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Abstract - The present paper analyses the transient of a PM synchronous motor and proposes a method to improve its starting performances. Three banks of capacitors are connected in series between the three-phase source and the motor. A stability analysis is performed to demonstrate that a range of capacitance exists that is useful to improve the motor dynamics. Motor line starting is then analyzed using the calculated value of capacitance and several simulation and experimental runs are performed.

Introduction

Permanent Magnet ac (PMAC) motors have several advantages over induction motors for line start and inverter-fed applications. The presence of the magnets means that the magnetizing current is unnecessary; this improves the power factor of PMAC motors. The absence of magnetizing current and the much lower rotor losses once synchronized, make for a higher efficiency machine. The fact that the source of the machine's losses is confined largely to the stator, from where it is more easily removed, is a particularly attractive feature of PMAC motors.

Heat is generated in the cage of the line start motor during start-up. This heat is more of a problem in PMAC motors than in induction machines because of the proximity of the cage to the magnets. Both the residual flux density and coercivity of the popular NeBoFe and SmCo₃ magnets reduce as a function of temperature. Indeed if the temperature excursion is beyond a certain value, permanent demagnetization can result. The capital cost of PMAC machines is also higher, but the overall cost over the operational lifetime, which includes savings due to higher efficiency, can be lower for PMAC motors in the fractional to 30 kW size.

The magnitude of the mechanical load dictates a specific excitation to minimize stator currents and motor losses or alternatively, to optimize motor efficiency and power factor. The magnet choice and design clearly depend on such considerations but other constraints must be taken into account like the start-up stator currents and motor torque.

As in other electrical machines line starting is the most significant test to size the motor according to the performances required by the user. In case of PM synchronous motors three different torques appear during run-up from zero speed [1]:

- 1) braking torque due to the magnet;
- 2) pulsating torque due to rotor saliency and also acting as a braking torque;
- 3) cage torque due to the rotor bars.

It is clear that design of the magnet and rotor cage should be a compromise between starting and steady-state performances. The present paper analyses the transient and steady-state performance of a PM

synchronous motor and proposes a method to improve its starting performance.

Three banks of capacitors are connected in series between the three-phase source and the motor as shown in Fig.1. The effect of the capacitors on the motor dynamics is essentially due to a voltage increase on the motor terminals and to a consequent variation of the electromagnetic torques produced at the air-gap. It is demonstrated that a range of capacitance exists that is useful to improve the motor dynamics. Motor line starting is then analyzed using the calculated value of capacitance and several simulation and experimental runs are performed to verify the convenience of the proposed series capacitor connection.

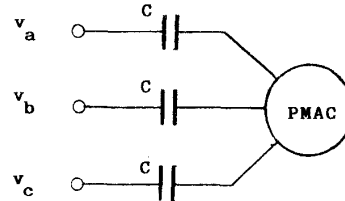


Figure 1. Schematic of a PM synchronous motor supplied through series capacitors.

System Modeling

Transient behavior of PM synchronous motors is usually modelled by a set of equations written in a rotor reference frame, including two rotor equations which account for the short-circuited damper windings, respectively on the q and d-axis.

The presence of three capacitor banks connected in series can be accounted for by considering the v_{qc} and v_{dc} transformed voltages across the capacitors [2]. The system equations are:

$$v_q = r i_q + p \lambda_q + \omega \lambda_d - v_{qc} \quad (1)$$

$$v_d = r i_d + p \lambda_d - \omega \lambda_q - v_{dc} \quad (2)$$

$$0 = r_{1q} i_{1q} + p \lambda_{1q} \quad (3)$$

$$0 = r_{1d} i_{1d} + p \lambda_{1d} \quad (4)$$

where:

$$v_{qc} = -\frac{i_d}{\omega C} + \frac{1}{\omega} p v_{dc} \quad (5)$$

$$v_{dc} = +\frac{i_q}{\omega C} - \frac{1}{\omega} p v_{qc} \quad (6)$$

and p is the derivative operator.

