge about our GIS domain using description logic (DL) theory that has also been proven as a useful formalism for modeling in technical domains (see e.g. [22] and [14] for example applications). In addition, the formal properties of description logics have been extensively studied (see e.g. [15] and [21]).

The following sections give a brief introduction to some aspects of DL theory. We do not attempt to give a thorough overview and formal account of DL theory. However, we try to summarize the notions important for this paper and refer to [5, 3, 11] for more complete information about description logic theory.

2.2.1 DL: The Abstract Domain

In a DL a factual world consists of named individuals and their relationships that are asserted through binary relations. Hierarchical descriptions about sets of individuals form the terminological knowledge. Descriptions (or terms) about sets of individuals are called *concepts* and binary relations are called *roles*. Descriptions consist of identifiers denoting concepts, roles, and individuals, and of description constructors. Concepts or roles may be either *primitive* or *defined*. A specification of a primitive concept is denoted with the declaration operator '$\sqsubseteq$' and represents membership conditions that are necessary but *not* sufficient. The specification of a defined concept is denoted by '$\doteq$' and represents conditions that are both necessary *and* sufficient. For any individual x the set $\{y|\mathsf{r}(x, y)\}$ is called the set of *fillers* of role r.

Concept specifications may consist of concept terms and concept names. Unary (e.g. $\neg$) are used as modifiers and binary operators (e.g. $\wedge$, $\vee$) are used as connectives. A concept term can also be given as a restriction for role fillers. *Value restrictions* constrain the range of roles and allow only fillers that are individuals of a specific concept (e.g. ($\forall$ has_father male)). *Number restrictions* specify the maximum or minimum number of allowed fillers (e.g. ($\exists_{\leq 3}$ has_child), ($\exists_{\geq 1}$ has_father)). Roles with an implicit '$\exists_{\leq 1}$' number restriction are called *attributes*. These concept specifications are only a subset of all possible specifications. Section 2.2.3 lists the model-theoretic semantics of DL elements mentioned in this paper. The semantics defines the reasoning services a DL inference engine has to provide. In most description logics, the terminology must not contain cyclic definitions because the semantics of cycles cause tremendous theoretical and practical difficulties. Furthermore, a concept name must occur only once on the left-hand side in the definitions of a terminology. The expressiveness of a DL and the tractability of reasoning algorithms for a particular DL depends on the type and possible combinations of connectives and restrictions (see e.g. [21]).

DL systems (i.e. implementations of a DL) usually distinguish two separate reasoning components. The *terminological reasoner* or *classifier* classifies concepts with respect to subsumption relationships between these concepts and organizes them into a taxonomy. The TBox language is designed to facilitate the construction of concept expressions describing classes (types) of individuals. The classifier automatically performs consistency checking of concept definitions and offers retrieval facilities about the classification hierarchy. The (forward-chaining) *assertional reasoner* or *realizer* recognizes and maintains the "type" (i.e. concept membership) of individuals. The purpose of the ABox language is to state constraints or facts (usually restricted to unary or binary predications) that apply to a particular domain or world. Assertional reasoners support a query language in order to access stated and deduced constraints. Some query languages offer the expressiveness of the full first-order calculus.

2.2.2 DL: The Concrete Domain

Baader and Hanschke [2, 11] have explored the idea of separating the domain of a description logic into an *abstract* and *concrete* part. An important objective of our approach is to develop a DL formalization of space with two separate domains: the *abstract* and *space domain*. The concept specifications for the abstract domain are used to represent terminological knowledge about spatial objects (e.g. in geography) at an abstract logical level. The (concrete) space domain extends the abstract domain by adding structured mathematical entities for polygons and allows access to efficient reasoning algorithms for concrete spatial regions (e.g. polygons in maps).

A DL with a concrete domain extends a standard DL by adding predicates and individuals for the concrete domain. The predicates can be used to define new concepts in the abstract domain. Role and attribute fillers can be restricted by predicates of the concrete domain.

