Resonance Frequencies and Mode Shapes of Switched Reluctance Motors

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Abstract -The 3D vibration modes and resonant frequencies of s switched reluctance motor(SRM) are demonstrated and calculated for low acoustic noise design and control of the SRMs in the paper. The effects of different frames on vibration modes and resonant frequencies are compared. Accelerometer test results of a 4kW SRM with 8/6 poles verify some of the numerical computations.

I. INTRODUCTION

Switched reluctance motor drives are being used in domestic and industrial applications from power steering to washing machines to traction. They are based on their advantages over competitive motors in manufacturing, reliability and robustness as well as lifetime. Their wide application is mainly obstructed by two disadvantages: acoustic noise and torque ripple.

Analytical and experimental results have widely confirmed a magnetic origin as the dominant noise source of the SRM[1~5]. When the phase windings of the SRM conducts current, the magnetic attraction between stator and rotor poles is produced periodically. While the tangential component of the forces on pole surfaces contributes to torque production, the radial magnetic attractions cause radial deformation of the SRM stator. The latter leads to stator vibration and acoustic noise. The phenomenon becomes particularly severe when the frequency of vibration approaches or coincides with that of the mechanical resonance of the stator[3]. The resonant frequencies with the low order mode shapes, especially 0^{th}, 2^{nd} and 4^{th} order, play an important roll in vibration production of stator in a 4 phase SRM with 8/6 poles. To suppress the acoustic noise in the SRM, smoothing or shaping phase current and switch angle control as well as active control strategy was presented earlier[3,4,6]. From the viewpoint of both design and control, it is essential to predict the natural resonant frequencies of the stator accurately, to understand the effect of different factors on these frequencies.

With the aid of the structural finite element method and elasticity theory, 3D vibration modes and their corresponding resonant frequencies are calculated for a 4kW 8/6 SRM with 4 phases. The modal analysis of stator lamination stacks with smooth frame and with ribbed frame is performed. The effects of the encased and inner ribbed frame on the vibration modes and the resonant frequencies of SRM stator are compared. Accelerometer tests on a 4kw SRM verify some of the numerical results.

II. NUMERICAL MODELS OF STATOR VIBRATION

Analysis Basis

The analysis is based on the 3D elasticity theory. The Hamilton's principle is introduced to determine the motion equation of the vibration system[14]:

$$\int_{2}^{2}[\delta(T-V)+\delta W_{nc}]dt=0 \qquad (1)$$

where the potential energy U and the kinetic energy T in the 3D solid of volume V enclosed by surface S at time t can be expressed as the function of the independent displacements and the velocity, respectively. The virtual work δW_{nc} in Equation (1), done by the non-conservative forces on the surface S, vanishes in determining the natural frequencies and mode shapes of free vibration of the SRM stator. The motion equation for an undamped system can be expressed in matrix notation as follows when the free vibration is periodic form:

$$([\mathbf{K}]-\omega_i^2[\mathbf{M}])\{\phi\}_i=\{\mathbf{0}\} \qquad (2)$$

where the structure stiffness matrix [K] including prestress effects is positive definite or positive semi-definite, and the inertia matrix [M] is positive definite. They are formed from the elementary stiffness matrix and the elementary inertia matrix, respectively[7]. Eigenvector $\{\phi\}_i$ represents the mode shape at the ith natural angular frequency. Obviously, the condition that the Equation (2) has a non-trival solution is that the determinant of coefficients should vanish, i.e.

$$\det([\mathbf{K}]-\omega_i^2[\mathbf{M}])=0 \qquad (3)$$

The polynomial equation has n engenvalues, i.e., roots ω^2_i (i=1,2,...,n), which correspond n natural frequencies:

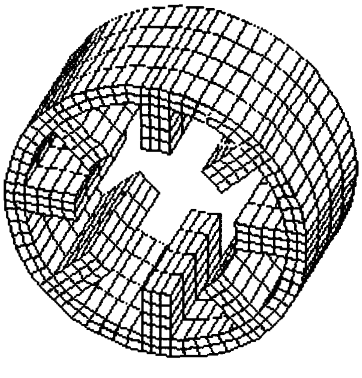
$$f_i=\frac{\omega_i}{2\pi}, \qquad i=1,2,\ldots,n \qquad (4)$$

Furthermore, the engenvectors $\{\phi\}_i$ in Equation (2) can be solved up to n vectors where n is the degree of freedom of the system.