The flux linkages with stator and rotor circuits are expressed by:

$$\lambda_q = L_{lq} i_q + m_q (i_q + i_{1q}) \quad (7)$$

$$\lambda_d = L_{ld} i_d + m_d (i_d + I_{mo} + i_{1d}) \quad (8)$$

$$\lambda_{1q} = m_q (i_q + i_{1q}) + L_{l1q} i_{1q} \quad (9)$$

$$\lambda_{1d} = m_d (i_d + I_{mo} + i_{1d}) + L_{l1d} i_{1d} \quad (10)$$

where I_{mo} represents the constant value of a fictitious current source used to model the permanent magnet; L_{lq} , L_{ld} , L_{l1q} and L_{l1d} are the constant leakage inductances of the stator and rotor circuits; m_q and m_d are the variable mutual inductances between stator and rotor circuits, respectively on the q and d-axis. According to previous established results [3], [4], m_q and m_d are influenced by saturation of the flux paths due to the small iron portions offered to stator flux on the q-axis. This produces a flux redistribution on the air-gap and the variation of the magnet operating point along its demagnetization characteristic.

By adding the mechanical equation and the expression of the total electromagnetic torque T_e produced by the motor the complete system modeling is obtained:

$$T_e = Jp\omega_r + T_l \quad (11)$$

$$T_e = \frac{3}{2} \bar{p} (\lambda_d i_q - \lambda_q i_d) \quad (12)$$

$$p\delta = \omega_o - \omega_r \quad (13)$$

where T_l is the load torque, J the system inertia and \bar{p} the pole pair number.

Equations (1)-(13) can be reduced to eight independent relations and successively arranged in terms of state variables:

$$p\lambda_q = -\omega_r \lambda_d - \frac{rL_{lq}}{D_q} \lambda_q + \frac{rm_q}{D_q} \lambda_{1q} + v_q + v_{qc} \quad (14)$$

$$p\lambda_d = -\frac{rL_{ld}}{D_d} \lambda_d + \omega_r \lambda_q + \frac{rm_d}{D_d} \lambda_{1d} + \frac{rm_d(L_{ld} - m_d)}{D_d} I_{mo} + v_d + v_{dc} \quad (15)$$

$$p\lambda_{1q} = \frac{r_{1q} m_q}{D_q} \lambda_q - \frac{r_{1q} L_{lq}}{D_q} \lambda_{1q} \quad (16)$$

$$p\lambda_{1d} = \frac{r_{1d} m_d}{D_d} \lambda_d - \frac{r_{1d} L_{ld}}{D_d} \lambda_{1d} + \frac{r_{1d} m_d (L_{ld} - m_d)}{D_d} I_{mo} \quad (17)$$

$$pv_{qc} = \frac{1}{C} \frac{L_{lq}}{D_q} \lambda_q - \frac{1}{C} \frac{m_q}{D_q} \lambda_{1q} - \omega_r v_{dc} \quad (18)$$

$$pv_{dc} = \frac{1}{C} \frac{L_{ld}}{D_d} \lambda_d - \frac{1}{C} \frac{m_d}{D_d} \lambda_{1d} - \frac{1}{C} \frac{m_d (L_{ld} - m_d)}{D_d} I_{mo} + \omega_r v_{qc} \quad (19)$$

$$p\delta = \omega_o - \omega_r \quad (20)$$

$$p\omega_r = \frac{1}{J} (T_e - T_l) \quad (21)$$

where D_d and D_q are defined through the synchronous reactances L_d and L_q and the total rotor inductances L_{1d} and L_{1q} :

$$D_d = L_d L_{1d} - m_d^2 \quad (22)$$

$$D_q = L_q L_{1q} - m_q^2 \quad (23)$$

Stability Analysis

The stability analysis can be performed by the small variation technique. By linearizing around a fixed steady-state operating point the following non linear eighth order differential system is obtained:

$$\Delta \dot{\lambda}_q = -\frac{rL_{lq}}{D_q} \Delta \lambda_q - \omega_o \Delta \lambda_d + \frac{rm_q}{D_q} \Delta \lambda_{1q} - \lambda_{do} \Delta \omega_r + v_{do} \Delta \delta - \Delta v_{qc} + \Delta v \cos \delta_o \quad (24)$$

