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Cooperative localization against GPS signal loss in multiple UAVs flight

Yaohong Qu[1,2] and Youmin Zhang[2,*]

1. School of Automation, Northwestern Polytechnical University, Xi'an 710072, P. R. China;
2. Department of Mechanical and Industrial Engineering, Concordia University, Montreal H3G 1M8, Canada

Abstract: Based on multiple unmanned aerial vehicles (UAVs) flight at a constant altitude, a fault-tolerant cooperative localization algorithm against global positioning system (GPS) signal loss due to GPS receiver malfunction is proposed. Contrast to the traditional means with single UAV, the proposed method is based on the use of inter-UAV relative range measurements against GPS signal loss and more suitable for the small-size and low-cost UAV applications. Firstly, for re-localizing an UAV with a malfunction in its GPS receiver, an algorithm which makes use of any other three healthy UAVs in the cooperative flight as the reference points for re-localization is proposed. Secondly, by using the relative ranges from the faulty UAV to the other three UAVs, its horizontal location can be determined after the GPS signal is lost. In order to improve an accuracy of the localization, a Kalman filter is further exploited to provide the estimated location of the UAV with the GPS signal loss. The Kalman filter calculates the variance of observations in terms of horizontal dilution of positioning (HDOP) automatically. Then, during each discrete computing time step, the best reference points are selected adaptively by minimizing the HDOP. Finally, two simulation examples in Matlab/Simulink environment with five UAVs in cooperative flight are shown to evaluate the effectiveness of the proposed method.

Keywords: unmanned aerial vehicle (UAV), cooperative localization, Kalman filter, horizontal dilution of positioning (HDOP).

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1. Introduction

Nowadays, multiple unmanned aerial vehicles (UAVs) and aerial robotics are increasingly used in military and scientific research field [1]. Among these applications, active cooperation of several low-cost small-size UAVs has important advantages [2]. Some relevant research results have been published in recent years. Examples include spacecraft formation flight [3–5], UAV formation flight [6–8], coordinated rendezvous of UAVs [9], coordinated path planning [10], task coordination of UAVs [10,11], etc. All these researches are based on the premise that the navigation systems are working in normal and good conditions. As it is well-known, reliability is an important concern in safety-critical systems including aircrafts and UAVs. In addition to the extensive investigations and developments in the area of fault-tolerant control (FTC) systems [12], there are numerous publications recently investigating on fault-tolerant navigation and localization methods for single UAV which are based on integration of inertial navigation system (INS) and global positioning system (GPS), e.g., see [13,14]. However, considering constraints on the cost and payload size, many small-size UAVs or micro UAVs are equipped with GPS only for localization and navigation [15,16]. Therefore, GPS transient failures, which may occur in certain scenarios [17], may have catastrophic effects on UAVs during mission flight.

With the rapid development of wireless communication techniques, a larger amount of information available can be utilized and shared by each of multi-UAV working cooperatively, including relative range measurements which provides relative positioning between UAVs. The relative range measurements can be obtained using either ultra-wideband (UWB) radio technology or optical systems. Fault-tolerant localization for UAVs using inter-UAV range measurements was initially studied in [17]. Since only one of the distance measurements was utilized by the method in [17], the localization accuracy was not good enough. This fact motivates this paper to investigate the fault-tolerant localization problem of UAVs with more information in multi-UAV flight conditions. Similar to the principle of GPS, in which the user's location can be determined by ranges between the positioning satellites and the user, the location of an UAV in multi-UAV cooperative flight can be calculated through measuring the relative ranges from one UAV to other neighboring UAVs at known locations. Although the localization accuracy of UAV is not better than positioning satellites, the use of Kalman

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*Corresponding author.
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filtering can help to improve the accuracy since it is not difficult for us to establish an UAV's motion model [18]. Nevertheless, the accuracy is still not good enough for the localization and navigation of the UAV. Therefore, by taking the calculation results as observation data, an adaptive Kalman filter algorithm for the UAV in the presence of GPS receiver malfunction is also proposed in this paper. Meanwhile, in order to further improve the accuracy of estimated location, selection of the reference points is also optimized.

2. Calculation of the location

2.1 Problem formulation

Assume that multiple UAVs, which are equipped respectively with a GPS receiver, are flying at a same constant altitude, and an UAV's GPS receiver does not work properly due to a failure during the flight. Furthermore, it is assumed that the UAV with faulty GPS can still measure the relative distances to other UAVs by inter-UAV range measurements. According to the composition of motions of a rigid body, one knows that in a two-dimensional (2D) plane, the location of an UAV with malfunction can be calculated by any three other UAVs which are non-collinear and whose locations are known. Similar to the principle of GPS positioning, the three UAVs can be taken as the "positioning satellites" and the UAV with malfunction as the "user". Thus, in principle, when an UAV's GPS does not work, it can still be located by the other three UAVs. Here, a cooperative localization example with four UAVs is shown in Fig. 1.