Basically, (i) the set of predicate names defined by a concrete domain has to be closed under negation and has to contain a predicate name for domain membership, and (ii) the satisfiability problem for finite conjunctions of corresponding predicates has to be decidable (see [11] for a detailed definition).

2.2.3 Semantics of DL Elements

Let $\mathcal{C}$ be the set of concepts, $\mathcal{R}$ the set of roles, and $\mathcal{P}$ the set of concrete predicates in a DL theory. The model-theoretic semantics of a DL is based on the notion of an *interpretation* which is defined as a pair $\langle \mathcal{D}, \xi \rangle$ where $\mathcal{D} = \mathcal{D}_\mathcal{C} \cup \mathcal{D}_\mathcal{P}, \mathcal{D}_\mathcal{C} \cap \mathcal{D}_\mathcal{P} = \emptyset$ and ξ is an assignment function such that $\xi : \mathcal{C} \longrightarrow 2^{(\mathcal{D}_\mathcal{C} \cup \mathcal{D}_\mathcal{P})}$, $\xi : \mathcal{R} \longrightarrow 2^{\mathcal{R}'}$ where $\mathcal{R}' = (\mathcal{D}_\mathcal{C} \times (\mathcal{D}_\mathcal{C} \cup \mathcal{D}_\mathcal{P}))$. ξ must satisfy the following conditions for mapping syntactical terms to semantical entities (concept names are denoted by c, role names by r, and concrete predicate names by p). We only list semantics for DL elements mentioned in this paper.

$$
\begin{aligned}
\xi[(\top)] &= \mathcal{D}_\mathcal{C} \\
\xi[(\bot)] &= \emptyset \\
\xi[\textsf{concept name}] &\subseteq \mathcal{D}_\mathcal{C} \text{ or} \\
\xi[\textsf{concept name}] &\subseteq \mathcal{D}_\mathcal{P} \\
\xi[\textsf{role name}] &\subseteq \mathcal{D}_\mathcal{C} \times \mathcal{D}_\mathcal{C} \text{ or} \\
\xi[\textsf{role name}] &\subseteq \mathcal{D}_\mathcal{C} \times \mathcal{D}_\mathcal{P} \\
\xi[\textsf{predicate name}] &\subseteq \mathcal{D}_\mathcal{P} \\
\xi[(c_1 \wedge \ldots \wedge c_n)] &= \cap_{i=1}^{n} \xi[c_i] \\
\xi[(\exists_{\geq n}\ \mathsf{r})] &= \{x \mid \|\{(x,y) \mid (x,y) \in \xi[\mathsf{r}]\}\| \geq n\} \\
\xi[(\exists_{\leq n}\ \mathsf{r})] &= \{x \mid \|\{(x,y) \mid (x,y) \in \xi[\mathsf{r}]\}\| \leq n\} \\
\xi[(\forall\ \mathsf{r}\ \mathsf{c})] &= \{x \mid \forall y : (x,y) \in \xi[\mathsf{r}] \Rightarrow y \in \xi[\mathsf{c}]\} \\
\xi[(\forall\ \mathsf{r}\ \mathsf{p})] &= \{x \mid \forall y : (x,y) \in \xi[\mathsf{r}] \Rightarrow y \in \xi[\mathsf{p}]\}
\end{aligned}
$$

In the TBox the two special symbols '$\doteq$' and '$\sqsubseteq$' are used for introducing defined and primitive concepts, respectively. The definitions are mapped onto set-inclusion axioms.

