Effects of Material Properties on Switched Reluctance Motor Vibration Determination

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Abstract: The switched reluctance motor is well known for its higher acoustic noise, caused by stator vibrations. Techniques for noise reduction require knowledge of the modal frequencies, which depend on mechanical shapes and dimensions as well as material properties, for example, Young's modulus, Poisson's ratio, mass density, etc. It is found that the generally accepted value of Young's modulus is not valid for a machine with laminations and no frame. This paper introduces a simple and nondestructive method for the measurement of Young's modulus; it is then used in a finite element program to determine the resonant frequencies of SRM stator vibration. The effects of mass density and Poisson's ratios are also discussed. The FE results are validated by vibration tests, which show good accuracy.

I. Introduction

Vibration and acoustic noise in the switched reluctance motor is caused by the ovalizing deformation of the stator lamination stack due to its radial magnetic attraction to the rotor [1]. This is a big disadvantage for industrial applications. There are several ways of determining stator resonant frequencies and mode shapes of the SRM: analytical calculations, numerical computation (usually by the finite element method), and/or experimental measurements [2]. There is considerable advantage and interest in using numerical methods for the computation of resonant frequencies and mode shapes during motor design. The material properties, like mass density and Young's modulus are key factors besides stator dimensions and shapes. In the approximate formula for stator natural frequency, the stator resonant frequency has the relation [3]

$$f_r \propto \sqrt{\frac{E}{\rho}} \qquad (1)$$

Where f_r is the stator natural frequency, E and ρ are the Young's modulus and mass density of the stator material respectively.

The density of the lamination core can be easily determined, however Young's modulus deserves some consideration. The Young's modulus value of stainless steel ($E = 2.07 \times 10^{11}$ N/m^2) has been widely used by researchers in electric machine vibrations area. However in order to allow theoretical prediction to converge to experimental results, some researchers even varied the value until the predicted natural frequencies converged to the measured values [4]. The fact is that the stator lamination steel of an electric machine is different from stainless steel, in addition, the material properties may change during core lamination or motor manufacturing. Because of the important role it plays in the determination of stator natural frequencies and its complexity, the treatment of Young's modulus has to be considered seriously.

II. Young's Modulus Measurement

2.1 Methodology

Many useful mechanical properties of materials, like Young's modulus, are obtained from tension tests and/or compression tests [5]. However these tests are not trivial for an electric machine stator. Firstly, the force load may change the electromagnetic properties of the stator laminations. Secondly, the test may damage the stator lamination and/or windings. Finally the odd shape of an electric machine stator makes it difficult for mounting.

The other method widely used by mechanical and material researchers is an ultrasonic method, which is nondestructive. This is especially good for an electric machine. It will be discussed in detail in this paper.

The ultrasonic technique is a method for measuring the elastic properties of a material. The sample to be measured is placed between an ultrasonic transmitter (pulser) and receiver, and the time for an ultrasonic pulse to traverse the media between the transducers is measured. The block diagram is shown in Fig.1.

Stress waves from 20Hz to 20kHz are perceived as sound. Waves above 20kHZ are referred to as ultrasonic; ultrasonic frequencies between 1MHz and 10MHz are commonly used in the nondestructive evaluation of engineering materials, for materials characterization.
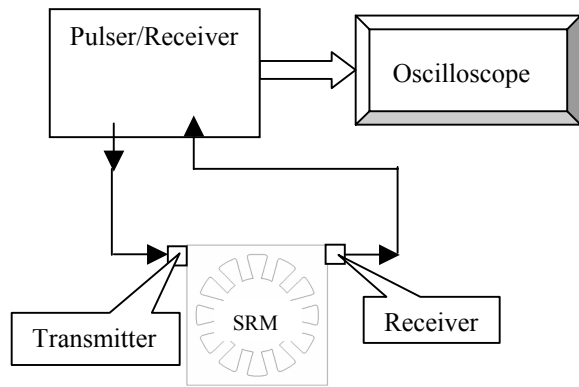
Fig.1 Block diagram of Ultrasonic method

Ultrasonic wave speed, c, depends on the stiffness and on the density, ρ, of the materials under study [6,7]. For longitudinal waves

$$c = \sqrt{\frac{E}{\rho}} \qquad (2)$$

Where E is the Young's modulus. This is valid for a long homogeneous rod of length much longer than the wavelength, and width much less than the wavelength. If the width is much larger than the wavelength, then wave speed is governed by the tensorial modulus

$$c = \sqrt{\frac{1-\upsilon}{(1+\upsilon)(1-2\upsilon)}\frac{E}{\rho}} \qquad (3)$$