$$\Delta \dot{\lambda}_d = \omega_o \Delta \lambda_q - \frac{rL_{ld}}{D_d} \Delta \lambda_d + \frac{rm_d}{D_d} \Delta \lambda_{1d} + \lambda_{qo} \Delta \omega_r - v_{qo} \Delta \delta - \Delta v_{dc} - \Delta v \sin \delta_o \quad (25)$$

$$\Delta \dot{\lambda}_{1q} = \frac{r_{1q} m_q}{D_q} \Delta \lambda_q - \frac{r_{1q} L_{lq}}{D_q} \Delta \lambda_{1q} \quad (26)$$

$$\Delta \dot{\lambda}_{1d} = \frac{r_{1d} m_d}{D_d} \Delta \lambda_d - \frac{r_{1d} L_{ld}}{D_d} \Delta \lambda_{1d} \quad (27)$$

$$\Delta \dot{v}_{qc} = \frac{1}{C} \frac{L_{lq}}{D_q} \Delta \lambda_q - \frac{1}{C} \frac{m_q}{D_q} \Delta \lambda_{1q} - \omega_o \Delta v_{dc} - v_{dco} \Delta \omega_r \quad (28)$$

$$\Delta \dot{v}_{dc} = \frac{1}{C} \frac{L_{ld}}{D_d} \Delta \lambda_d - \frac{1}{C} \frac{m_d}{D_d} \Delta \lambda_{1d} + \omega_o \Delta v_{qc} + v_{qco} \Delta \omega_r \quad (29)$$

$$\Delta \dot{\delta} = -\Delta \omega_r \quad (30)$$

$$\Delta \dot{\omega}_r = \frac{1}{J} \left\{ \frac{3}{2} \bar{p} \left[\left(\frac{L_{1q}}{D_q} - \frac{L_{1d}}{D_d} \right) (\lambda_{qo} \Delta \lambda_d + \lambda_{do} \Delta \lambda_q) + \frac{m_q}{D_q} (\lambda_{1qo} \Delta \lambda_d + \lambda_{do} \Delta \lambda_{1q}) + \frac{m_d}{D_d} (\lambda_{1do} + L_{l1d} I_{mo}) \Delta \lambda_q + \frac{m_d}{D_d} \lambda_{qo} \Delta \lambda_{1d} \right] - \Delta T_l \right\} \quad (31)$$

Steady-state values appearing with the added subscript o in equations (24)-(31) can be calculated by solving (14)-(21) written at steady-state conditions for the specific chosen operating point.

Linearization allows us to study the system stability by eigenvalue analysis. System stability is assured if there are no eigenvalues with positive real part. The basic equation is:

$$\det |\lambda I - A| = 0 \quad (32)$$

where λ indicates the roots, or eigenvalues, of the characteristic equation, I is the identity matrix, and A is the state transition matrix. Many preliminary calculations are needed to evaluate the stability. In fact, due to the difficult task of expressing the eigenvalues as analytical functions of parameters, the stability criterion requires that equation (32) must be numerically solved for different values of the capacitors connected in series. Moreover, each value of series capacitors influences the initial steady-state conditions so that recalculation of the steady-state operating point is required for every capacitor value.

In Table I the steady-state initial conditions are reported for several values of series capacitors

Table I

Steady-state Initial Conditions Calculated for Different Capacitances.

C [F]	0.005	0.0075	0.010	0.015
i_{q0} [A]	6.697	6.633	6.602	6.571
i_{d0} [A]	-5.053	-4.995	-4.967	-4.940
λ_{q0} [Wb]	0.465	0.461	0.459	0.456
λ_{d0} [Wb]	0.138	0.141	0.143	0.145
λ_{1q0} [Wb]	0.426	0.422	0.420	0.418
λ_{1d0} [Wb]	0.167	0.171	0.172	0.174
v_{qco} [V]	3.217	2.120	1.581	1.048
v_{dco} [V]	4.263	2.815	2.101	1.394

calculated by equations (14)-(21). All calculations and simulations are performed on a 2hp, three-phase, star-connected, four-pole Permanent Magnet synchronous motor. The motor is rated at 192V, 5.3A with 50Hz supply frequency. It is evident that the capacitor value has a low influence on the initial conditions. Table II reports the calculated eigenvalues using the

Table II

Eigenvalues of the Linearized Model Calculated for the Initial Conditions of Table I.

