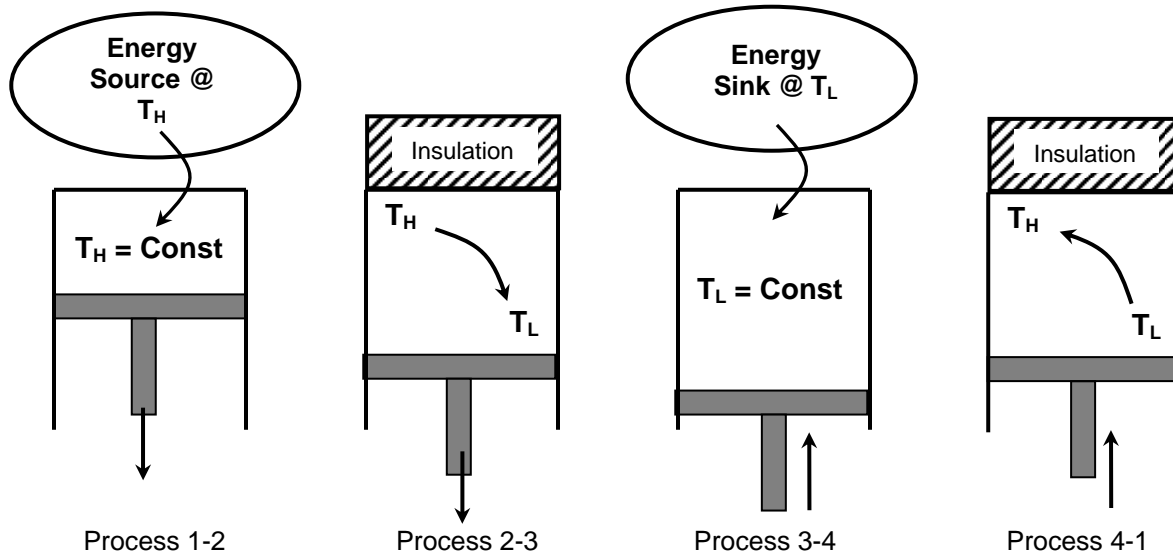


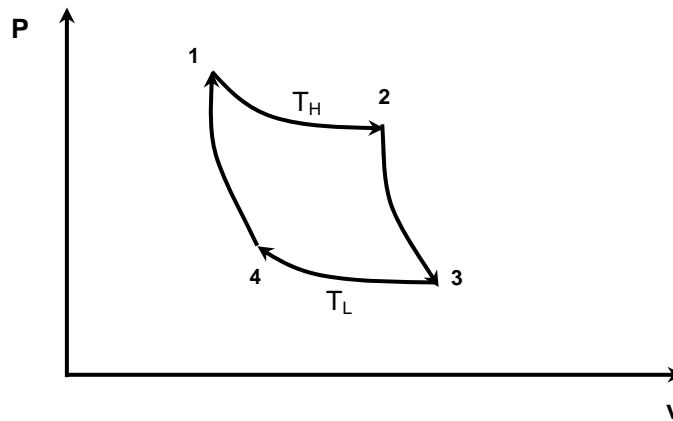
II. Carnot Cycle

The Carnot cycle was first proposed in 1824, by Sadi Carnot. The interest in the cycle is largely theoretical, as no practical Carnot cycle engine has yet been built.

Nevertheless, **it can be shown to be the most efficient cycle possible**, so that considerable attention has been given at discovering ways of making the more practical cycles look, as much as possible, like the Carnot.



- (Process 1-2) A constant temperature heat addition.
- (Process 2-3) An adiabatic expansion
- (Process 3-4) A constant temperature heat rejection.
- (Process 4-1) An adiabatic compression