Where υ is the Poisson's ratio. The test motor stator used in this paper belongs to this category. So the Young's modulus of the motor stator can be described as:

$$E = \frac{(1+\upsilon)(1-2\upsilon)}{(1-\upsilon)}\rho c^2 \qquad (4)$$

Velocity (wave speed c) can be measured by determining the time delay for the wave to pass through a sample of material. The velocity is the distance (thickness) divided by the time delay. In this method, one transducer sends the waves and another one receives them.

2.2 Test Setup

The prototype SRM used in this paper is shown in Fig.2 (the stator part with windings). Fig.3 shows the above-described ultrasonic method that is used to measure Young's modulus of this 12/8 3-phase SRM stator core lamination. A 200MHz Computer Controlled Pulser/Receiver together with two transducers (KB-Aerotech, GAMA 2.25MHz/.50) are used. The test data is then recorded in a digital oscilloscope and analyzed in a computer program to obtain the wave speed and Young's modulus.
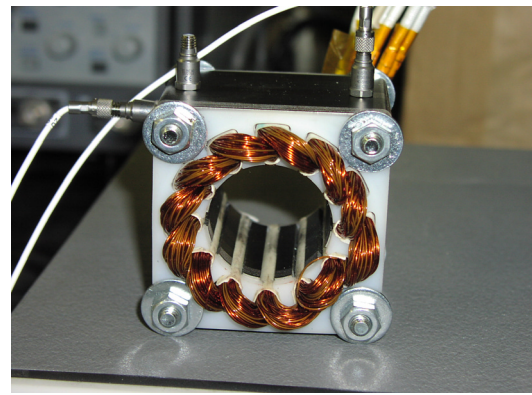

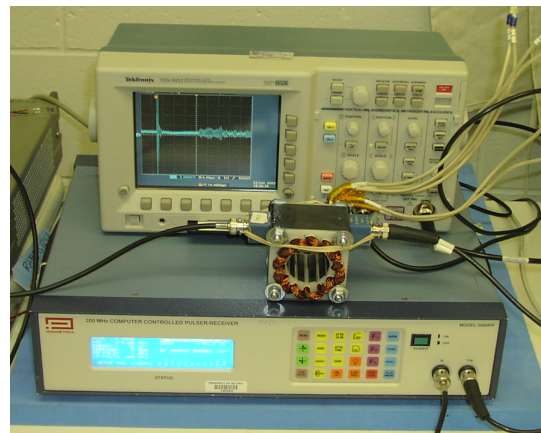Fig.2 Stator of the Prototype SRM

Fig.3 Test setup

Comparing with the traditional tension /compression method, the ultrasonic method has several advantages. Firstly, it is fast, convenient and simple and no special hydraulic equipment is needed. Secondly, no special mounting is needed, the transducers are attached to the test object where needed. Then no damage is done to the stator or to the windings. Finally it is easy to change the test location, for example, in horizontal direction or vertical direction, or from the inner side of stator pole.

2.3 Test Results

Fig.4 records the travel time of the ultrasonic signal inside the stator core lamination. The wave speed c can then be calculated, knowing the length of the test object (SRM stator). Using equation (4), Young's modulus E of the stator lamination is obtained in both the vertical and lateral directions (as viewed in Fig.2). Where, Poisson's ratio υ = 0.3 [4,5,6], mass density ρ = $0.935 \times 7.8 \times 10^3$ kg/m^3, 0.935 is the stacking factor (generally it is 0.92~0.95) of the stator core lamination. The measured value is 1.521×10^{11} N/m^2, and is the same in both directions, quite different from 2.07×10^{11} N/m^2 used by most researchers.

