X Lecture 5, Oct 6, 2010.

- Detection of Deterministic Signals in Coloured Gaussian Noise

Assume that $\underline{s}_j = (s_{j1}, \dots s_{jn})^T$ where $j = 0,1$ and $\underline{s}_j \in \mathbb{R}^n$ and $\underline{N}$ is an n-dimensional noise vector whose covariance matrix is Σ_N and has $\underline{0}$ mean.

In this case:

$$H_0 : \underline{y} = \underline{s}_0 + \underline{N} \Rightarrow \underline{y} \sim N(\Sigma_N, \underline{s}_0)$$

versus

$$H_1 : \underline{y} = \underline{s}_1 + \underline{N} \Rightarrow \underline{y} \sim N(\Sigma_N, \underline{s}_1)$$

for a Gaussian $\underline{X} \sim N(\Sigma, \underline{\mu})$ we have

$$p_{\underline{X}}(\underline{x}) = \frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(\underline{x} - \underline{\mu})^T \Sigma^{-1} (\underline{x} - \underline{\mu})\right]$$

where $|\Sigma|$ is the determinant of Σ.

So:

$$p(\underline{y} | H_j) = \frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} \exp\left[-\frac{1}{2}(\underline{x} - \underline{s}_j)^T \Sigma_N^{-1} (\underline{x} - \underline{s}_j)\right]$$

$$j = 0,1$$

The likelihood ratio is:

$$L(\underline{y}) = \frac{p(\underline{y}|H_1)}{p(\underline{y}|H_0)} = \frac{\frac{1}{(2\pi)^{n/2}|\Sigma_N|^{1/2}} \exp[-(\underline{y}-\underline{s}_1)^T \Sigma_N^{-1}(\underline{y}-\underline{s}_1)]}{\frac{1}{(2\pi)^{n/2}|\Sigma_N|^{1/2}} \exp[-(\underline{y}-\underline{s}_0)^T \Sigma_N^{-1}(\underline{y}-\underline{s}_0)]}$$

$$= \exp\left[(\underline{s}_1 - \underline{s}_0)^T \Sigma_N^{-1} \left(\underline{y} - \frac{\underline{s}_0 + \underline{s}_1}{2}\right)\right\}$$

$$\underline{y} \in \mathbb{R}^n.$$

to get to the last line, you need to use the fact that Σ_N and therefore, Σ_N^{-1} is symmetric and as a result

$$\underline{s}_j^T \Sigma_N^{-1} \underline{y} = \underline{y}^T \Sigma_N^{-1} \underline{s}_j .$$

Note the similarity between $L(\underline{y})$ above and the scalar form we saw before.

The likelihood ratio test will be:

$$\tilde{\delta}_0(\underline{y}) = \begin{cases} 1 & \\ \gamma & \text{if } \exp\left[(\underline{s}_1 - \underline{s}_0)^T \Sigma_N^{-1} \left(\underline{y} - \frac{\underline{s}_0 + \underline{s}_1}{2}\right)\right] \begin{matrix} > \\ = \\ < \end{matrix} \tau \\ 0 & \end{cases}$$

or taking the logarithm, we can get the following decision rule, called <u>Log Likelihood Ratio</u> (LLR) Test.

$$\tilde{\delta}_0(\underline{y}) = \begin{cases} 1 \\ \gamma \\ 0 \end{cases} \quad \text{if } (\underline{s}_1 - \underline{s}_0)^T \Sigma_N^{-1} \underline{y} \quad \begin{matrix} > \\ = \\ < \end{matrix} \ \tau'$$

where
$$\tau' = \log \tau + \frac{1}{2}(\underline{s}_1 - \underline{s}_0)\Sigma_N^{-1}(\underline{s}_0 + \underline{s}_1)$$

Let
$$(\underline{s}_1 - \underline{s}_0)\Sigma_N^{-1} = \underline{\tilde{s}}$$

Then
$$(\underline{s}_1 - \underline{s}_0)\Sigma_N^{-1} \underline{y} = \underline{\tilde{s}}^T \underline{y} = \sum_{k=1}^{n} \tilde{s}_k y_k$$

So, the dector has the structure of a correlation detector.