	C = 0.005 F	C = 0.0075 F
$\lambda_{1,2}$	$-84.97 \pm j 143.9$	$-82.1 \pm j 147.8$
$\lambda_{3,4}$	$-64.5 \pm j 434.7$	$-61.7 \pm j 404.5$
$\lambda_{5,6}$	$10.1 \pm j 153.2$	$-7.2 \pm j 174.4$
λ_7	-23.4	-9.11
λ_8	-145.2	-136.07
	C = 0.010 F	C = 0.015 F
$\lambda_{1,2}$	$-67.9 \pm j 154.6$	$-54.7 \pm j 363.2$
$\lambda_{3,4}$	$-59.1 \pm j 386.3$	$-50.1 \pm j 214.7$
$\lambda_{5,6}$	$-26.0 \pm j 183.9$	$-49.8 \pm j 145.2$
λ_7	-9.02	-9.26
λ_8	-132.4	-128.8

proper initial conditions of Table I. It is clearly shown in Table II the existence of instability as the series capacitors assume a value below 0.0075F. At $C=0.005F$ the λ_5 and λ_6 eigenvalues have a positive real part.

Dynamic Behavior with Line Series Capacitors

The dynamic performance of permanent magnet synchronous motors can improve by means of the use of line series capacitors. With the aim of verifying such useful actions, the dynamic behavior of a permanent magnet synchronous motor has been simulated with and without capacitors. In particular, with the motor fully loaded, a sudden 20% load torque decrease has been considered.

In Fig 2 the simulated traces of rotor speed are reported. In particular, Fig.2a (without capacitors)

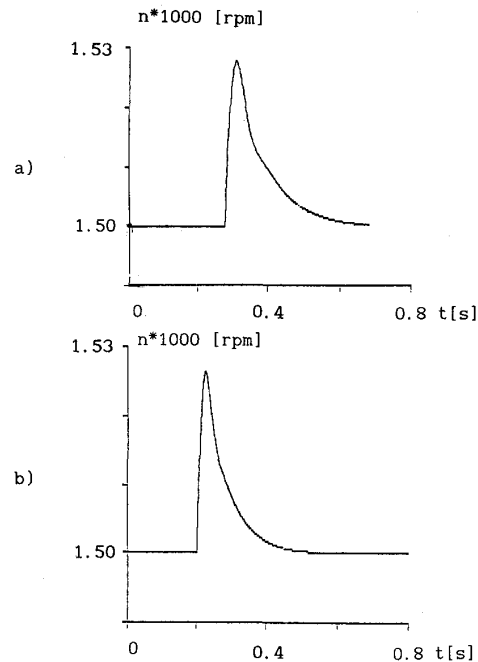


Figure 2. Simulated rotor speed during a sudden load torque decrease:
a) without series capacitors;
b) with $C=0.015F$.

shows that the load reduction causes an instantaneous rise in speed. This is damped in about 0.3s by the inherent stability of the system. Fig.2b, which shows the same transient with a bank of 0.015F series capacitors connected, demonstrates the same overspeed peak but a reduction to 0.2s of the time needed to reach again the synchronous speed. Fig.3 reports the load and electromagnetic torque during the same transient simulation respectively without capacitors (Fig. 3a) and with capacitors (Fig. 3b). By simulation it is possible to show that lower capacitor values can cause system oscillations. Fig.4 and Fig.5 show the rotor speed and torques with a capacitor value of 0.005F. The speed trace shows, after the overspeed peak, a high frequency oscillatory trend superimposed on the synchronous speed, with a peak to peak value of 0.3% of the synchronous speed. The torque traces show a similar phenomenon as is evident from Fig.5.