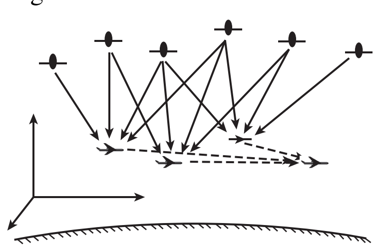

Fig. 1 Cooperative localization example with four UAVs

The basic steps of the proposed method in this paper are as follows. Firstly, the location of an UAV in the presence of malfunction is calculated using the relative range measurements based on any other three UAVs at known locations (reference points). Then, to reduce the measurement noises and uncertainties, a Kalman filter is applied to the UAV by taking the calculated result as observation. We assume that the calculated result is Gauss random distribution with zero mean and its variance can be estimated by the value of horizontal dilution of positioning (HDOP) in each computing step, thus the Kalman filter is adaptive. Finally, the reference points are determined by selecting the minimal HDOP in the UAV team.

2.2 Localization error equations of GPS

In inertial coordinate system, the observation equations in three-dimensional (3D) space for each satellite with known coordinates (x_i, y_i, z_i) and unknown user coordinates (X, Y, Z) are given by

$$\rho_i = \sqrt{(x_i - X)^2 + (y_i - Y)^2 + (z_i - Z)^2} + C_b \quad (1)$$

where C_b is receiver clock bias. Here, the unknown variables are X, Y, Z, C_b respectively. Thus, to obtain a 3D position, at least four satellites are required by GPS, as shown in Fig. 2.

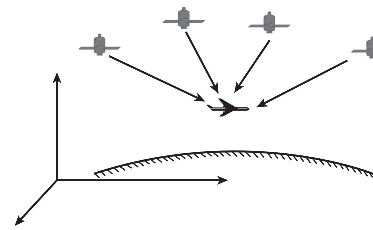

Fig. 2 A GPS user can be located by no less than four satellites

From (1), we know that it is a nonlinear equation that can be linearized using Taylor series expansion. Let the vector of ranges be $\rho = h(x)$, where a nonlinear function $h(x)$ of the four dimensional vector x represents user position and receiver clock bias, and expand the left-hand side of this equation in a Taylor series about certain nominal solution x_{nom} for the unknown vector

$$x = [X, Y, Z, C_b]^{\text{T}} \quad (2)$$

for which

$$\begin{cases} \rho = h(x) = h(x_{\text{nom}}) + \dfrac{\partial h(x)}{\partial x}\Big|_{x=x_{\text{nom}}} \delta x + H.O.T \\ \delta x = x - x_{\text{nom}} \\ \delta \rho = h(x) - h(x_{\text{nom}}) \end{cases} \quad (3)$$

where $H.O.T$ stands for "higher-order terms".

These equations become

$$\begin{cases} \delta \rho = \dfrac{\partial h(x)}{\partial x}\Big|_{x=x_{\text{nom}}} \delta x = H^{(1)} \delta x \\ \delta x = X - X_{\text{nom}},\ \delta y = Y - Y_{\text{nom}},\ \delta z = Z - Z_{\text{nom}} \end{cases} \quad (4)$$

where $H^{(1)}$ is the first-order term in the Taylor series expansion

$$\delta \rho = \rho(X, Y, Z) - \rho(X_{\text{nom}}, Y_{\text{nom}}, Z_{\text{nom}}) \approx \frac{\partial \rho}{\partial x}\Big|_{X_{\text{nom}}, Y_{\text{nom}}, Z_{\text{nom}}} \delta x + v_\rho \quad (5)$$

where v_ρ represents noise in receiver measurements. This vector equation can be written in scalar form as

$$\begin{cases} \dfrac{\partial\rho_i}{\partial X}=\dfrac{-(x_i-X_{\text{nom}})}{\sqrt{(x_i-X_{\text{nom}})^2+(y_i-Y_{\text{nom}})^2+(z_i-Z_{\text{nom}})^2}} \\ \dfrac{\partial\rho_i}{\partial Y}=\dfrac{-(y_i-Y_{\text{nom}})}{\sqrt{(x_i-X_{\text{nom}})^2+(y_i-Y_{\text{nom}})^2+(z_i-Z_{\text{nom}})^2}} \\ \dfrac{\partial\rho_i}{\partial Z}=\dfrac{-(z_i-Z_{\text{nom}})}{\sqrt{(x_i-X_{\text{nom}})^2+(y_i-Y_{\text{nom}})^2+(z_i-Z_{\text{nom}})^2}} \end{cases} \tag{6}$$

where i denotes the ith satellite with

$$i=1,2,3,4 \tag{7}$$

By combining (5) and (6), an error equation in matrix/vector format can be obtained

$$\begin{bmatrix}\delta\rho_1\\ \delta\rho_2\\ \delta\rho_3\\ \delta\rho_4\end{bmatrix}=\begin{bmatrix}\dfrac{\partial\rho_1}{\partial X} & \dfrac{\partial\rho_1}{\partial Y} & \dfrac{\partial\rho_1}{\partial Z} & 1\\ \dfrac{\partial\rho_2}{\partial X} & \dfrac{\partial\rho_2}{\partial Y} & \dfrac{\partial\rho_2}{\partial Z} & 1\\ \dfrac{\partial\rho_3}{\partial X} & \dfrac{\partial\rho_3}{\partial Y} & \dfrac{\partial\rho_3}{\partial Z} & 1\\ \dfrac{\partial\rho_4}{\partial X} & \dfrac{\partial\rho_4}{\partial Y} & \dfrac{\partial\rho_4}{\partial Z} & 1\end{bmatrix}\begin{bmatrix}\delta X\\ \delta Y\\ \delta Z\\ \delta C_b\end{bmatrix}+\begin{bmatrix}v_{\rho 1}\\ v_{\rho 2}\\ v_{\rho 3}\\ v_{\rho 4}\end{bmatrix} \tag{8}$$