Some other Poisson's ratio values are also used to calculate the Young's modulus, which are listed in Table 1. They will also be used later in FE program for determination of resonant frequency for comparison purpose.
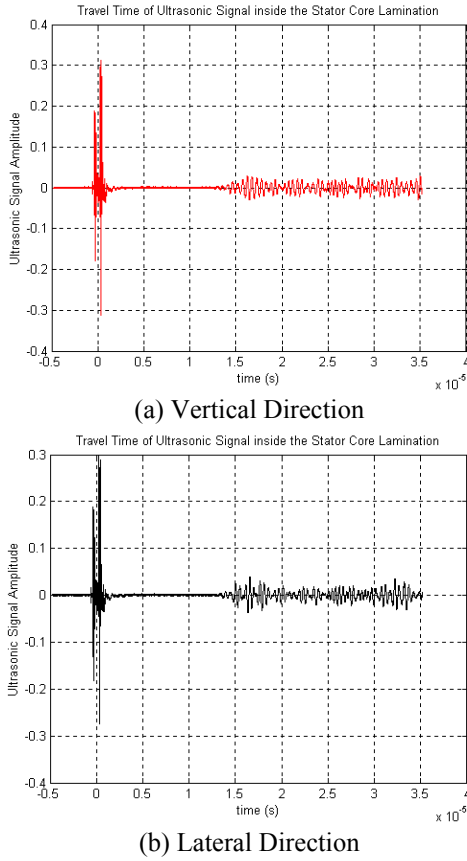

(a) Vertical Direction

(b) Lateral Direction

Fig.4 Travel Time of Ultrasonic Signal inside the Stator Core Lamination

Table 1 Measured Young's Modulus

Poisson's Ratio υ	Young's Modulus E ($\times 10^{11}$N/m^2)
0.3 (Commonly Used)	1.521
0.27	1.638
0.34	1.371
0 (One-dimensional)	2.047

III. Calculation Results

3.1 Equivalent Mass Density Determination

The effect of phase windings on the vibration of electric machines has been discussed by many studies [9,10,11]. Generally it is believed that the windings have effects of both additional mass and vibration damping on the vibrational behavior of electric machines. In this paper, the mass of phase windings is treated as increase of the mass density of the pole to which the winding is attached. According to [11], the concentrated windings of the SRM are installed on the stator poles. There is insulation between a pole and winding. For conventional motors, the contact between pole and winding assembly is tight enough to allow the windings to move with the pole, but cannot add an extra stiffness due to the existence of the insulation. Although the windings may contribute to stiffness if the slot fill factor is high, the increase in the stiffness is low when compared to the mass added by the windings to the stator assembly. Therefore, the effect of windings is equated to an increase of the pole mass. As a result, the geometrical model for finite element computation is the same as the model without considering the winding effect, but the poles are treated as a different material with specific mass density. The equivalent mass density of the pole with winding effect will be explained explicitly as follows.

The equivalent mass density ρ_e can be expressed as $\rho_e = \dfrac{m_{pole} + m_{coil}}{V_{pole}}$, where m_{pole} is the mass of one pole, m_{coil} is the mass of the coil attached to one pole, V_{pole} is the volume of the pole.

The volume of the pole $V_{pole} = L_{stack} \times H_{pole} \times W_{pole}$, where L_{stack} is the length of the pole (which is the same as stator stack length), H_{pole} is the height if the pole and W_{pole} is the width of the pole.

The mass of one pole is $m_{pole} = \rho_{Fe} \times f_{stack} \times V_{pole}$, where ρ_{Fe} is the mass density of steel, f_{stack} is stacking factor of the stator lamination, generally it is 0.92~0.95.

The mass of the coil can be calculated as $m_{coil} = N_{turn} \times m_{turn}$, where N_{turn} is the turns of winding per pole and m_{turn} is the mass of each turn. The mass of each turn of the coil $m_{turn} = \rho_{cu} \times L_{turn} \times A_{turn}$, where ρ_{cu} is the mass density of copper, L_{turn} is the length of each turn and A_{turn} is the intersection area of copper wire. Fig.4 shows the shape of one turn of the coil, from which the length of each turn can be calculated, $L_{turn} = 2\pi R_{oh} + 2L_{copper}$, where R_{oh} is the radius of the winding overhang, L_{copper} is the length of the straight line shown in Fig.5, which is longer than the stator stack length. The intersection area of copper wire can be calculated as $A_{turn} = \pi\left(\dfrac{D_{barewire}}{2}\right)^2$, where $D_{barewire}$ is the diameter of the bare copper wire with corresponding A.W.G.