<u>Performance evaluation</u>

Let
$$T(\underline{Y}) = (\underline{s}_1 - \underline{s}_0)^T \Sigma_N^{-1} \underline{Y}$$

and note that $T(\underline{Y})$ is a linear combination of Gaussian samples (the components of $\underline{Y}$). So, it is a Gaussian random variable.

$$E[T(\underline{Y}) \mid H_j] = E[\underline{\tilde{s}}^T \underline{Y} \mid H_j] = \underline{\tilde{s}}^T \cdot E[\underline{Y} \mid H_j]$$

$$= \tilde{s}^T E[(\underline{N} + \underline{s}_j) \mid H_j] = \tilde{s}\, \cancel{E[\underline{N} \mid H_j]}^{\,0} + \tilde{s}\, E[\underline{s}_j \mid H_j]$$

$$= \underline{\tilde{s}}^T \underline{s}_j \triangleq \tilde{\mu}_j$$

Similarly,

$$Var(T(\underline{y})|H_j) = E[(\underline{\tilde{s}}^T\underline{y} - \underline{\tilde{s}}^T\underline{s}_j)^2 | H_j]$$
$$= E[(\underline{\tilde{s}}^T\underline{N})^2]$$
$$= E[\underline{\tilde{s}}^T\underline{N}\,\underline{N}^T\underline{\tilde{s}}] = \underline{\tilde{s}}^T E[\underline{N}\,\underline{N}^T]\,\underline{\tilde{s}}$$
$$= (\underline{s}_1 - \underline{s}_0)^T \Sigma_N^{-1} (\underline{s}_1 - \underline{s}_0) \triangleq d^2$$

So $T(y)$ is Gaussian with mean $\tilde{\mu}_j$, $j=0,1$ and variance d^2 (under both Hypothesis the variance is the same). This shows that randomization is irrelevant.

The probability of choosing H_1 under H_j is

$$P_j(\Gamma_1) = \frac{1}{\sqrt{2\pi}\,d} \int_{\tau'}^{\infty} e^{-(x-\tilde{\mu}_j)^2/2d^2}\,dx$$

$$= 1 - \Phi\left(\frac{\tau' - \tilde{\mu}_j}{d}\right) = Q\left(\frac{\tau' - \tilde{\mu}_j}{d}\right)$$

That is:

$$P_j(\Gamma_1) = \begin{cases} 1 - \Phi\left(\frac{\log\tau}{d} + \frac{d}{2}\right) & \text{for } j=0 \\ 1 - \Phi\left(\frac{\log\tau}{d} - \frac{d}{2}\right) & \text{for } j=1 \end{cases}$$

We see that the Bayesian performance for this Gaussian case is the similar to the Scalar case. The d used in the scalar case is the one-dimensional version of $\underline{d}$ used here.

Consider the α-level Neyman-Pearson test.

$$P_F(\tilde{\delta}_0) = P_0(\Gamma_1) = \alpha$$

$$P_0(\Gamma_1) = 1 - \Phi\left(\frac{\log \tau}{d} + \frac{d}{2}\right) = \alpha$$

So,

$$\frac{\log \tau}{d} + \frac{d}{2} = \Phi^{-1}(1-\alpha)$$

$$\tau' = \log \tau + \frac{1}{2}(\underline{s}_1 - \underline{s}_0)^T \Sigma_N^{-1}(\underline{s}_0 + \underline{s}_1)$$

$$= d\,\Phi^{-1}(1-\alpha) - \frac{d^2}{2} + \frac{1}{2}(\underline{s}_1 - \underline{s}_0)\Sigma_N^{-1}(\underline{s}_0 + \underline{s}_1)$$

$$\boxed{= d\,\Phi^{-1}(1-\alpha) + \tilde{\mu}_0}$$

The corresponding detection probability is:

$$P_D(\tilde{\delta}_{NP}) = P_1(\Gamma_1) = 1 - \Phi\left(\frac{\tau' - \tilde{\mu}_1}{d}\right)$$

$$= 1 - \Phi\left(\Phi^{-1}(1-\alpha) - d\right)$$

- Interpretation of d^2:

The formula for $P_D(\tilde{\delta}_{NP})$ shows that the performance improves as d increases.