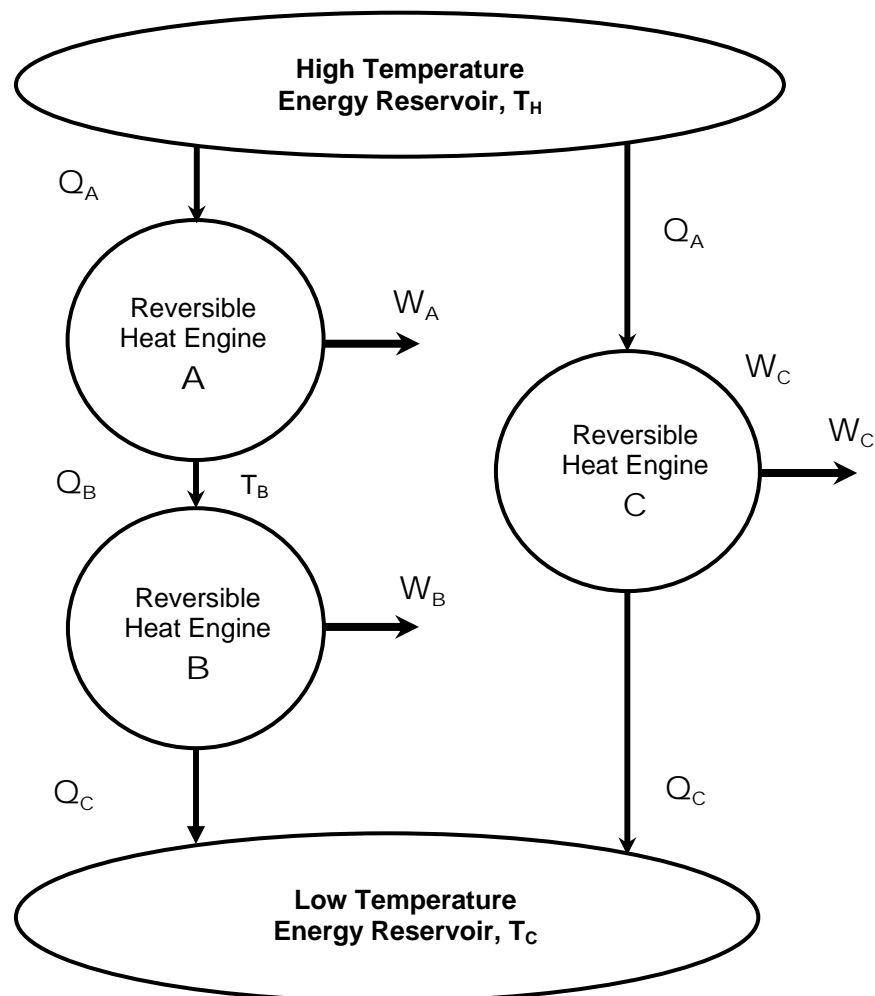
II.1. Carnot Cycle Efficiency

Like other heat engines, the Carnot cycle efficiency can be attained from the relationship:

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{W_{out} - W_{in}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

II.1.1. Carnot cycle principles

1. The efficiency of an irreversible heat engine is always less than the efficiency of a reversible one operating between the same two reservoirs.
2. The efficiencies of all reversible heat engines operating between the same two reservoirs are the same.



If we consider the Carnot cycle above, since a control volume could be drawn about engines A & B together, they may be considered as a single reversible engine.

The efficiency of Engine A&B must be the same as that of Engine C since both are reversible.

Also

$$W_A + W_B = W_C$$

Since energy reservoirs are characterized by their temperatures, the thermal efficiency of reversible heat engines is a function of the reservoir temperatures only. That is,

$$\frac{Q_L}{Q_H} = f(T_H, T_L)$$

Applying this idea to the three engines separately,

$$Q_A/Q_C = f(T_A, T_C), \quad Q_A/Q_B = f(T_A, T_B), \quad Q_B/Q_C = f(T_B, T_C)$$

For the efficiency of A&B to be equal to that of C

$$f(T_A, T_C) = f(T_A, T_B) \cdot f(T_B, T_C)$$

A careful examination of this equation reveals that the left hand side is a function of T_A and T_C , and therefore the right hand side must also be a function of T_A and T_C only and not, T_B . This condition will be satisfied only if the function has the following form:

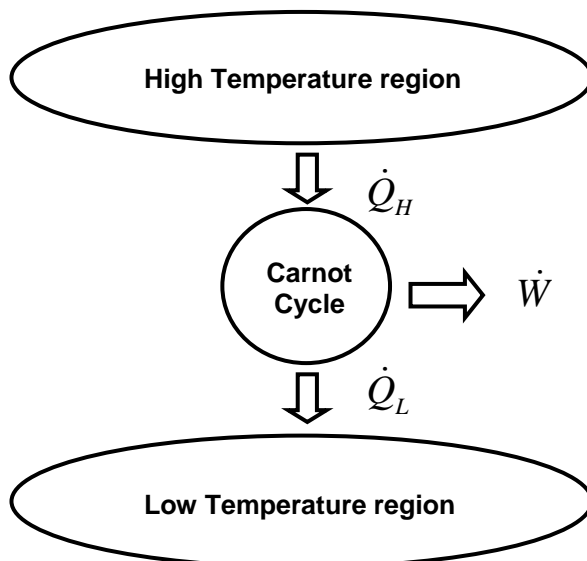
$$\Phi(T_A)/\Phi(T_C) = [\Phi(T_A)/\Phi(T_B)] \cdot [\Phi(T_B)/\Phi(T_C)]$$

From this relationship Kelvin proposed a **temperature scale** in which $\Phi(T) = T$, such that:

$$\eta = 1 - \frac{T_L}{T_H}$$

II.1.2. Applications of the Carnot Cycle

a. Carnot Cycle Engine



From the first law of thermodynamics:

$$\dot{Q}_H = \dot{Q}_L + \dot{W}$$

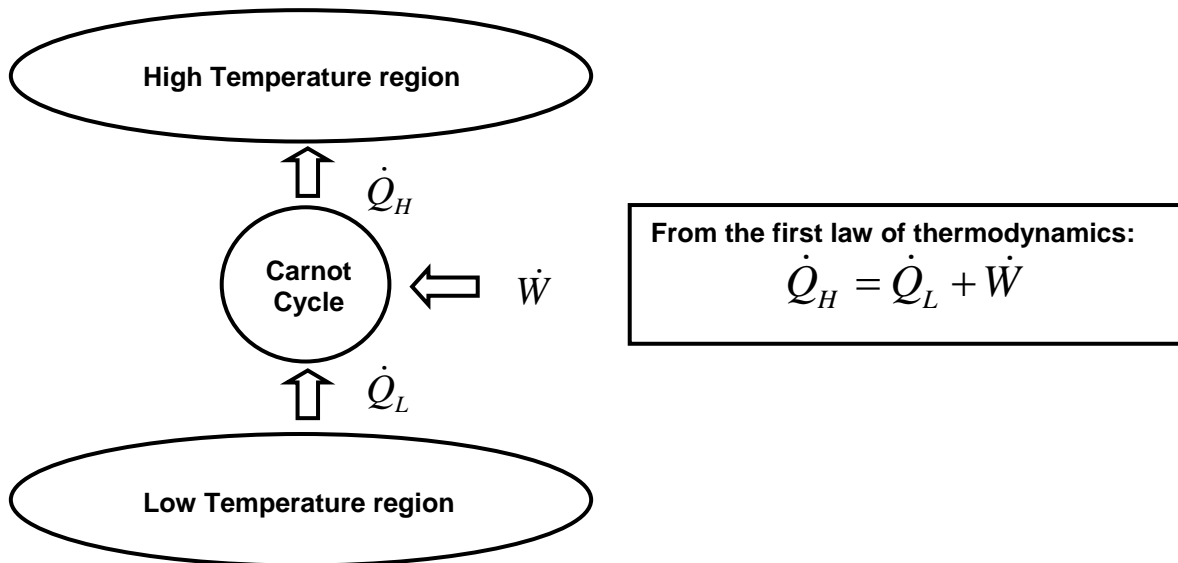
From the definition of efficiency, we find for the Carnot engine:

$$\eta = 1 - \frac{\dot{Q}_L}{\dot{Q}_H}$$

Using the thermodynamic temperature scale:

$$\eta = 1 - \frac{T_L}{T_H}$$

b. Carnot Cycle Refrigerator



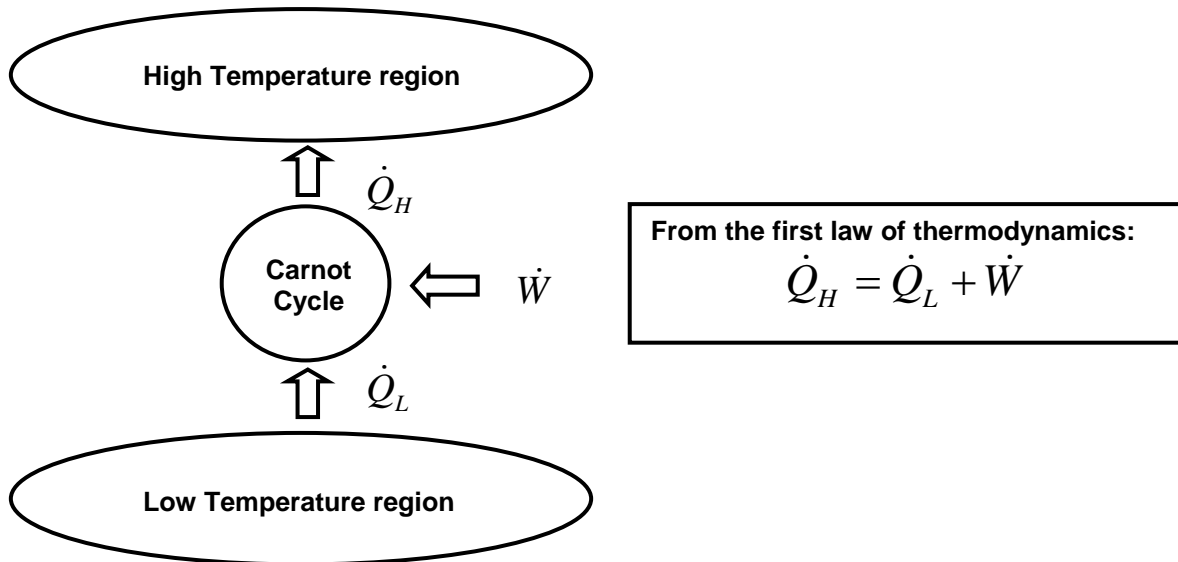
From the definition of Coefficient of Performance (COP), we find for the Carnot refrigeration cycle:

$$COP = \frac{\dot{Q}_L}{\dot{W}} = \frac{\dot{Q}_L}{\dot{Q}_H - \dot{Q}_L} = \frac{1}{\frac{\dot{Q}_H}{\dot{Q}_L} - 1}$$

Using the thermodynamic temperature scale:

$$COP = \frac{1}{\frac{T_H}{T_L} - 1}$$

C. Carnot Cycle Heat Pump



From the definition of Coefficient of Performance (COP), we find for the Carnot refrigeration cycle:

$$COP = \frac{\dot{Q}_H}{\dot{W}} = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L} = \frac{1}{1 - \frac{\dot{Q}_L}{\dot{Q}_H}}$$

Using the thermodynamic temperature scale:

$$COP = \frac{1}{1 - \frac{T_L}{T_H}}$$

II.2. Applications

Example 1

An inventor claims to have developed a power cycle capable of delivering a net work output of 410 kJ for an energy input by heat transfer of 1000 kJ. The system undergoing the cycle receives the heat transfer from hot gases at a temperature of 500 K and discharges energy by heat transfer to the atmosphere at 300 K. Evaluate this claim.

Example 2

When a fridge stands, in a room at 20°C, the motor has to extract 500 W of heat from the cabinet at 4°C to compensate for less than perfect insulation. How much power must be supplied to the motor if its efficiency is 80% of the maximum efficiency?

Example 3

An ideal or Carnot heat pump is used to heat a house to $T_H=294$ K, how much work must be done by the pump to deliver $Q_H=3350$ J of heat into the house when the outdoor temperature is 273 K and 252 K.

Example 4

A Carnot power cycle using air as a working fluid has a thermal efficiency of 40%. At the beginning of isothermal expansion, the pressure is 620 kPa, and the specific volume is $0.1 \text{ m}^3/\text{kg}$. If the heat input for the cycle is 50 kJ/kg, determine:

- The highest and the lowest temperature for the cycle.
- The work and heat for each process of the cycle.

Assume air to be an ideal gas with constant specific heats.