Finally, the equation for equivalent mass density of the pole is:

$$\rho_e = \frac{\rho_{Fe} \times f_{stack} \times \left(L_{stack} \times H_{pole} \times W_{pole} \right) + N_{turn} \times \left\{ \rho_{cu} \times (2\pi R_{oh} + 2L_{stack}) \times \left[\pi \left(\frac{D_{barewire}}{2} \right)^2 \right] \right\}}{\left(L_{stack} \times H_{pole} \times W_{pole} \right)}$$
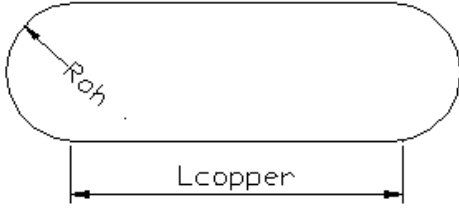
$$(5)$$

Fig.5 One Turn of the Coil

The equivalent density of the pole is $20.29 \times 10^3 \mathrm{kg/m}^3$.

3.2 Finite Element Analysis

A 3-D FE model is constructed for this motor, as shown in Fig.6. The measured Young's modulus E is used with the ANSYS 3-D model to determine the natural frequencies, with the 2nd modes being at 1563.8Hz and 2578.1Hz.
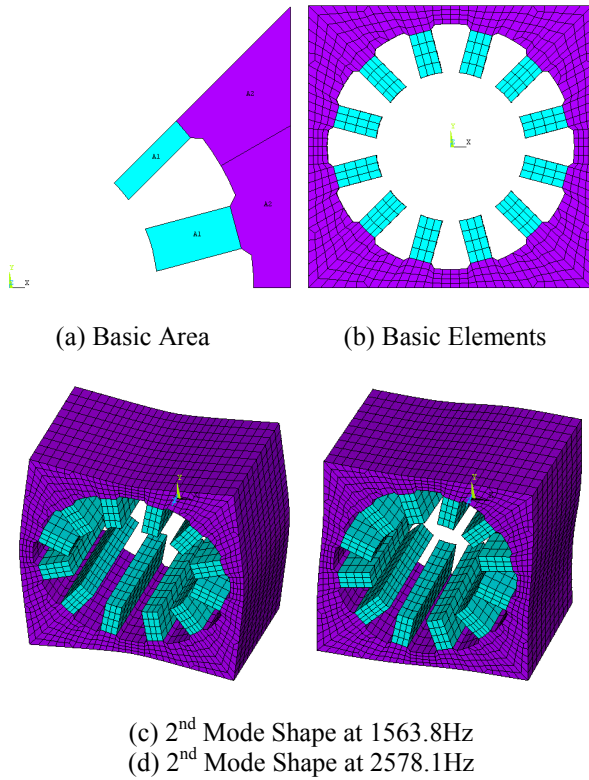

(a) Basic Area   (b) Basic Elements

(c) 2nd Mode Shape at 1563.8Hz
(d) 2nd Mode Shape at 2578.1Hz

Fig.6 FE Basic Area, Mesh and 2nd Mode Shapes (with measured Young's modulus)

The commonly used Young's modulus (2.07×10^{11} N/m²) is also used in the FE program to re-evaluate the resonant

frequencies. Both are given in Table 2. To evaluate the effect of Poisson's ratio on the resonant frequencies, a different Poisson's ratio value, $\upsilon = 0.34$ and 0.27 [6,12], is used in the FE program, with all the other material properties kept the same. No effect has been observed by changing this value.

IV. Experimental Validation

In order to validate the FE calculations of resonant frequencies, a force hammer test is carried out for the SRM. Fig.7 shows the time domain hammer impulse force, damped transient stator acceleration and frequency domain acceleration. It can be seen clearly that the 2nd mode resonant frequencies, which are 1497Hz and 2655Hz respectively, are very close to the calculated ones with the measured Young's modulus (the peak of 2095Hz appeared in this figure is the resonant frequency of the test setup). The errors between the calculated and measured resonant frequencies are less than 4.5% for both resonant frequencies, as shown in Table 2, row 1). The errors of the calculated resonant frequencies using commonly used Young's modulus are 21.9% and 13.3% respectively, as listed in Table 2, row 2).
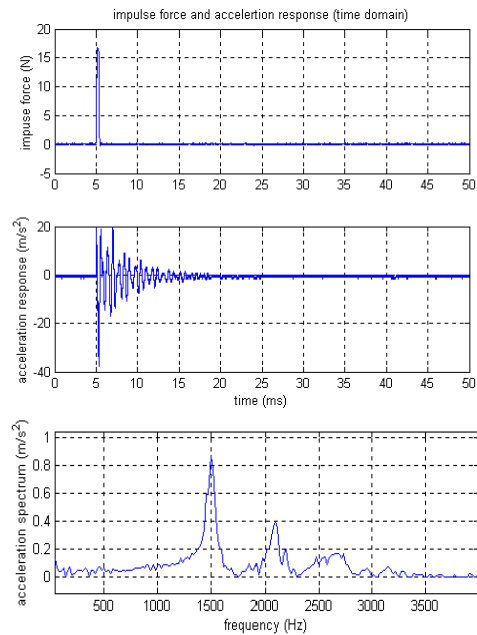