or in a symbolic form

$$\delta\rho=H^{(1)}\delta x+v_k \tag{9}$$

To calculate $H^{(1)}$, satellite positions and the nominal value of the user's position are necessary. Thus, the geometric dilution of precision (GDOP) (approximately) can be calculated by the following equation

$$\delta\rho=H^{(1)}\delta x \tag{10}$$

where $\delta\rho$ and $H^{(1)}$ are known from the pseudorange, satellite position, and nominal value of the user's position. The correction δx is the unknown vector.

If both sides of (10) by $H^{(1)\mathrm{T}}$ are pre-multiplied, (10) will become

$$H^{(1)\mathrm{T}}\delta\rho=(H^{(1)\mathrm{T}}H^{(1)})\delta x \tag{11}$$

Then, pre-multiplying (11) by $(H^{(1)\mathrm{T}}H^{(1)})^{-1}$ will lead to

$$\delta x=(H^{(1)\mathrm{T}}H^{(1)})^{-1}H^{(1)\mathrm{T}}\delta\rho \tag{12}$$

If δx and $\delta\rho$ are assumed random with zero mean, and the error covariance

$$\begin{aligned} &E<(\delta x)(\delta x)^{\mathrm{T}}>= \\ &E<(H^{(1)\mathrm{T}}H^{(1)})^{-1}H^{(1)\mathrm{T}}\delta\rho\cdot \\ &[(H^{(1)\mathrm{T}}H^{(1)})^{-1}H^{(1)\mathrm{T}}\delta\rho]^{\mathrm{T}}>= \\ &(H^{(1)\mathrm{T}}H^{(1)})^{-1}H^{(1)\mathrm{T}}E<\delta\rho\delta\rho^{\mathrm{T}}>\cdot \\ &H^{(1)}(H^{(1)\mathrm{T}}H^{(1)})^{-1} \end{aligned} \tag{13}$$

The pseudorange measurement covariance is assumed uncorrelated satellite-to-satellite with variance σ^2

$$E<(\delta\rho)(\delta\rho)^{\mathrm{T}}>=\sigma^2 I_4 \tag{14}$$

a 4×4 matrix.

Substituting (14) into (13) gives

$$E<(\delta x)(\delta x)^{\mathrm{T}}>=\sigma^2(H^{(1)\mathrm{T}}H^{(1)})^{-1} \tag{15}$$

From (15), all the DOPs (including GDOP, HDOP, vertical DOP (VDOP), time DOP (TDOP)) which represent sensitivities to pseudorange errors can be obtained. In fact, we are principally interested in the diagonal elements of $(H^{(1)\mathrm{T}}H^{(1)})^{-1}$with $\sigma^2=1$ m^2.

2.3 Location and calculation

At a constant altitude flight, the relative distance between two UAVs can be expressed as

$$\rho_i=\sqrt{(x_i-X)^2+(y_i-Y)^2} \tag{16}$$

where (X,Y) are the unknown coordinates, (x_i,y_i) are known coordinates, and $i=1,2,3$ is the index of UAVs at a known location.

Taking square for both sides of (16), then we have

$$\rho_i^2=(X^2+Y^2)+(x_i^2+y_i^2)-2x_iX-2y_iY \tag{17}$$

Let

$$A=(X^2+Y^2) \tag{18}$$

Substitute the relevant terms in (17) with (18) and express each equation as

$$\begin{cases}\rho_1^2-(x_1^2+y_1^2)=A-2x_1X-2y_1Y\\ \rho_2^2-(x_2^2+y_2^2)=A-2x_2X-2y_2Y\\ \rho_3^2-(x_3^2+y_3^2)=A-2x_3X-2y_3Y\end{cases} \tag{19}$$

(19) can be expressed in vector form

$$M=Hx \tag{20}$$

where

$$M=\begin{bmatrix}\rho_1^2-(x_1^2+y_1^2)\\ \rho_2^2-(x_2^2+y_2^2)\\ \rho_3^2-(x_3^2+y_3^2)\end{bmatrix},\quad H=\begin{bmatrix}-2x_1 & -2y_1 & 1\\ -2x_2 & -2y_2 & 1\\ -2x_3 & -2y_3 & 1\end{bmatrix}$$

$$x=\begin{bmatrix} X & Y & A\end{bmatrix}^{\mathrm{T}} \tag{21}$$

By pre-multiplying both sides of (20) by H^{-1}, the following equation can be obtained

$$x = H^{-1}M \tag{22}$$

If the rank of H, the number of linearly independent columns of the matrix H, is less than three, then H will not be invertible. In this case, its determinant is given by

$$\det H = |H| = 0 \tag{23}$$

This paper only investigates the cases where any three UAVs are not collinear, which also means that H is invertible.