To see the physical significance of d, let's consider the i.i.d. case with noise being i.i.d. $\sim N(0, \sigma^2)$ and $\underline{s}_0 = \underline{0}$ and $\underline{s}_1 = \underline{s}$.

$$\Sigma_N = \begin{bmatrix} \sigma^2 & 0 & 0 & \cdots & 0 \\ 0 & \sigma^2 & \cdots & \cdots & 0 \\ 0 & 0 & & & 0 \\ \vdots & \vdots & & & \vdots \\ & & & & \sigma^2 \end{bmatrix} = \sigma^2 I_N$$

$$d^2 = (\underline{s}_1 - \underline{s}_0)^T \Sigma_N^{-1} (\underline{s}_1 - \underline{s}_0) = \frac{\underline{s}^T I_N \underline{s}}{\sigma^2} = \frac{\underline{s}^T \cdot \underline{s}}{\sigma^2}$$

$$= \frac{1}{\sigma^2} \sum_{k=1}^{n} s_k^2 = n \frac{\overline{s^2}}{\sigma^2}$$

where $\overline{s^2}$ is the average signal power given by

$$\overline{s^2} = \frac{1}{n} \sum_{k=1}^{n} s_k^2$$

So, it can be seen that d^2 is signal-to-noi ratio (SNR) multiplied by the number of samples. So, the performance can be improved by either increasing n (increasing the observation time, SNR, i.e., $\frac{\bar{s}^2}{\sigma^2}$. The latter can be done by either increasing the signal power (which is what we usually can do to some extent) or reducing the noise.

Reduction to i.i.d. noise

Σ_N is an $n \times n$ matrix which is symmetric and positive definite, i.e.

$$\underline{x}^T \Sigma_N \underline{x} > 0 \quad \text{for} \quad \forall\, \underline{x} \neq \underline{0} \in \mathbb{R}^n.$$

So, it has positive eigen values $\{\lambda_k\}$.

Denoting the eigenvector corresponding to $\{\lambda_k\}$ as $\{\underline{v}_k\}$, we can choose $\{\underline{v}_k\}$ such that

$$\underline{v}_k^T \cdot \underline{v}_\ell = \delta_{k\ell} = \begin{cases} 0 & k \neq \ell \\ 1 & k = \ell. \end{cases}$$

Σ_N can be written as

$$\Sigma_N = \sum_{k=1}^{n} \lambda_k \underline{v}_k \underline{v}_k^T$$

This represents the spectral decomposition of Σ_N.

The inverse of Σ_N is

$$\Sigma_N^{-1} = \sum_{k=1}^{n} \lambda_k^{-1} \underline{v}_k \underline{v}_k^T$$

$$T(\underline{y}) = (\underline{s}_1 - \underline{s}_0)^T \Sigma_N^{-1} \underline{y} .$$

Let's apply the following transformation:

$$\hat{s}_{jk} = \underline{\nu}_k^T \underline{s}_j / \sqrt{\lambda_k} \qquad k = 1, \cdots, n \qquad j = 0, 1$$

and

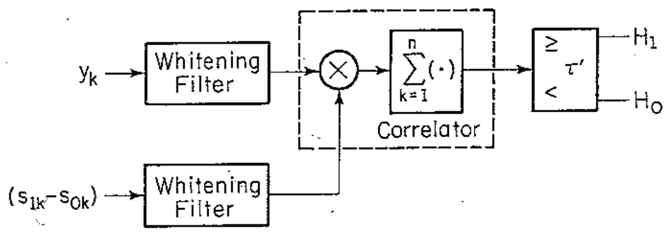
$$\hat{\underline{y}}_k = \underline{\nu}_k^T \underline{y} / \sqrt{\lambda_k} \qquad k = 1, \cdots, n$$