Fig.7 Force Hammer Test Result: Force Impulse and Stator Acceleration

V. Comparisons

The effects of Young's modulus, equivalent mass, Poisson's ratio and stator core lamination stacking factor on the resonant frequencies are studied. These results are compared with impulse hammer test results, as shown in Table 2.

Row 1) shows the calculated resonant frequencies with measured Young's modulus, commonly used Poisson's ratio, consideration of winding effect and stacking factor. Both errors are less than 4.5% compared with the impulse force hammer tests.

Row 2) shows the calculated results with commonly used Young's modulus (2.07×10^{11} N/m^2), the calculation error are not acceptable although the winding effect and stacking factor are considered, with 21.9% and 13.3% respectively.

Row 3), 4), 5) show the effect of Poisson's ratio on the resonant frequencies, no major effect has been found. The

Poisson's ratio $\nu = 0$ here refers to the so-called "one-dimensional" material.

Row 6), 7), 8) show the role of Poisson's ratio on the accurate determination of Young's modulus during ultrasonic measurements, which is based on equation (4). Comparing with row 1), the commonly used Poisson's ratio $\nu = 0.3$ has the best result, while $\nu = 0.27$ and 0.34 have acceptable results, both errors are around 8%. $\nu = 0$, which refers to homogeneous one-dimensional structure, leads to 20% error. The Young's modulus determination of electric machine stators can't be treated as homogeneous structure.

Table 2 Effect of Young's Modulus on the Resonant Frequencies of the 2nd Mode Shapes

	E	ρ_1	ρ_2	ν	f_2	f_2 (error %)	f_2'	f_2' (error %)
1)	**1.521**	**20.29**	**7.8×0.935**	**0.3**	**1563.8**	**4.5**	**2578.1**	**-2.9**
2)	2.07	20.29	7.8×0.935	0.3	1824.3	21.9	3007.7	13.3
3)	1.521	20.29	7.8×0.935	0.27	1562.9	4.4	2577.4	-2.9
4)	1.521	20.29	7.8×0.935	0.34	1565.1	4.6	2579.2	-2.9
5)	1.521	20.29	7.8×0.935	0	1558.6	4.1	2573.2	-3.1
6)	1.638	20.29	7.8×0.935	0.27	1621.9	8.3	2674.7	0.7
7)	1.371	20.29	7.8×0.935	0.34	1485.9	-0.7	2448.7	-7.8
8)	2.047	20.29	7.8×0.935	0	1808.1	20.8	2985.7	12.5
9)	1.627	20.80	7.8	0.3	1586.4	6.0	2608.6	-1.7
10)	2.07	7.8	7.8	0.3	2344.9	56.6	3621.3	36.4
11)	2.07	7.8×0.935	7.8×0.935	0.3	2425.1	62.0	3745.0	41.1
12)	1.521	7.8×0.935	7.8×0.935	0.3	2078.8	38.9	3210.2	20.9
13)	1.521	28.59	7.8×0.935	0.3	1383.5	-7.6	2326.7	-12.4

Note: The measured 2nd resonant frequencies are 1497Hz and 2655Hz respectively (see also the next section), which are used as basis for calculating the percentage errors.
E = Young's modulus ($\times 10^{11}$ N/m^2)
ρ_1 = mass density of stator pole ($\times 10^3$ kg/m^3)
ρ_2 = mass density of stator core lamination ($\times 10^3$ kg/m^3)
ν = Poisson's ratio
f_2, f_2' = 2nd mode resonant frequencies
0.935 is the stator core lamination stacking factor (generally it is 0.92~0.95)

Rows 9) reveals the effect of stator core lamination stacking factor on the resonant frequency. The properties of stacked lamination are definitely different from that of iron or steel, as can be seen from the Young's modulus measurements. The existence of stacking factor will change the mass of the object, hence the resonant frequencies.