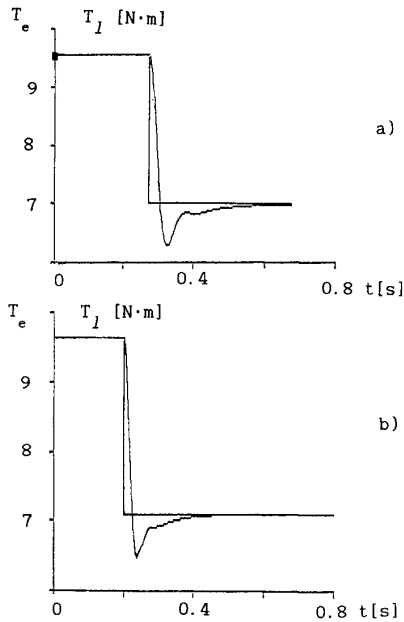


Figure 3. Motor and load torques during a sudden decrease of the load amplitude:
a) without series capacitors;
b) with $C=0.015F$.

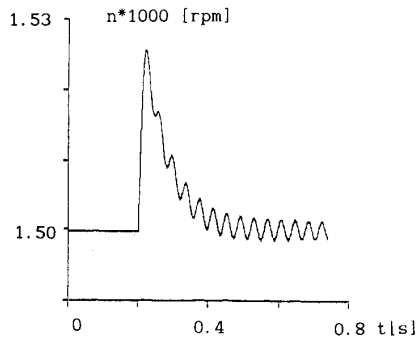


Figure 4. Rotor speed during a sudden load torque decrease with $C=0.005F$.

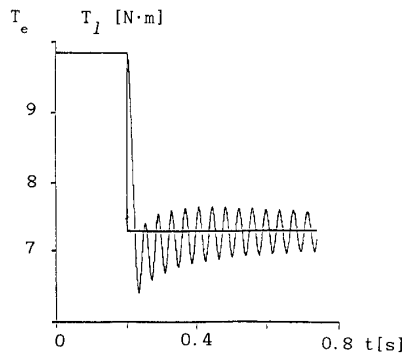


Figure 5. Motor and load torques during a sudden decrease of the load amplitude with $C=0.005F$.

Start-up of Permanent Magnet Synchronous Motor

The start-up transient of Permanent Magnet synchronous motors presents similar characteristics to standard induction machines during such a transient. In fact, the rotor cage develops an asynchronous torque similar to that produced in an induction machine, that only works when PM synchronous motor is not synchronized or when the load angle δ changes.

During run-up, while the rotor cage acts like in an induction motor, the permanent magnets on the rotor act like a generator, resulting in a braking torque. Hence the induction motor torque produced by the cage must overcome this braking torque, any braking torque produced by reluctance variation on the rotor and the load before it can accelerate the motor up to speed. The voltages generated by the magnets start off at dc and increase in frequency towards 50Hz as the rotor accelerates. A capacitor in the line would impede the flow of the low frequency currents during starting and hence reduce the braking torque. Once synchronized, both the magnet and reluctance torques contribute to the useful output torque of the machine with the cage having no effect on the net torque.

For every one of this torque components an analytical expression can be written in terms of the instantaneous values. Substituting relations (7) and (8) in (12) the torque equation may be split into three terms:

$$T_M = \frac{3}{2} \bar{p} m_d I_{mo} i_q \quad (33)$$

$$T_A = \frac{3}{2} \bar{p} (m_d i_{ld} i_q - m_q i_{lq} i_d) \quad (34)$$

$$T_R = \frac{3}{2} \bar{p} (L_d - L_q) i_d i_q \quad (35)$$

where T_M , T_A and T_R express respectively the magnet torque, the torque due the rotor bars and the reluctance torque. Simulation runs of start-up, with regards to the torque components above depicted, are shown in Fig.6, from which the contribution of every torque component on the synchronization process can be observed. Fig.6a reports the simulations runs with a capacitor bank of 0.01F whereas Fig.6b is the case without capacitors.

Fig.7 shows the computer simulations of rotor speed and phase stator current when the motor starts with no-load from rest. A global start-up evaluation from such traces can be formulated by using some particular factors such as starting time, or the ratio between maximum and rated current. The corresponding experimental traces are shown in Fig.8. As the three capacitor banks are connected between the main supply and the permanent magnet synchronous motor some differences arise during the run-up transient, as it is shown in Fig.9 and in Fig.10. In particular the starting time decreases.