Then,

$$T(y) = (\underline{s}_1 - \underline{s}_0)^T \Sigma_N^{-1} \underline{y} = \sum_{k=1}^{n} (\hat{s}_{1k} - \hat{s}_{0k}) \hat{y}_k$$

Now, the Hypothesis test becomes:

$$H_0 : \hat{\underline{Y}} = \hat{\underline{N}} + \hat{\underline{s}}_0$$



versus

$$H_1 : \hat{\underline{Y}} = \hat{\underline{N}} + \hat{\underline{s}}_1$$

where $\hat{N}_k = \underline{\nu}_k^T \underline{N} / \sqrt{\lambda_k}$

$$\begin{aligned} E[\hat{\underline{N}}_k \hat{\underline{N}}_\ell] &= E\{\underline{\nu}_k^T \underline{N}\, \underline{\nu}_\ell^T \underline{N}\} / \sqrt{\lambda_k \lambda_\ell} \\ &= E\{\underline{\nu}_k^T \underline{N}\, \underline{N}^T \underline{\nu}_\ell\} / \sqrt{\lambda_k \lambda_\ell} \\ &= \underline{\nu}_k^T E\{\underline{N}\, \underline{N}^T\} \underline{\nu}_\ell / \sqrt{\lambda_k \lambda_\ell} \\ &= \underline{\nu}_k^T \Sigma_N \underline{\nu}_\ell / \sqrt{\lambda_k \lambda_\ell} = \underline{\nu}_k^T \underline{\nu}_\ell \sqrt{\lambda_\ell / \lambda_k} \end{aligned}$$

where we have used the definition of eigenvector

$$\Sigma_N \underline{\nu}_\ell = \lambda_\ell \underline{\nu}_\ell .$$

So, we have:

$$E\{\hat{N}_k \hat{N}_\ell\} = \begin{cases} 1 & \text{if } k=\ell \\ 0 & \text{if } k\neq \ell \end{cases}$$

i.e., $\{\hat{N}_k\}$ are i.i.d. $N(0,1)$.

Note that, with this transformation (whitening),

$$d^2 = (\underline{s}_1 - \underline{s}_0)\Sigma_N^{-1}(\underline{s}_1 - \underline{s}_0) = \|\hat{\underline{s}}_1 - \hat{\underline{s}}_0\|^2$$

i.e., it is equal to square of the distance between the two signals in the new coordinate system.

Signal Selection

In order to improve the performance of the detection scheme, we should choose the signals such that

$$(\underline{s}_1 - \underline{s}_0)^T \Sigma_N^{-1}(\underline{s}_1 - \underline{s}_0)$$

is maximized.

Using the spectral decomposition

$$\Sigma_N^{-1} = \sum_{k=1}^{n} \lambda_k^{-1} v_k v_k^T$$

We have

$$d^2 = (\underline{s}_1 - \underline{s}_0)^T \Sigma_N^{-1} (\underline{s}_1 - \underline{s}_0) = (\underline{s}_1 - \underline{s}_0)^T \sum \lambda_k^{-1} \underline{\nu}_k \underline{\nu}_k^T (\underline{s}_1 - \underline{s}_0)$$

$$= \sum_{k=1}^{n} \lambda_k^{-1} (\underline{s}_1 - \underline{s}_0)^T \nu_k \nu_k^T (\underline{s}_1 - \underline{s}_0)$$

$$\leq \lambda_{min}^{-1} \sum (\underline{s}_1 - \underline{s}_0)^T \underline{\nu}_k \nu_k^T (\underline{s}_1 - \underline{s}_0)$$

