ELEC 6131 – Error Detecting and Correcting Codes

Midterm

March 5, 2019

Time: 90 minutes

1) Consider the polynomial $p(x) = x^4 + x + 1$ over GF(2).
   a) Show that $p(x)$ is primitive (2 Marks).
   b) List all elements of GF(16) generated by $p(x)$ in power and polynomial form (4 Marks).
   c) Show that α^3 a root of $f(x) = x^4 + x^3 + x^2 + x + 1$ (2 Marks).
   d) What are other roots? (2 Marks)

2) Consider a (7, 4) code with the following Generating matrix:

$$G = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}.$$

Transform it to systematic form (3 Marks).

3) Consider an (8, 4) code with following equations:

$$c_0 = u_0 + u_1 + u_2$$
$$c_1 = u_1 + u_2 + u_3$$
$$c_2 = u_0 + u_1 + u_3$$
$$c_3 = u_0 + u_2 + u_3$$
$$c_j = u_{7-j} \text{ for } j = 4, 5, 6, 7.$$

   a) Find the generating matrix of the code. (1 Mark).
   b) Find the parity check matrix of the code (2 Marks).
   c) What is the minimum distance of the code (2 Marks).
   d) How many errors can this code correct (1 Mark)? How many errors it can detect (1 Mark)?

4) Show that for any (n, k) binary linear code with minimum distance 2t+1, following inequality holds (5 Marks):

$$n - k \geq \log_2 \left[1 + \binom{n}{1} + \binom{n}{1} + ... + \binom{n}{t} \right].$$

5) A code has the following parity check matrix:

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Find the transmitted codeword if the received bits are:
$r = [1, e_1, 0, 1, 1, e_2, 0, 1, e_3, 0]$ where, e_1, e_2, e_3 are erasures. (5 Marks).