Larger values of series capacitor banks have no effect on the motor behavior since they work as a low impedance line parameter. Lower values of series capacitors show that an oscillatory electromechanical phenomenon arises after that speed increases from zero. In fact, as shown in Fig.11 for series capacitor equal to 0.003F the rotor does not reach the synchronous speed but shows a hunting trend. The amplitude of such oscillations increases as the series capacitor value decreases.

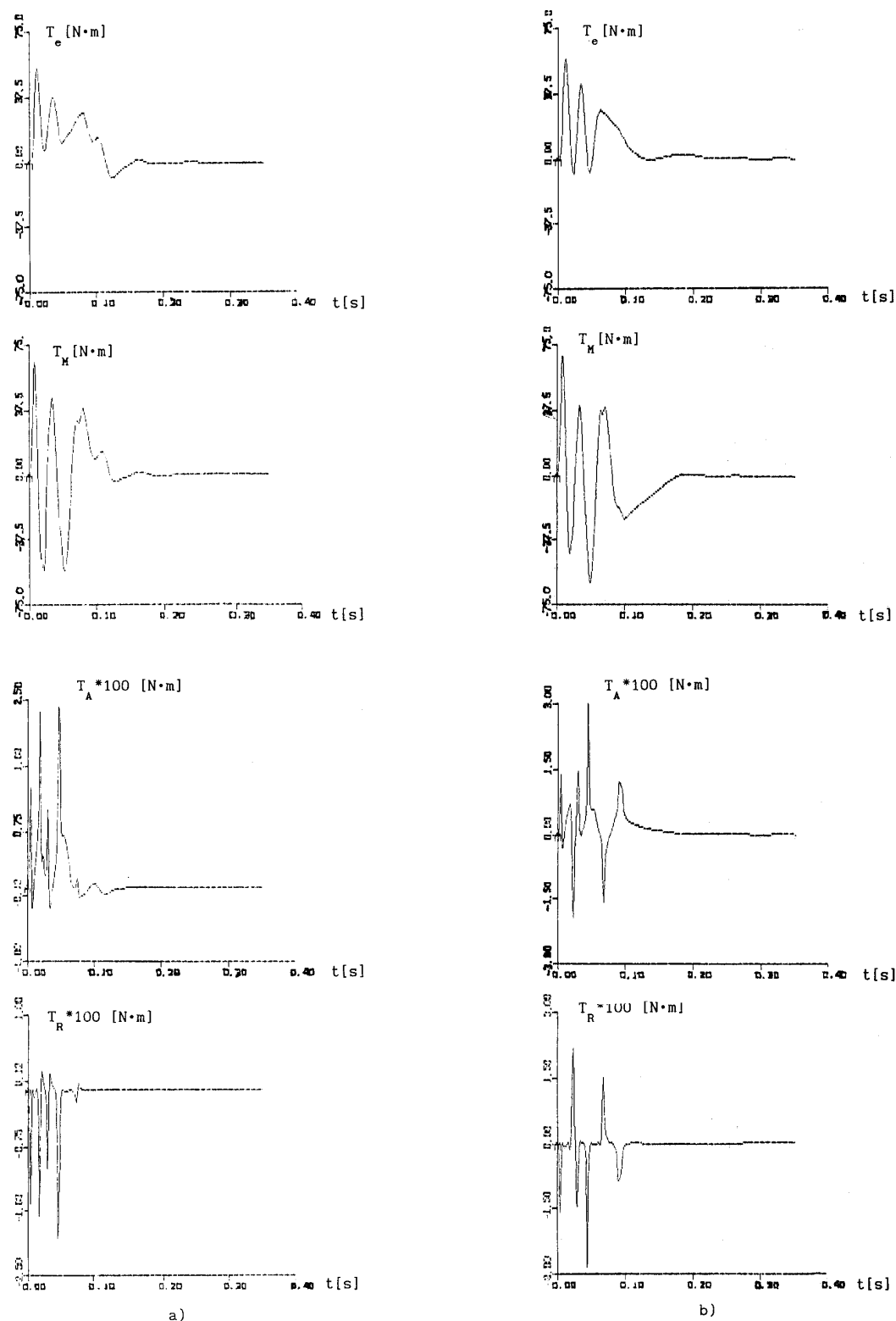


Figure 6. Motor torque T_e and torque components T_M , T_A , T_R during a no-load run-up from rest to synchronous speed:
a) with $C=0.01F$;
b) without series capacitors.