Rows 10) and 11) are the calculated results with commonly used Young's modulus without considering winding effects [13]. This is a commonly used method during the electric machine design stage because of the lack of winding parameters. The results reveal the fact that winding effects cannot be neglected, otherwise unacceptable errors will occur. In row 10) errors of 56.6% and 36.4%

occur when neglecting the stacking factor, and in row11) errors are 62.0% and 41.1% when considering the stacking factor. The calculation results are still unacceptable if winding effects are not considered, even though a correct Young's Modulus is used, as seen in row 12), where errors are 38.9% and 20.9% for the two 2nd mode frequencies.

Row 13) describes an easily made mistake when determining the equivalent mass density of the stator pole — using the total wire diameter instead of the bare wire diameter (notice that there is an insulation layer for winding copper wires), hence increasing the equivalent mass and lowering the resonant frequencies.

VI. Study on a 2nd SRM

A 4kW 8/6 SRM is used as a 2nd motor in this paper to study the effects of material properties. Fig.8 shows the 3-D FE model and a 2nd mode modal shape (end-bells and rotor are not included). Different materials properties are used in the FE model to calculate the 2nd mode resonant frequency, and then compare to the measured one. The measured 2nd mode resonant frequency is 1060.4Hz. This is used as a base for calculation error percentage. Table 3 shows FE calculation results with different material properties.
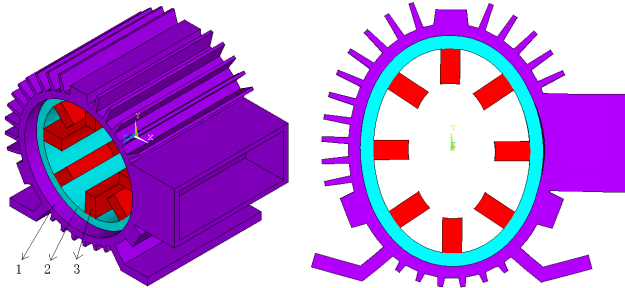
(1—casting iron frame; 2—lamination, 3—equivalent material with winding effect considered)

Fig.8 3-D FE Model and 2nd Mode Shape

As compared in Table 3, Case 3 (measured Young's Modulus 1.521×10^{11} N/m^2 used, with winding effect, with stacking factor considered) doesn't change the resonant frequency as much as that in the 12/8 SRM described above. This means that the Young's modulus is less important for the determination of resonant frequencies in the SRM with a cast iron frame. Case 4 shows again that winding effect has to be considered.

Table 3 Effect of material properties on Oulton SRM

	Case 1	Case 2	Case 3	Case 4
f_2 (Hz)	1040.6	1054.9	1027.9	1369.7
f_2 (Error %)	-1.9	-0.5	-3.1	29.2
E_1 ($\times 10^{11}$ N/m^2)	1.6	1.6	1.6	1.6
ν_1	0.275	0.275	0.275	0.275
ρ_1 ($\times 10^3$ kg/m^3)	7.8	7.8	7.8	7.8
E_2 ($\times 10^{11}$ N/m^2)	2.07	2.07	1.521	2.07
ν_2	0.3	0.3	0.3	0.3
ρ_2 ($\times 10^3$ kg/m^3)	18.59	18.083	18.083	7.293
E_3 ($\times 10^{11}$ N/m^2)	2.07	2.07	1.521	2.07
ν_3	0.3	0.3	0.3	0.3
ρ_3 ($\times 10^3$ kg/m^3)	7.8	7.293	7.293	7.293

Note:
Case 1 — Commonly used material properties, with winding effect;
Case 2 — Commonly used material properties, with winding effect, with stacking factor considered;
Case 3 — Measured Young's Modulus used, with winding effect, with stacking factor considered;
Case 2 — Commonly used material properties, without winding effect, with stacking factor considered.

VII. Conclusions

A convenient and nondestructive method for the measurement of Young's modulus of stacked lamination in electric motor stators is introduced in this paper. It shows the importance of Young's modulus in the calculation of stator resonant frequencies. The commonly used Young's modulus value is not appropriate for the calculation of resonant frequencies in the motor without a frame, however it produces lower errors for the motor with frame. The results are validated by a 3-D FE calculation together with force hammer vibration tests.

The Poisson's ratio is important for determining the Young's modulus using ultrasonic tests. It is necessary to include the stator core lamination stacking factor, the winding effects, together with appropriate Young's modulus value for resonant frequencies determination during the design phase of SRMs.

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