Considering the fact of unavoidable measurement noises and that localization accuracy of an UAV from GPS is not better than the satellite, a Kalman filter is then designed on each UAV to improve its accuracy. The location of the UAV with malfunction is calculated using the relative range measurements. Then, taking the calculation result as an observation input, another Kalman filter is applied to the UAV. Assume that the calculation result is Gauss random distribution with zero mean and its variance can be estimated by HDOP in every computing step, so the Kalman filter works adaptively. The schematic structure of the proposed cooperative localization method is shown in Fig. 3.

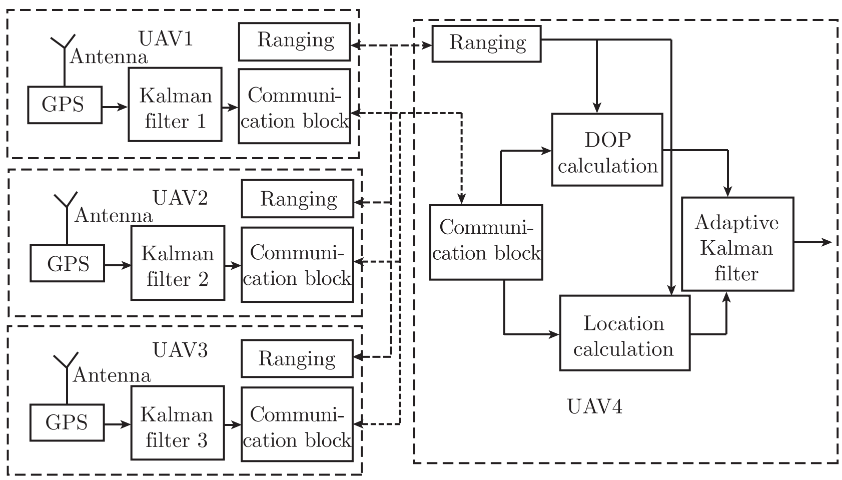

Fig. 3 Schematic structure of the proposed cooperative localization method

2.4 Error equations of multi-UAV cooperative localization

Similar to (6) and (8), the error equations of the multi-UAV cooperative localization algorithm in 2D plane can be written as

$$\left\{\begin{array}{l} \dfrac{\partial\rho_i}{\partial X} = \dfrac{-(x_i - X_{\mathrm{nom}})}{\sqrt{(x_i - X_{\mathrm{nom}})^2 + (y_i - Y_{\mathrm{nom}})^2}} \\ \dfrac{\partial\rho_i}{\partial Y} = \dfrac{-(y_i - Y_{\mathrm{nom}})}{\sqrt{(x_i - X_{\mathrm{nom}})^2 + (y_i - Y_{\mathrm{nom}})^2}} \end{array}\right. \tag{24}$$

where

$$i = 1, 2, 3 \tag{25}$$

or

$$\begin{bmatrix} \delta\rho_1 \\ \delta\rho_2 \\ \delta\rho_3 \end{bmatrix} = \begin{bmatrix} \dfrac{\partial\rho_1}{\partial X} & \dfrac{\partial\rho_1}{\partial Y} & 1 \\ \dfrac{\partial\rho_2}{\partial X} & \dfrac{\partial\rho_1}{\partial Y} & 1 \\ \dfrac{\partial\rho_3}{\partial X} & \dfrac{\partial\rho_1}{\partial Y} & 1 \end{bmatrix} \begin{bmatrix} \delta X \\ \delta Y \\ \delta C_b \end{bmatrix} + \begin{bmatrix} v_{\rho 1} \\ v_{\rho 2} \\ v_{\rho 3} \end{bmatrix} \tag{26}$$

(26) can also be written as

$$\delta\rho = H^{(1)}\delta x + v_k \tag{27}$$

In (24), the values of X_{nom} and Y_{nom} may be chosen as the estimation of model in Kalman filter.

Considering that the variances of three UAVs' locations are different after Kalman filter, the variance of δx is given by

$$E < (\delta x)(\delta x)^{\mathrm{T}} >= \\ [H^{(1)}]^{-1} \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix} ([H^{(1)}]^{-1})^{\mathrm{T}} \tag{28}$$

where σ_1^2, σ_2^2, σ_3^2 are respectively from the estimations of three Kalman filters.

Note that the small increment of the vector x is $\delta x = [\Delta X\ \Delta Y\ \Delta C_b]^{\mathrm{T}}$, therefore another form of (28) can also be written as

$$A = (A_{ij}) = E[(\delta x)(\delta x)^{\mathrm{T}}] =$$

$$\begin{bmatrix} E(\Delta X^2) & E(\Delta X\Delta Y) & E(\Delta X\Delta C_b) \\ E(\Delta Y\Delta X) & E(\Delta Y^2) & E(\Delta Y\Delta C_b) \\ E(\Delta C_b\Delta X) & E(\Delta C_b\Delta Y) & E(\Delta C_b) \end{bmatrix} \tag{29}$$

From (29), all the DOPs (including GDOP, HDOP, and TDOP) can be obtained, which represent sensitivities to relative range errors.

