ELEC 261 – COMPLEX VARIABLES
(MID TERM TEST)

DATE: Feb 17, 2006 **MARKS: 15**
TIME: 60 mins

NAME: **ID No.:**

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1. Solve the following equation for z:

$$(1+i)z^3 + i - 1 = 0$$ (3 marks)

Express the result in the polar form and sketch it in the complex z-plane.

2. Solve $\sinh(z + \mathbf{Log}\, i) = 3\cosh(z + \mathbf{Log}\, i)$ (3 marks)

3. Determine all possible points at which the following function is analytic:

$$f(z) = \left(x + \frac{i}{x}\right)e^{xy}, \quad x \neq 0$$

Give reasons for your answers. (3 marks)

4. Evaluate $\oint_C \frac{2z}{z^2 + \pi^2} dz$ along the following paths:

   a. $C: |z - i| = 3$ in the counterclockwise direction
   b. $C: |z - 4| = 3$ in the counterclockwise direction. (3 marks)

5. Evaluate $\int_C |z|^2 \, dz$ along the upper half of the path $C: |z - 1| = 1$. (3 marks)

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DATE: March 17, 2006 **MARKS: 15**
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1. Given

$$f(z)=\frac{z^2+3z+8}{(z-3i)(z-1)^2(z+1+i)^3},$$

(a) Determine the different regions for Taylor and Laurent expansions around the point $z = 1$ and sketch the various regions in the z-plane. Also, state the type of series you get in each of these regions. **DO NOT OBTAIN THE SERIES.**
(b) Are all the singularities for $f(z)$ poles? If so, determine the order of each pole.

(5 marks)

2. (a) Find the center and radius of convergence of the power series:

$$\sum_{n=0}^{\infty}\frac{n^{\alpha}(z+i)^{3n}}{(2n+3)8^n}$$

Sketch the region of convergence in the z-plane.

(b) Determine if the following series is convergent or divergent:

$$\sum_{n=0}^{\infty}\frac{(n+2)(n+3)}{(n+1)(2n-1)}$$

(5 marks)

3. Find the Taylor's series for the function $f(z)=e^{z-1}+\dfrac{1}{2z+1}$ around the point $z=-1$ and find the region of convergence.

(5 marks)

Course		Number	Section
Complex Variables for Electrical and Computer Engineers		**ELEC 261**	U, W
Examination	Date	Time	# of pages
Final Examination	Winter 2006	3 Hours	3
Instructor(s)			
Dr. M.N.S. Swamy			

Materials allowed: [X] No [ ] Yes (Please specify)

Calculators allowed: [ ] No [X] Yes

Students are allowed to use silent, non-programmable calculators without text display.

Special Instructions:

CELL PHONES ARE NOT PERMITTED.

Attempt all questions. Please number and begin each question on a new page.
Show all steps clearly in neat and legible handwriting.
Students are required to return question paper with exam booklet(s).

1. (a) Find all the roots of the equation $\sin z = 2$

(3 marks)

(b) Given $f(z) = z^{1/4}$, find all the values of $f(-2+i2\sqrt{3})$ in polar form and sketch them in the complex plane.

(3 marks)

2. (a) Show that $u(x, y) = x^2 - y^2 - x$ is a harmonic function. Also determine its conjugate harmonic function, $v(x, y)$.

(4 marks)

(b) Find the centre and radius of convergence for the power series:

$$\sum_{0}^{\infty} \frac{(2-2i)^{n+3}}{(n+3)^3 8^n}(z+2i)^{3n}$$

(3 marks)

3. Integrate $\int_C \frac{dz}{\sqrt{z}}$ along the following contours:

(a) C: the upper half of the circle $|z| = 1$ in the counter-clockwise direction.

(b) C: the lower half of the circle $|z| = 1$ in the clockwise direction.

(c) Are the resultant solutions in parts (a) and (b) the same? Give reasons to support your answer.

(6 marks)

4. Given the region bounded by $|z| = 1$, $|z| = 2$, the line $\theta = \pi/4$ and the line $\theta = \pi/8$ in the z – plane, find the image of this region in the w – plane for the transformation $w = z^2 e^{i(\pi/4)}$. Show clearly the given region in the z-plane and its corresponding image in the w-plane.

(6 marks)

5. Evaluate $\int_C \frac{e^z dz}{z(1+z)^3}$, where C is a contour in the counter-clockwise direction and is given by

(i) $|z| = 2$.

(ii) a triangle with vertices at $i, -1-i, 1-i$.

(6 marks)

6. Given $f(z) = \dfrac{1}{z^2 - 1}$

(a) Find the Taylor series around the point $z = i$. Determine its radius of convergence.

(5 marks)

(b) Expand $f(z)$ in Laurent series valid in the region $0 < |z - 1| < 1$. Using this series determine the residue of $f(z)$ at $z = 1$.

(6 marks)

7. (a) Given $L\{f(t)\} = F(s)$, show that $L\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$

(4 marks)

(b) Solve the following differential equation using Laplace transforms:

$$y'' + 4y = 0,\ y(0) = 1,\ y'(0) = 6.$$

(4 marks)

8. (a) If $L\{f(t)\} = F(s)$, show that $L\{e^{at} f(t)\} = F(s - a)$

(2 marks)

(b) Using the above result, find the inverse Laplace transform of

$$\frac{3s - 1}{(s - 2)^2}$$

(4 marks)

(c) Using the convolution theorem, find the inverse Laplace transform of

$$\frac{1}{(s - 2)(s + 3)}$$

(4 marks)