- $\mathsf{Cname} \doteq \mathsf{C}$ is mapped onto $\xi[\mathsf{Cname}] = \xi[\mathsf{C}]$
- $\mathsf{Cname} \sqsubseteq \mathsf{C}$ is mapped onto $\xi[\mathsf{Cname}] \subseteq \xi[\mathsf{C}]$

The semantics of ABox assertions is defined analogously:

- $\mathsf{Iname} : \mathsf{C}$ is mapped onto $\mathsf{Iname} \in \xi[\mathsf{C}]$
- $\langle \mathsf{Iname}_1, \mathsf{Iname}_2 \rangle : \mathsf{Rname}$ is mapped onto $(\mathsf{Iname}_1, \mathsf{Iname}_2) \in \xi[\mathsf{Rname}]$

An interpretation that satisfies all axioms in a terminology is called a *model*. The notion of a model is used to define the reasoning services a DL inference engine has to provide: subsumption and consistency checking which are closely related. A term A *subsumes* another term B if and only if for every model $\langle \mathcal{D}, \xi \rangle$ $\xi[\mathsf{B}] \subseteq \xi[\mathsf{A}]$ holds. A term A is *coherent* if and only if there exists a model $\langle \mathcal{D}, \xi \rangle$ such that $\xi[\mathsf{A}] \neq \emptyset$.

3 A Space Box for Polygons

This section introduces a space box reasoner that realizes inference services over 2D polygons. We demonstrate that the reasoning services provided by current description logics with a concrete domain extension are insufficient for the formalization of space and propose several extensions.

The fundamental idea of the SBox reasoner is the treatment of spatial regions as subsets of $\Re^2$ and to define subsumption between polygons with respect to the relation g_contains as defined in Section 2.1 (see also Figure 2). The relation g_contains has the properties of an order relation (reflexive, antisymmetric, transitive), i.e. it has the same properties as the subsumption relation for concepts. With this definition of spatial subsumption we can reduce the satisfiability problem to the decision whether a set of polygons is connected (i.e. there exists a non-empty intersection) or disjoint.

We restrict the concrete predicates to the description of polygons. With the polygon restriction we gain applicability of efficient algorithms (e.g. the simplex procedure) for solving the satisfiability problem. We use concrete predicates for expressing equality (equal$_{p}$) or containment (g_inside$_{p}$) of a polygon with respect to the reference polygon p which is used as the second argument of the relation. We assume an attribute has_area whose filler is from the spatial domain (i.e. a concrete predicate).

For instance, we can now define a concept northern_german_region by using the 'for-all' constructor ($\forall$ r P):

northern_german_region $\doteq$ ($\forall$ has_area g_inside$_{p_5}$)

For northern_german_region the possible filler of has_area is restricted to a polygon inside of p_5. The polygon p_5 defines the area of Northern Germany. The construct g_inside$_{p_5}$ subsumes every region of Northern Germany whose associated polygon is g_inside of p_5. Additionally, we need a concrete predicate for expressing equality of polygons since subsumption of arbitrary subregions is not always desired. For instance, the concept federal_state_hh (HH is part of the car license number for Hamburg) contains the equality condition in order to prevent subsumption with subregions of the city of Hamburg area (see also Figure 5):

federal_state_hh $\doteq \ldots \wedge$ ($\forall$ has_area equal$_{p_2}$)

The polygon p_2 defines the area of the federal state Hamburg. equal$_{p_2}$ does not subsume any subregion of p_2. Note that due to the definition of g_inside, g_inside$_{p_2}$ subsumes equal$_{p_2}$.

We also define similar concrete predicates with respect to the other spatial relations mentioned in the previous section. The idea is to convert a spatial relation into a concept (a one-place predicate) by providing the second argument of the relation as a constant. The reference polygon is required due to the syntax of the concept term constructor. The set of predicates is closed under negation and fulfills the decidability requirement of concrete domains (cf. the notion of 'admissibility' of concrete domains).

However, we need more expressivity in order to adequately characterize spatial relations between certain individuals. For instance, the concept definition for hh_border_district specifies that every subsumed individual is associated with a polygon that is t_inside of another polygon that, in turn, is referred to by an object which is subsumed by the concept federal_state_hh.