$$= \lambda_{min}^{-1} (\underline{s}_1 - s_0)^T (\sum \underline{\nu}_k \nu_k^T) (\underline{s}_1 - s_0)$$

$$= \lambda_{min}^{-1} (\underline{s}_1 - \underline{s}_0)^T (\underline{s}_1 - \underline{s}_0) = \lambda_{min}^{-1} \|\underline{s}_1 - \underline{s}_0\|^2$$

where

$$\lambda_{min} = \min(\lambda_1, \lambda_2, \cdots, \lambda_n).$$

So, we have

$$d^2 \leq \lambda_{min}^{-1} \|\underline{s}_1 - \underline{s}_0\|^2$$

This means that with fixed $\|\underline{s}_1 - \underline{s}_0\|$, the best Thing to do is to select the signal along the eigenvector Corresponding to the smallest eigenvalue. This means sending the information in the direction of smallest noise.

Example: Two Sample Detection

$$\Sigma_N = \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

$$\lambda_1 = \sigma^2(1-\rho) \quad \text{and} \quad \lambda_2 = \sigma^2(1+\rho)$$

$$\underline{v}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad \underline{v}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

if $\rho > 0$, $\lambda_{min} = \lambda_1$ and the optimum signal is

$$\underline{s}_1 = \sqrt{\frac{P}{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad \underline{s}_0 = \sqrt{\frac{P}{2}}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

if $\rho < 0$, $\lambda_{min} = \lambda_2$ and

$$\underline{s}_1 = \sqrt{\frac{P}{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \underline{s}_0 = \sqrt{\frac{P}{2}}\begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

where P is the signal power, i.e.,

$$P = \|\underline{s}_1\|^2 = \|\underline{s}_0\|^2$$

Then d^2 is:

$$d^2 = \frac{1}{\lambda_{min}}(\underline{s}_1 - \underline{s}_0)^T(\underline{s}_1 - \underline{s}_0)$$

$$= \frac{4P}{(1-|\rho|)\sigma^2}$$

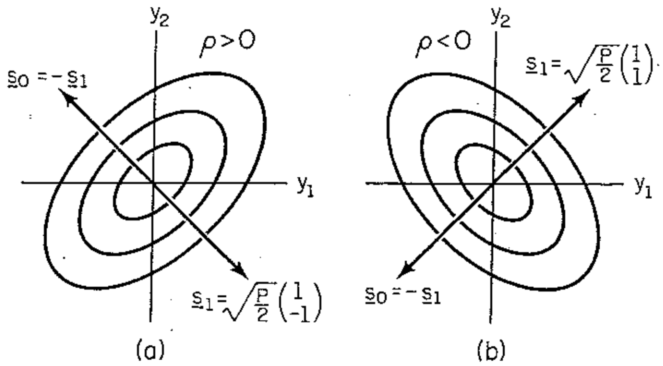


FIGURE III.B.9. Illustration of optimum signals for Gaussian noise with $\mathbf{\Sigma}_N = \sigma^2\begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix}$.

Detection of Signals with unknown parameters

$$H_0: \; Y_k = N_k + s_{0k}(\theta) \qquad k=1,\ldots,n$$

versus

$$H_1: \; Y_k = N_k + s_{1k}(\theta) \qquad k=1,\ldots,n$$

Assume that $\theta \sim w_j \quad j=0,1.$

Then,

$$L(\underline{y}) = \frac{E_1\{p_N(\underline{y}-\underline{s}_1(\theta))\}}{E_0\{p_N(\underline{y}-\underline{s}_0(\theta))\}}$$

$$= \frac{\int_\Lambda p_N(\underline{y}-s_1(\theta))\,w_1(\theta)\,d\theta}{\int_\Lambda p_N(\underline{y}-\underline{s}_0(\theta))\,w_0(\theta)\,d\theta}$$

Without loss of generality, we can assume

$$\underline{s}_0(\theta) \equiv \underline{0} \quad \text{and} \quad \underline{s}_1(\theta) \equiv \underline{s}(\theta)$$