2.5 Design of Kalman filter

UAVs' flight can be taken as a constant velocity (CV) model in every computation instant because the computation frequency is generally so quick that the UAV's heading angle and speed can be considered nearly as constants in one computation step. Thus, we can simply a discrete motion model of UAV as

$$\chi(k+1) = \varPhi\aleph(k) + \varGamma(k)W(k) \tag{30}$$

where

$$\chi = \begin{bmatrix} x & v_x & y & v_y \end{bmatrix}^{\mathrm{T}}, \quad \varPhi = \begin{bmatrix} 1 & \Delta T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta T \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\varGamma = I_{4\times 4}, \quad W(k) = \begin{bmatrix} 0 & w_{ve}(k) & 0 & w_{vn} & (k) \end{bmatrix}^{\mathrm{T}} \tag{31}$$

and x is east location of UAV in east-north-up (ENU) coordinate system, y is north location of UAV in ENU coordinate system, v_x is east speed of UAV in ENU coordinate system, v_y is north speed of UAV in ENU coordinate system, ΔT is time of computing cycle or sampling period, w_{ve} and w_{vn} are uncorrelated Gaussian white noises.

In this paper, we assume that all the UAVs have a same model including ΔT and w_{ve}, w_{vn}.

The observations of three fault-free UAVs come from their own GPS receivers respectively. To simplify the problem, we assume that the location is provided directly in rectangular coordinate system (ENU coordinate system). Thus, the observation equation can be easily expressed as

$$\zeta(k+1) = H(k+1)\chi(k+1) + V(k+1) =$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \chi(k+1) + \begin{bmatrix} v_1(k+1) \\ v_2(k+1) \end{bmatrix} \tag{32}$$

where V is the observation noise vector.

In order to apply Kalman filters, we assume that $W(k)$ and $V(k)$ satisfy the following conditions

$$\begin{cases} E\left[W(k)\right] = 0, \\ \mathrm{Cov}<W(k), W(j)> = E < W(k)W(j)^{\mathrm{T}} >= Q(k)\delta_{kj} \\ E\left[V(k)\right] = 0, \\ \mathrm{Cov} < V(k), V(j) > = E < V(k)V(j)^{\mathrm{T}} >= R(k)\delta_{kj} \\ \mathrm{Cov} < W(k), V(j) > = E < W(k)V(j)^{\mathrm{T}} >= 0 \end{cases} \tag{33}$$

where δ_{ij} is the Dirac Delta function.

Then, the discrete Kalman filter can be implemented as

$$\begin{cases} \hat{\chi}(k+1/k) = \varPhi\hat{\chi}(k) \\ P(k+1/k) = \varPhi P(k)\varPhi^{\mathrm{T}} + Q \\ K(k+1) = P(k+1/k)H^{\mathrm{T}}[HP(k+1/k)H^{\mathrm{T}} + R]^{-1} \\ \hat{\chi}(k+1) = \hat{\chi}(k+1/k) + K(k+1)[\zeta(k+1) - \\ \qquad H\hat{\chi}(k+1/k)] \\ P(k+1) = [I - K(k+1)H]P(k+1/k) \end{cases} \tag{34}$$

As introduced above, the observation of UAV with malfunction comes from the result of location calculation. Now, we are mainly interested in the diagonal elements of (29). Because A_{11} and A_{22} will reflect the accuracy of X coordinate and Y coordinate respectively, we define $\varPi_{\mathrm{HDOP}} = \sqrt{A_{11} + A_{22}}$, which reflects the accuracy of location in the 2D plane. For the UAV with malfunction, the observation variance matrix is expressed as

$$R(k) = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix} \tag{35}$$

Consequently, an adaptive Kalman filter can be applied to the UAV.

2.6 Selection of reference points

According to the above analysis, the value of HDOP is related to the positions of reference points, i.e. the smaller HDOP, the better accuracy of calculated location. Therefore, we can take the UAV group with minimal HDOP as the reference points to calculate the location of UAV with malfunction. In fact, when there are not too many UAVs, an online enumeration method can be adopted by comparing all values of HDOP.

3. Simulation and analysis

3.1 Assumption

To validate the proposed method by simulation, the following assumptions are made:

Assumption 1 Five UAVs with 50 m/s airspeed are flying at the same altitude and their desired roll angles are respectively different. An UAV's GPS stops work during the cooperative flight but it can measure the relative ranges to other UAVs and can obtain location information of other UAVs in real-time by data link communications. The wind effect is not considered during the simulation.

Assumption 2 The accuracy of GPS is 10 m in the circular error probability (CEP). The accuracy of the range is 20 m.

Assumption 3 The computing cycle and sampling period for the system are the same as 0.02 s.

Assumption 4 In Kalman filtering implementation, the initial matrix of observation variance is

$$R = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}$$

and the variance matrix of the system model is

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix}$$

3.2 UAV's lateral dynamic model

In the simulations, the linearized lateral dynamic model of an UAV is adopted in this paper.