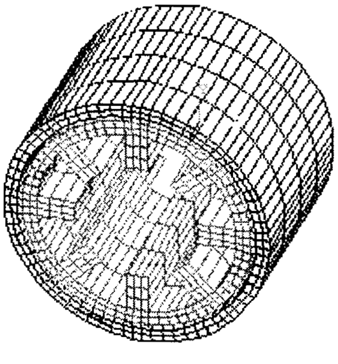
Computation Models

Five basic models are built in the paper to study and access the resonant frequencies and mode shapes of the SRM stator. These models shown in Fig.1 represent the structure of SRM stators in different industrial applications. The computation results are compared and analyzed to find the effects of the various mechanical constructions on the vibration behavior of the stator.

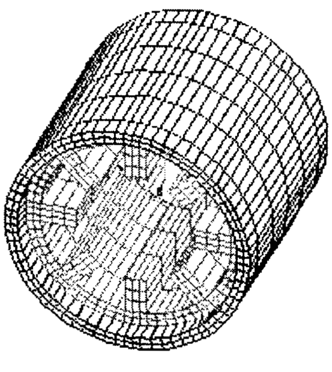
All the models in the paper are based on 3D finite elements. Hexahedral elements are chosen in all models to guarantee the free deformation in all degrees of freedom. The model I in Fig.1(a) is the stator lamination stack; Model II has the longitudinal keys between the lamination stack and the smooth frame; The lamination stack with s perfect friction contact to the smooth frame and with different frame length is in model III; The effects of a ribbed frame is considered in model IV; Finally model V includes the details of a ribbed frame structure with terminal box.



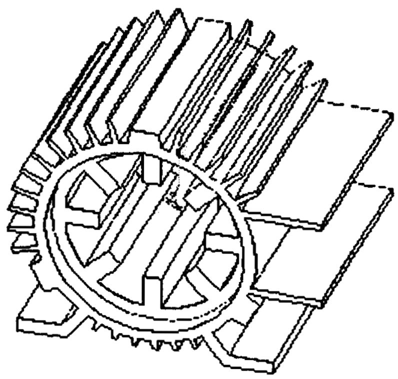
(a) Model I



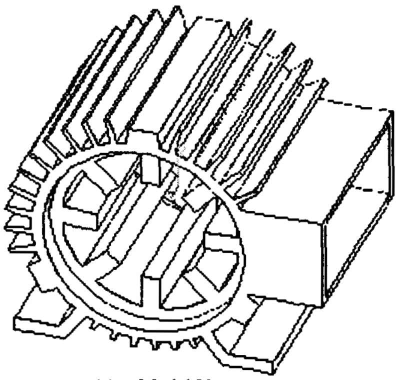
(b) Model II



(c) Model III



(d) Model IV



(e) Model V

Fig. 1 Different three dimensional models

III. Computation and Measurement Results

Numerical Results of Model I

In Fig.1(a), the stator stack without frame is modeled. The effects of both length and radius of stator yoke on vibration modes and frequencies are presented here. The results show only a slight influence of the length of the stack on the natural frequencies of the first several in-plane mode shapes. However, there are large effects on the modes and the frequencies of stack of the rest 3D vibration. The resonant frequency of the stator stack will decrease roughly linearly with the increase of the yoke radius. Furthermore, the existence of the stator poles will lower the frequencies of the first few order modes, and increase the higher order natural frequencies.

The Effects of the Frame of a Type in Model II

This stator model is composed of the lamination stack and the frame. The stack is installed with the use of longitudinal keys on the inner surface of a smooth frame, shown in Fig1(b). The stack and the frame are made of the different materials. The calculation results demonstrate: (1)

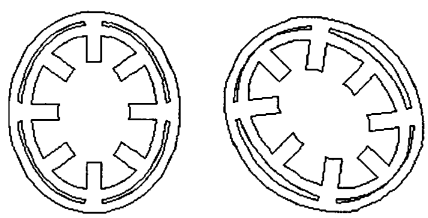
The frequencies of in-plane mode shapes may decrease or increase with the existence of this frame. (2) The same resonant frequency of some mode shapes in model I (say 823Hz of the 2nd order mode) become separated from each other in model II, shown in Fig.2. (3) Some extra vibration modes appear with the condition.