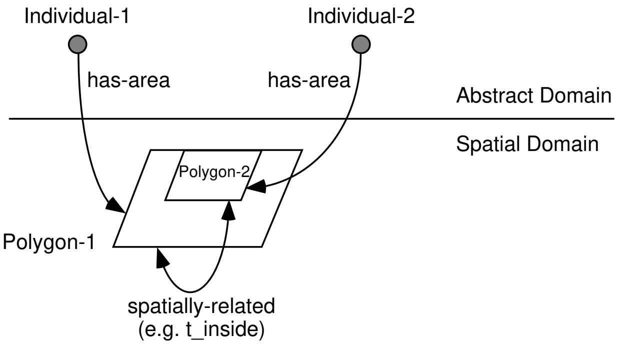

Figure 4: Relationship between abstract and spatial domain

This situation is illustrated in Figure 4.

hh_border_district $\doteq \ldots \wedge$
($\bigcirc$ t_inside federal_state_hh)

In order to support this constructor we have to extend existing description logics beyond concrete domains (i.e. we have to extend the description logics defined by Hanschke [11]). In a first approach, we propose a new concept-forming operator $\bigcirc$ that captures the intuition indicated above. The assignment function is extended for $\bigcirc$ concept terms (be sr a name for an elementary spatial relation as defined in Section 2.1 and c be a concept term):

$$\begin{aligned}\xi[(\bigcirc\ \mathsf{sr}\ \mathsf{c})] = \{x \mid\ & \exists y_1, y_2, z : (x, y_1) \in \xi[\mathsf{has\_area}],\\ & (z, y_2) \in \xi[\mathsf{has\_area}],\\ & (y_1, y_2) \in \xi[\mathsf{sr}], x \neq z,\\ & z \in \xi[\mathsf{c}]\}\end{aligned}$$

Please note that this concept-forming operator is restricted since only elementary spatial relations and not abstract roles are allowed in place of sr. We are currently investigating the consequences of this new operator with respect to the completeness of a DL inference algorithm.

Another possible solution to the problems discussed above is the proposal of a role-forming operator that combines abstract attributes and concrete predicates. For instance, by using a *defined* abstract role t_inside_role we could express the above example as follows:

t_inside_role $\doteq \exists$(has_area)(has_area).t_inside

hh_border_district $\doteq$
$\ldots \wedge$ ($\forall$ t_inside_role federal_state_hh)

The operator's formal properties and its applicability are currently under investigation (see also Section 6). The semantics of this new construct is defined as

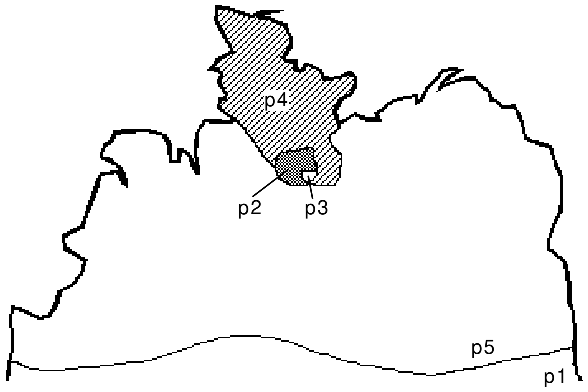

Figure 5: A sketch of the northern part of Germany with polygons for Germany (p_1), Northern Germany (p_5), the federal states Schleswig-Holstein (p_4) and Hamburg (p_2) as well as a small district of Hamburg (p_3). Polygon p_3 is assumed to be inside p_2 but p_2 is not inside p_4.

follows (be P a concrete predicate and $\mathsf{r}_1, \mathsf{r}_2$ attributes with concrete individuals as fillers):

$$\xi[\exists(\mathsf{r}_1)(\mathsf{r}_2).\mathsf{P}] = \{(x,y) \mid \exists z_1, z_2 : (x,z_1) \in \xi[\mathsf{r}_1], (y,z_2) \in \xi[\mathsf{r}_2], (z_1,z_2) \in \xi[\mathsf{P}]\}$$

4 Spatioterminological Inferences: an Extended Example

The use of the constructs presented in the previous section is demonstrated with the map example from Figure 1. The surrounding area is presented in Figure 5. Hamburg (represented by polygon p_2) is located in Germany (p_1), especially in Northern Germany (p_5) and directly near the federal state Schleswig-Holstein (p_4). The district Öjendorf (p_3) of the map in Figure 1 is inside Hamburg. Actually, it is a border district to Schleswig-Holstein.