Let the state vector be $\chi = [\beta,\ p,\ \omega,\ \varphi]$, and the lateral continuous-time model of UAV can be written by

$$\dot{\aleph} = G(t)\aleph + F(t)u \tag{36}$$

where β denotes the sideslip angle, p represents the roll angle speed, ω is the heading angle speed, and φ represents the roll angle.

The matrices in (36) are given respectively as

$$G(t)=\begin{bmatrix} -0.343\,6 & -0.031\,29 & -0.823\,3 & 0.125\,2 \\ -66.47 & -44.61 & 5.971 & -90.57 \\ 13.23 & 1.529 & -8.712 & 2.867 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$F(t) = \begin{bmatrix} 0.070\,79 \\ 90.57 \\ -2.867 \\ 0 \end{bmatrix}$$

The sampling time is 0.02, therefore the discrete model can be written as

$$\chi(k+1) = A\chi(k) + Bu(k) \tag{37}$$

where

$$A=\begin{bmatrix} 0.991\,5 & -0.000\,572\,1 & -0.015\,08 & 0.002\,583 \\ -0.859\,5 & 0.400\,8 & 0.078\,88 & -1.191 \\ 0.227\,3 & 0.018\,57 & 0.839\,5 & 0.032\,97 \\ -0.009\,95 & 0.013\,16 & 0.000\,906 & 0.986\,3 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 0.001\,321 \\ 1.189 \\ -0.032\,5 \\ 0.013\,68 \end{bmatrix}$$

3.3 Flight trajectory calculation

In the ENU coordinate system, the locations of all the UAVs in the simulation are provided by its dead reckoning (DR), i.e.

$$\begin{cases} x = x_0 + \displaystyle\int_{t_0}^{t} V_a \sin\psi \mathrm{d}t \\ y = y_0 + \displaystyle\int_{t_0}^{t} V_a \cos\psi \mathrm{d}t \end{cases} \tag{38}$$

where (x_0, y_0) is the initial position of UAV, t_0 is start time, t is flight end time, ψ is the heading angle (equals to path angle if neglecting the sideslip angle), V_a is airspeed of UAV (constant in this paper, equals to ground speed with neglecting wind).

The discrete form of (38) can be expressed as

$$\begin{cases} x(k) = x_0 + \displaystyle\sum_{1}^{k} V_a \sin[\psi(k)]\Delta T \\ y(k) = y_0 + \displaystyle\sum_{1}^{k} V_a \cos[\psi(k)]\Delta T \end{cases} \tag{39}$$

3.4 Simulation results

Case 1 Constant roll angle control

In order to realize the above proposed algorithm, an UAV linearized lateral motion model and a control law for each UAV are applied respectively in the simulation. Fig. 4 shows the structural diagram of a roll angle control system during the flight.

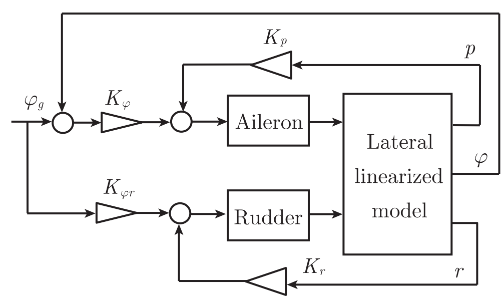

Fig. 4 Roll angle control structure of UAVs

In this paper, the control parameters in Fig. 4 are designed as

$$K_p = 0.2,\quad K_r = 0.6$$

$$K_\varphi = 1.0,\quad K_{\varphi r} = 0.3$$

In the simulation, assume that there are five UAVs in the 2D scenario, whose initial locations and desired roll angles are listed in Table 1.

Table 1 Initial location and desired roll angle of UAVs

No. of UAV	Initial location (east, north)/m	Desired roll angle/(°)
UAV	(0, 0)	28
UAV1	(−3 100, 4 000)	15
UAV2	(4 000, 4 100)	−10
UAV3	(−3 000, −4 100)	30
UAV4	(4 100, −4 000)	−16

An UAV with malfunction can be located by three other UAVs at known locations, as shown in Fig. 5.

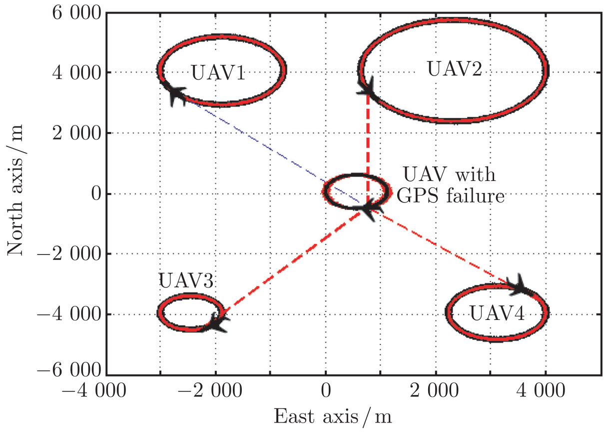

Fig. 5 Trajectories of all five UAVs

In Fig. 6, the HDOP curve of the reference UAV set and the switch curves by the proposed method are shown. The middle figure shows the minimal value of all HDOPs during flight.