The Effects of the Frame of a Type in Model III

When the lamination stack is installed into a cylindrical smooth frame, the finite element model can be built shown in Fig.1(c). As long as the stiffness of the frame is sufficient, the resonant frequencies of in-plane vibration modes will rise monotonously with the increase of yoke thickness. The calculations also show a slight decrease of the resonant frequency of in-plane modes with the increase of the length difference between the lamination stack and the frame. The decrease in the frequencies is less than 2 % for a conventional range of the frame lengths. The thicker the frame, the higher the resonant frequencies.

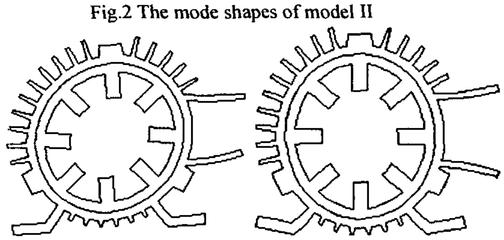
The Effects of the Ribbed Frame in Model IV and Model V

The resonant frequencies of the lower order modes are reduced with the addition of ribs to the in Fig.1(d), but the modal frequencies of much higher order are raised by the ribs. This demonstrates that the effect of the ribs is like an extra mass to the lower order vibrations and adds stiffness to the higher order vibrations. This conclusion contradicts some analytical formulas[10,12]. Here only the mode shapes of the 2nd order vibration are shown in Fig.3. It is not difficult to find the difference between Fig.2 and Fig.3. In addition, the existence of the ribs introduces many complex vibration mode shapes. All the results have been extracted through modal analysis.



(a) Mode at 776.6Hz (b) Mode at 952.7Hz

Fig.2 The mode shapes of model II



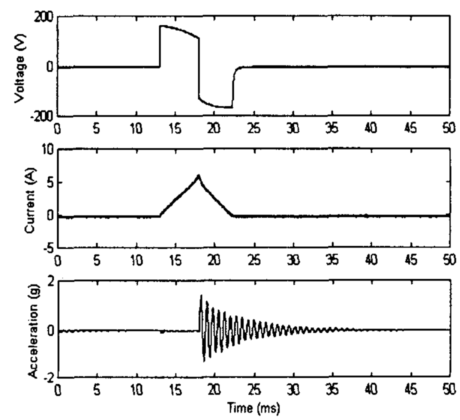
(a) Mode at 1275Hz (b) Mode at 1357Hz

Fig.3 Mode shapes with 2nd order deformation

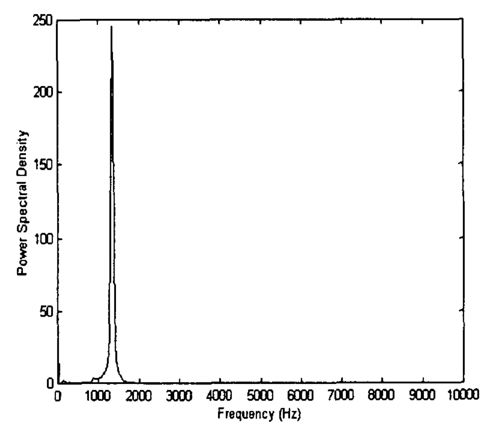
If the terminal box is added in Fig.1(d), the model IV become model V, shown in Fig.1(e). The results show that the mode shapes have additional changes, i.e., the every mode shape with integer order is accompanied by some rib deformation. The frequencies of some in-plane modes with low orders decrease slightly. In the paper, over 80 mode shapes are studied and compared with the results from Model IV.

Measurement Results

To validate the calculation results of the numerical model, the vibration of the 2nd order mode is measured with an accelerometer. The measurement curves of the voltage, current and acceleration are given in Fig.4(a), and the power spectrum density vs. frequency is obtained by FFT shown in Fig4(b). The measured resonant frequency of 1340 Hz is quite near the calculated result of 1349Hz in Model V. Actually the numerical result of 1357Hz from Model IV is accurate enough to satisfy the practical requirements. The use of Model III allows an error of less than 5%, but the error will become unacceptable if the frame is not included in the model.



(a) Voltage, current and accleration vs. time



(c) Power spectrum density vs. frequency

Fig.4 The measurement results

IV. CONCLUSION

Five basic stator structures of SRM are modeled to study and compare the effects of the different structures on the mode shapes and the resonant frequencies of SRM stator under free vibrations. The results from the finite element calculation are demonstrated, and the measured values of a real SRM validate the accuracy of numerical calculations. Several conclusions are drawn on the effects of stator structures on the vibration behavior.

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