Then,

$$L(\underline{y}) = \int_\Lambda \frac{p_N(\underline{y} - \underline{s}(\theta))}{p_N(\underline{y})} w(\theta)\, d\theta$$

$$= \int_\Lambda L_\theta(\underline{y})\, w(\theta)\, d\theta$$

For the i.i.d. Gaussian case with $N(0, \sigma^2)$,

$$L(\underline{y}) = \int_\Lambda \exp\left\{\left[\underline{s}^T(\theta)\underline{y} - \frac{1}{2}\|\underline{s}(\theta)\|^2\right]/\sigma^2\right\} w(\theta)\, d\theta$$

Example: Non-Coherent detection of a modulated sinusoid:

let $\underline{s}_0(\theta) = \underline{0}$ and $\underline{s}_1(\theta) = s(\theta)$

$\underline{s}(\theta)$'s components are,

$$s_k(\theta) = a_k \sin[(k-1)\omega_c T_s + \theta], \quad k = 1, \ldots, n$$

$a_1, \ldots, a_n$ are known constant amplitudes and θ is a random phase $\in [0, 2\pi]$ uniformly.

Assuming i.i.d. $\sim N(0, \sigma^2)$ noise

$$L(\underline{y}) = \frac{1}{2\pi}\int_0^{2\pi} \exp\left[\frac{1}{\sigma^2}\left(\sum_{k=1}^n y_k s_k(\theta) - \frac{1}{2}\sum_{k=1}^n s_k^2(\theta)\right)\right] d\theta$$

Using

$$\sin(\alpha+\beta) = \cos\alpha \sin\beta + \sin\alpha \cos\beta$$

we get,

$$\sum_{k=1}^{n} y_k s_\theta(\theta) = y_c \sin\theta + y_s \cos\theta$$

where

$$y_c = \sum_{k=1}^{n} a_k y_k \cos[(k-1)\omega_c T_s]$$

and

$$y_s = \sum_{k=1}^{n} a_k y_k \sin[(k-1)\omega_c T_s]$$

and Using

$$\sin^2\alpha = \frac{1}{2} - \frac{1}{2}\cos 2\alpha$$

we get,

$$-\frac{1}{2}\sum_{k=1}^{n} s_k^2(\theta) = -\frac{1}{4}\sum_{k=1}^{n} a_k^2 + \frac{1}{4}\sum_{k=1}^{n} a_k^2 \cos[2(k-1)\omega_c T_s + 2\theta].$$

Substituting these in $L(\underline{y})$, we get,

$$L(\underline{y}) = e^{-n\overline{a^2}/4\sigma^2} \times \frac{1}{2\pi}\int_0^{2\pi} \exp\left\{\frac{1}{\sigma^2}(y_c \sin\theta + y_s \cos\theta)\right\} d\theta$$

where $\overline{a^2} = \frac{1}{n}\sum_{k=1}^{n} a_k^2$

$$L(\underline{y}) = e^{-n\overline{a^2}/4\sigma^2} I_0\left(\frac{r}{\sigma^2}\right)$$

where $r = \sqrt{y_c^2 + y_s^2}$.

Since $I_0(\cdot)$ is monotone, we have

$$\tilde{\delta}_0(\underline{y}) = \begin{cases} 1 & > \\ \gamma & \text{if } r = \tau' = \sigma^2 I_0^{-1}\left(\tau e^{\frac{na^2}{4\sigma^2}}\right) \\ 0 & < \end{cases}$$