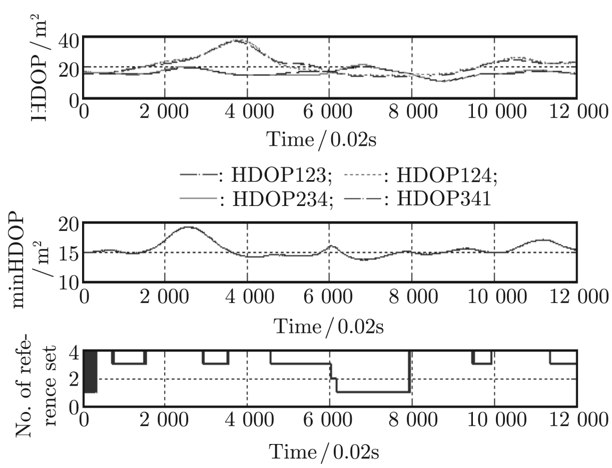

Fig. 6 HDOP curve of reference UAV set and switch curve by the proposed method

The distance error, which represents the error between the nominal trajectory and the trajectory by the proposed method, is shown in Fig. 7.

In order to show the effectiveness of the proposed method, Fig. 8 presents the distance error of the UAV with malfunction and HDOP value during flight without automatically selecting the reference points with minimal HDOP but just by taking UAV1, UAV2, and UAV3 as the reference points.

Fig. 9 presents the simulation results of flight trajectories of UAV with malfunction. The dash line is the calculated results only using the relative range measurement information and the solid line presents the results by the proposed algorithm.

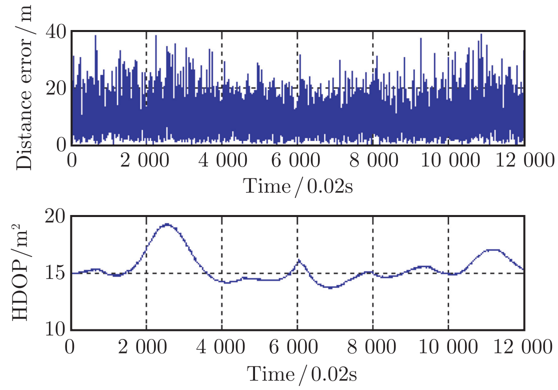

Fig. 7 Distance error of UAV with malfunction and HDOP curve by the proposed method

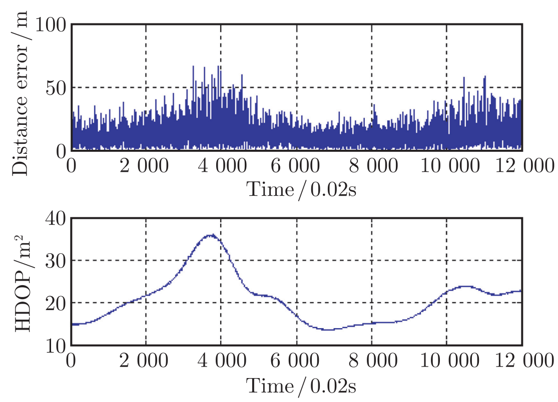

Fig. 8 Distance error of UAV with malfunction and HDOP curve by taking UAV1, UAV2, and UAV3 as the reference points

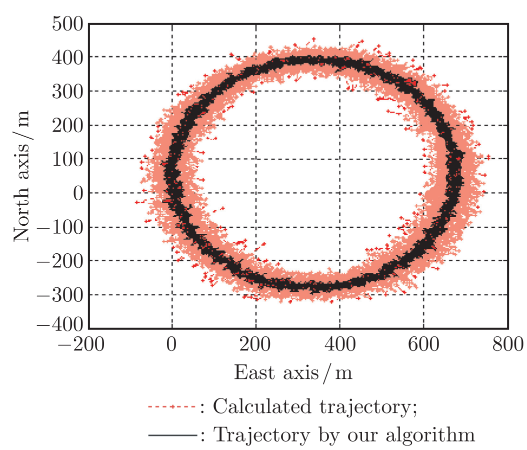

Fig. 9 Two different flight trajectories of UAV with malfunction

Case 2 Heading holder control

The control structure of heading holder during flight is shown as Fig. 10.

In this case, the following control parameters are used in simulation

$$K_p = 0.2, \quad K_r = 0.6, \quad K_\varphi = 1.0$$

$$K_{\varphi r} = 0.3, \quad K_\psi = 0.8$$

The guidance and tracking control law adopts a PID controller as

$$\psi_g = \psi_0 + K_P \Delta Z + K_I \int_0^t \Delta Z \mathrm{d}t \tag{40}$$

where the nonlinear part is expressed by

$$K_P \Delta Z + K_I \int_0^t \Delta Z \mathrm{d}t =$$

$$\begin{cases} -\dfrac{\pi}{2}, & K_P \Delta Z + K_I \displaystyle\int_0^t \Delta Z \mathrm{d}t < -\dfrac{\pi}{2} \\ \dfrac{\pi}{2}, & K_P \Delta Z + K_I \displaystyle\int_0^t \Delta Z \mathrm{d}t > \dfrac{\pi}{2} \\ K_P \Delta Z + K_I \displaystyle\int_0^t \Delta Z \mathrm{d}t, & \text{otherwise} \end{cases} \tag{41}$$