The formal model is presented with a description logic TBox which is presented below. The areas (polygons) for Germany and Hamburg are explicitly represented with named concepts. The Tbox classifier will detect the implicit subsumption relationship between german_area and hamburg_area. The concept german_federal_state is a primitive concept, i.e. it is defined only with necessary conditions. In our example domain, the filler of has_area must be a concrete object which is restricted to german_area which, in turn, describes a polygon inside p_1.

Because the concept german_federal_state is primitive, it does not subsume northern_german_region. However, due to space subsumption, the concept northern_german_region subsumes federal_state_hh and also federal_state_sh (the concept for Schleswig-Holstein). The concrete predicates $\mathsf{min_n}$ which means $[\mathsf{n} \ldots \infty]$ and $\mathsf{max_n}$ which means $[-\infty \ldots \mathsf{n}]$ are defined over natural numbers and are also provided by most DL systems.

Another northern_german_region is district_of_hh. Note again that the area of district_of_hh is not a german_federal_state because this concept is primitive (see above). The same holds for district_of_sh.

german_area $\doteq$ g_inside$_{\mathsf{p_1}}$

hamburg_area $\doteq$ g_inside$_{\mathsf{p_2}}$

german_federal_state $\sqsubseteq$
($\forall$ has_area german_area)

northern_german_region $\doteq$
($\forall$ has_area g_inside$_{\mathsf{p_5}}$)

federal_state_hh $\doteq$
german_federal_state $\wedge$ ($\forall$ has_area equal$_{\mathsf{p_2}}$)

federal_state_sh $\doteq$
german_federal_state $\wedge$ ($\forall$ has_area equal$_{\mathsf{p_4}}$)

district_of_hh $\doteq$
($\forall$ has_area hamburg_area)

district_of_sh $\doteq$
($\forall$ has_area g_inside$_{\mathsf{p_4}}$)

sh_border_district $\doteq$
($\forall$ has_area g_inside$_{\mathsf{p_4}}$) $\wedge$
($\bigcirc$ t_inside federal_state_sh)

hh_border_district $\doteq$
district_of_hh $\wedge$ ($\bigcirc$ t_inside federal_state_hh)

hh_border_district_to_sh $\doteq$
district_of_hh $\wedge$
($\bigcirc$ spatially_related federal_state_hh) $\wedge$
($\bigcirc$ touching federal_state_sh)

While the implicit subsumption relationships discussed above are quite obvious, the last two concept definitions provide more difficult examples. Based on the definitions given above, it can be proven that hh_border_district_to_sh is subsumed by hh_border_district. A hh_border_district is a district_of_hh which touches the area of federal_state_hh from the inside (relation t_inside). The polygon of federal_state_hh is explicitly given by the predicate equal$_{\mathsf{p_2}}$ (see the concept definition of federal_state_hh). If a district_of_hh touches the polygon of federal_state_sh, its corresponding area must be tangentially inside the polygon of federal_state_hh.

A TBox classifier that deals with the semantics of spatial relations must find these implicit subsumption relationships in order to correctly and completely classify the terminological knowledge base.

In most DL systems, a set of rules can be defined to assert additional constraints for ABox instances when certain conditions (represented by a concept term) are met. For instance, in Hamburg and Schleswig-Holstein, the mountain height is less than 1000 (meters). This relationship is asserted by a rule (operator $\rightarrow$) that fires for every individual that is classified as a member of the concept northern_german_region. The second rule adds additional constraints to instances of district_of_hh.

$$\begin{aligned}&\textbf{northern\_german\_region} \rightarrow\\&\quad(\forall\ \mathsf{mountain\_height}\ \mathsf{max}_{1000})\end{aligned}$$

$$\begin{aligned}&\textbf{district\_of\_hh} \rightarrow\\&\quad(\forall\ \mathsf{zip\_code}\ \mathsf{min}_{20000}) \wedge\\&\quad(\forall\ \mathsf{area\_descriptor}\ \mathsf{min}_{1000})\end{aligned}$$