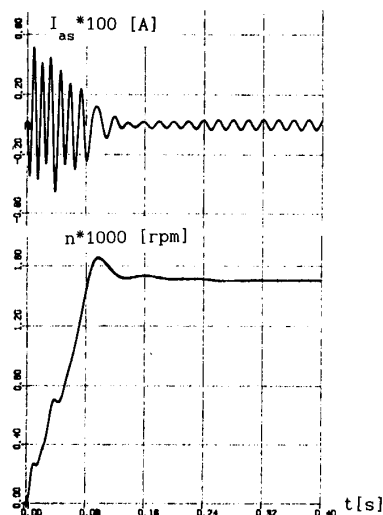


Figure 7. Simulated traces of rotor speed and stator current during the no-load run-up from rest without capacitors.

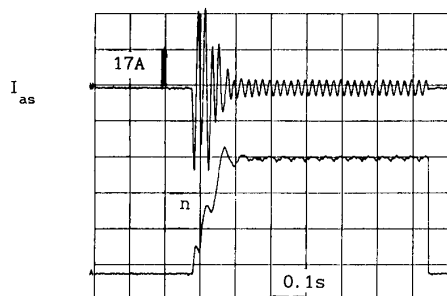


Figure 8. Experimental traces of stator current and rotor speed during the no-load run-up without capacitors.

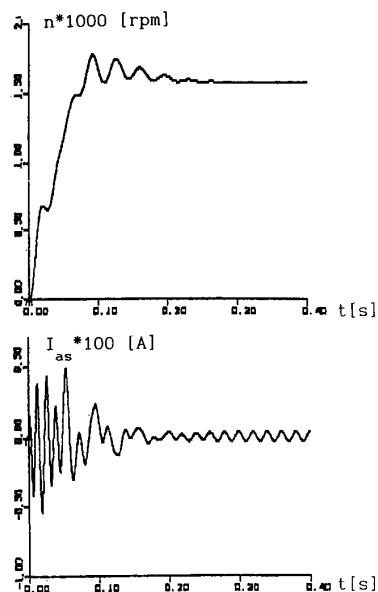


Figure 9. Simulated motor speed and stator current during start up with $C=0.0075F$.

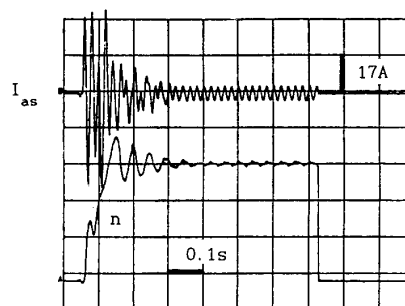


Figure 10. Experimental traces during start-up with $C=0.0075F$.

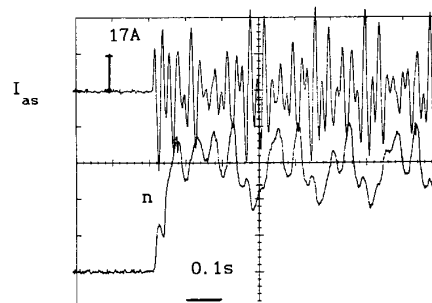


Figure 11. Experimental stator current and rotor speed with $C=0.003F$.

Conclusion

Permanent Magnet synchronous motors carrying a short-circuited rotor cage show starting as well as synchronization capability. In such machines excitation is provided with no energy consumption by the magnets buried in the rotor iron. In this paper a method has been proposed to improve the starting capability of the machine, consisting in connecting three banks of capacitors in series with the three-phase source. The system stability has been performed in such conditions via eigenvalue analysis and several simulation and experimental transient tests have been considered using the calculated value of capacitance. The convenience of the proposed connection has been verified.

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