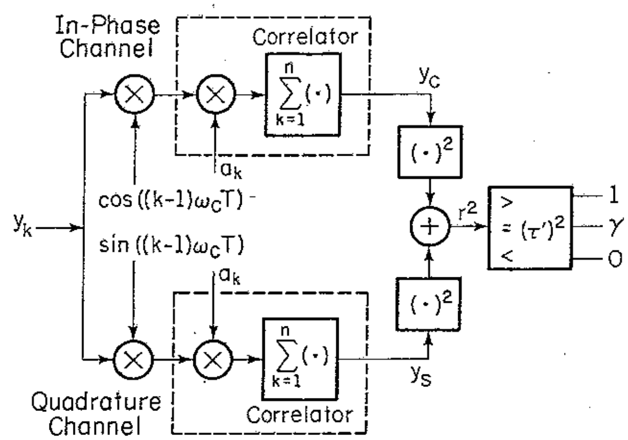


FIGURE III.B.10. Optimum system for noncoherent detection of a modulated sinusoid in i.i.d. Gaussian noise.

Detection of Stochastic Signals (Gaussian Case)

$$H_0: \underline{Y} \sim N(\underline{\mu}_0, \Sigma_0)$$

versus

$$H_1: \underline{Y} \sim N(\underline{\mu}_1, \Sigma_1)$$

The log likelihood Ratio is:

$$\begin{aligned}\log L(\underline{y}) &= \tfrac{1}{2}\log[|\Sigma_0|/|\Sigma_1|] + \tfrac{1}{2}(\underline{y}-\underline{\mu}_0)^T\Sigma_0^{-1}(\underline{y}-\underline{\mu}_0) \\ &\quad -\tfrac{1}{2}(\underline{y}-\underline{\mu}_1)^T\Sigma_1^{-1}(\underline{y}-\underline{\mu}_1) \\ &= \tfrac{1}{2}\underline{y}^T[\Sigma_0^{-1}-\Sigma_1^{-1}]\underline{y} + [\underline{\mu}_1^T\Sigma_1^{-1} - \underline{\mu}_0^T\Sigma_0^{-1}]\underline{y} + C,\end{aligned}$$

where C is the constant parts that can be absorbed in the threshold:

$$C = \frac{1}{2}\left[\log\left(\frac{|\Sigma_0|}{|\Sigma_1|}\right) + \underline{\mu}_0^T \Sigma_0^{-1} \underline{\mu}_0 - \underline{\mu}_1^T \Sigma_1^{-1} \underline{\mu}_1\right]$$

Apart from this part, $L(\underline{y})$ has a quadratic part and a linear part.

When the two populations have the same covariance, the quadratic term vanishes and we have a linear test as the case of coherent detection in Gaussian noise.

In the case where means are the same under both Hypotheses, i.e., $\underline{\mu}_1 = \underline{\mu}_0$ then we can take them to be zero without any loss of generality and we have a quadratic test.

This is the case of detecting zero-mean random signal in Gaussian noise:

$$H_0: \ \underline{Y} = \underline{N} \sim N(\underline{0}, \sigma^2 \underline{I})$$

versus

$$H_1: \ \underline{Y} = \underline{N} + \underline{S} \qquad \text{with } \underline{S} \sim N(\underline{0}, \Sigma_s)$$

Here $\Sigma_0 = \sigma^2 I$ and $\Sigma_1 = \sigma^2 I + \Sigma_s$

and the decision rule is:

$$\tilde{\delta}_0(\underline{y}) = \begin{cases} 1 & > \\ \gamma & \text{if } \underline{y}^T Q \underline{y} = \tau' \\ 0 & < \end{cases}$$

where

$$\tau' = 2(\log \tau - C)$$

and

$$Q = \sigma^{-2} I - (\sigma^2 I + \Sigma_s)^{-1} = \sigma^{-2} \Sigma_s (\sigma^2 I + \Sigma_s)^{-1}$$

As an example, if the random signal to be detected is i.i.d. $N(0, \sigma_s^2)$ then

$$\Sigma_s = \sigma_s^2 I \text{ and}$$

$$\underline{y}^T Q \underline{y} = \frac{\sigma_s^2}{\sigma^2(\sigma^2 + \sigma_s^2)} \sum_{k=1}^{n} y_k^2$$

This results in <u>energy detector</u> or <u>radiometer</u>.