Automatic classification is also important for assertional knowledge defined in the ABox. The following statements define partial information about individuals in our example domain.

$$\begin{aligned}\mathsf{hamburg} &: \mathsf{federal\_state\_hh}\\ \mathsf{öjendorf} &: \mathsf{district\_of\_hh}\\ \langle \mathsf{öjendorf}, \mathsf{p_3}\rangle &: \mathsf{has\_area}\\ \mathsf{vierlande} &: (\forall\ \mathsf{has\_area}\ \mathsf{g\_inside}_{\mathsf{p_4}}) \wedge\\ &\quad(\bigcirc\ \mathsf{touching}\ (\forall\ \mathsf{has\_area}\ \mathsf{equal}_{\mathsf{p_3}})) \wedge\\ &\quad(\bigcirc\ \mathsf{spatially\_related}\ \mathsf{federal\_state\_sh}))\end{aligned}$$

The individual hamburg is declared to be an instance of federal_state_hh. The individual öjendorf is a district_of_hh. The filler of the has_area role for öjendorf is $\mathsf{p_3}$. The ABox reasoner computes that the federal state hamburg and the district öjendorf are each subsumed by northern_german_region, i.e. the mountain heights in the associated areas are less than 1000 meters (see the rules defined above). This kind of derived information can be used to guide the map interpretation process by applying conceptual background knowledge. If the number 7434 in Figure 1 were asserted as a filler for the mountain height of öjendorf, the ABox would derive an inconsistency which indicates that another hypothesis has to be tried.

In the last assertion, another individual (named vierlande) which touches the polygon of öjendorf is defined. Since vierlande is by definition subsumed by ($\forall$ has_area $\mathsf{g\_inside}_{\mathsf{p_4}}$), it cannot be a district_of_hh but must be inside of district_of_sh. However, it touches the Öjendorf polygon ($\mathsf{p_3}$) and therefore, it must be automatically classified as a sh_border_district.

The examples illustrate the importance of complete inference algorithms for TBox, ABox and SBox classification. For instance, if the implicit subsumption relationship between hh_border_district and hh_border_district_to_sh were not detected, we could declare an instance of hh_border_district_to_sh and claim that a valid zip code in this area might be 7434 which is certainly inconsistent (cf. the rule definition for district_of_hh). Another hypothesis is that 7434 might be an area descriptor. This hypothesis is consistent with the terminological background knowledge defined in our TBox example and might be used as an intermediate result for further interpretation steps.

5 Related Work

The idea of incorporating conceptual knowledge (especially knowledge that can be modeled with a decidable description logic) into spatial reasoning and image interpretation problems has been proposed in [10]. Rather than Reiter and Mackworth (see the description of MAPSEE in [17]), who use first order predicate logic, we use a description logic as a basis for our image interpretation problems. In order to be able to validate the image interpretation knowledge itself (i.e. the TBox), we cannot include a domain closure axiom, i.e. we cannot enumerate all objects in every image to be interpreted. In other words: Neither can the problem be reduced to model checking nor to satisfiability checking in propositional logic. Lange and Schröder [12] also discuss the problem of image interpretation in a formal, logical framework. The incorporation of concrete domain predicates for image interpretation problems is presented by Schröder and Neumann [19].

Many other approaches for modeling spatial objects and their relations have been published. The ontological assumptions for the approach presented in this paper are based on a Newtonian conception of space (see [4]). In contrast to the Leibnizian conception (assuming space to be strictly dependent on the relations holding between physical objects), the cartesian structure of our concrete domain approach allows spatial relations to be defined by topological relations between areas defined by polygons (with an external or absolute reference system). The Leibnizian conception has been adopted in many approaches inspired by natural language interpretation problems. Due to space limitations, we cannot discuss the large amount of work on logical models of space in this area.

Grigni et al. [8] study the computational problems in developing an inference system for checking the satisfiability of (conjunctive) combinations of spatial relations. This work is important for us for checking the consistency of combinations of concept terms containing predicates based on spatial relations. Grigni et al. point out that in topological inferences the aspects of relational consistency and planarity interact in rather complex ways. They showed that besides the relational consistency problem a planarity problem has to be solved when areas are assumed to be disjoint. With this additional restriction, in many cases the complexity of the satisfiability problem becomes NP-hard. Lemon [13] showed that in some "spatial" logics based on convex regions one can construct consistent sentences that have no models in the intended geometrical interpretation, i.e. the logics are incomplete with respect to the intended geometrical interpretation (e.g. this has been proven for RCC introduced in [16]).

6 Conclusion and Ongoing Work

In this paper, we have demonstrated that topological relations directly influence the kind of conceptual or terminological knowledge that can (and must) be derived by a formal inference engine. On the other hand, assertions about concepts restrict the set of possible spatial relations between different individuals.

We have seen that the use of incomplete reasoning services in practical applications is problematic. For instance, in our application domain the reasoning service might be used to test whether the hypothesis "7434 is the zip code of Öjendorf" is consistent. In the example above we have seen that the correct answer depends on complete TBox classification algorithms. In this case, an incomplete reasoner that does not detect the implicit subsumption relationships in the TBox (see the discussion above) must answer "may be". However, if we pose the negated query "Is 7434 definitely not the zip code of Öjendorf" the answer must also be "may be" because an inconsistency cannot be derived due to incompleteness. The question is whether "may be" answers can be used for solving problems in a geographical information system, especially when "may be" happens to be interpreted as "no." Similar problems are likely to occur in incomplete approaches (see e.g. [18] for an image interpretation approach that uses an undecidable description logic).

One idea of the approach presented in this paper is to reduce the complexity of the reasoning algorithms by also considering quantitative spatial data which are available in many practical applications. If concrete polygons are given, no relational consistency checking (see above) is required but standard algorithms from computational geometry can be used. In our map interpretation scenario, the incorporation of a space box with a Newtonian view (i.e. with quantitative data) helps to avoid problems of so-called "spatial" logics. We have discussed some arguments that dealing only with qualitative relations like the ones used by Egenhofer neglects some aspect of space ([8], [13]) when, for instance, the qualitative calculus implies additional properties of geometric objects such as convexity or disjointness of regions.

The SBox extension proposed in this paper is no general geometrical theorem prover. The advantage of our approach which is based also on quantitative information about spatial regions is that the satisfiability test for finite conjunctions of predicates can be reduced to well-known algorithms in computational geometry (basically polygon intersection). Qualitative relations that are grounded in quantitative data provide a bridge to conceptual knowledge and allow more extensive reasoning services to be exploited for solving practical problems.

The treatment of predicate concept terms such as $(\forall\ \mathsf{has\_area}\ \mathsf{g\_inside}_{\mathsf{p_i}})$ and $(\forall\ \mathsf{has\_area}\ \mathsf{equal}_{\mathsf{p_i}})$ is supported by the work of Hanschke on concrete domain extensions. A prototype implementation using the CLASSIC [5] description logic (and its extension interface) demonstrates that the concept constructor $\bigcirc$ can be integrated into CLASSIC. However, it demonstrates also the disadvantages of this concept term because it cannot be freely combined with other language constructs. Therefore, we are currently investigating the formal properties of the role operator $\exists(\mathsf{r_1})(\mathsf{r_2}).\mathsf{P}$ which can be more flexibly used.

As Borgo et al. emphasize [4], for spatial reasoning we have to consider mereological aspects as well (e.g. part-whole relations). There are many proposals for integrating part-whole relations into description logics and we are investigating ways to combine these approaches with the spatial domain. The examples presented in this paper show that interesting application problems can be solved with an extended description logic supporting reasoning services about qualitative relations.

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