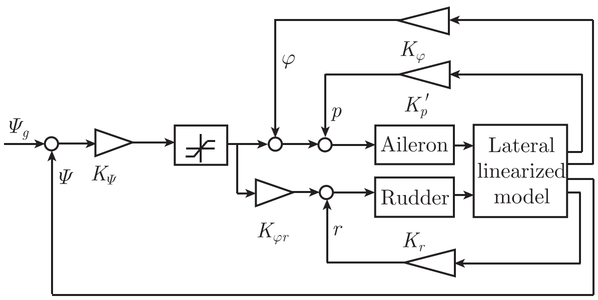

Fig. 10 Heading holder control structure of an UAV

During lateral path tracking control, the lateral deviation ΔZ is the main control signals. Let ΔZ be of the left-deviation positive and right-deviation negative. Assume that the two waypoints are A and B whose coordinates are (x_1, y_1) and (x_2, y_2) respectively. The current location of the UAV is P with coordinates (x_0, y_0), as shown in Fig. 11.

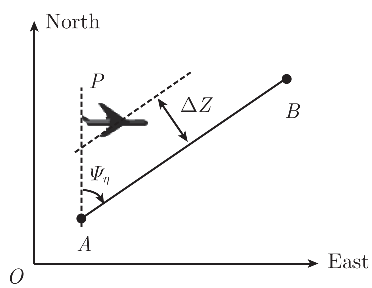

Fig. 11 Lateral deviation of UAV

Hence, the lateral deviation $|\Delta Z|$ can be calculated by

$$|\Delta Z| = \frac{|(y_2 - y_1)x_0 - (x_2 - x_1)y_0 + y_1 x_2 - y_2 x_1|}{\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}} \tag{42}$$

In ENU coordinate system, the three vectors in 3D space below are defined respectively

$$r_1 = \{x_0 - x_1, y_0 - y_1, 0\}$$

$$r_2 = \{x_2 - x_1, y_2 - y_1, 0\}$$

$$r_0 = \{0,\ 0,\ 1\}$$

Let

$$f = (r_1 \otimes r_2) r_0 =$$

$$(x_0 - x_1)(y_2 - y_1) - (x_2 - x_1)(y_0 - y_1) \tag{43}$$

Then, we can draw the following conclusions:

(i) If $f > 0$, point P is located in the planning routes on the left, $\Delta Z > 0$;

(ii) If $f = 0$, point P is located in the planning routes, $\Delta Z = 0$;

(iii) If $f < 0$, point P is located in the planning routes on the right, $\Delta Z < 0$.

Assume that five UAVs are flying in 2D horizontal plane, and the planned waypoints for the four reference UAVs are as shown in Table 2. The UAV with malfunction is flying at 15° desired roll angle with an initial location of (4 500, 2 500).

Table 2 Planned waypoints for UAVs

No. of UAV	Location of waypoints (in order)
UAV1	**1** (0, 0), **2** (0, 4 000), **3** (4 000, 4 000), **4** (4 000, 0)
UAV2	**1** (8 000, 0), **2** (4 000, 8 000), **3** (12 000, 4 000)
UAV3	**1** (7 000, 8 000), **2** (3 000, 9 000), **3** (8 000, 14 000)
UAV4	**1** (2 000, −4 000), **2** (6 000, −4 000), **3** (7 000, −7 000) **4** (5 000, −9 000), **5** (1 000, −6 000)

Assume that the GPS receiver of one UAV does not work due to a failure. This leads to the loss of GPS signals. Furthermore, in order to imitate measurement noises in GPS observation and range measurements, a white noise with zero mean and variance of (15 m)^2 has been added to the nominal trajectory and the white noise with zero mean and variance of (20 m)^2 is added to the nominal relative range measurements. The flight speed of all UAVs is 50 m/s. Fig. 12 shows the trajectories information of five

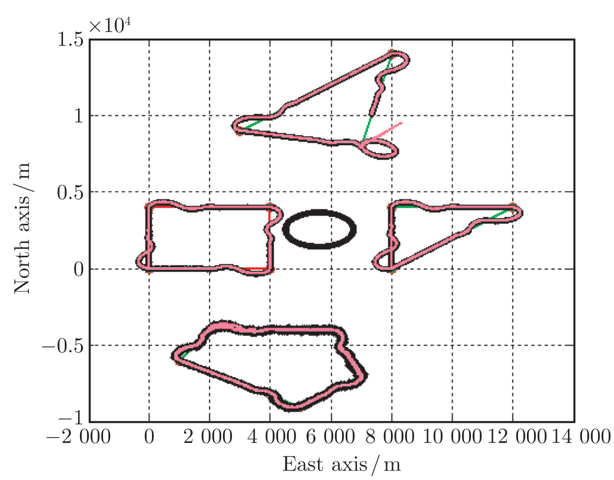

Fig. 12 Trajectories of all five UAVs

UAVs (including planned routes, nominal trajectories, observations, and results from KFs; The circle denotes the trajectory of the UAV after GPS receiver fails). The localization accuracy of the malfunction UAV through the proposed method is illustrated in Fig. 13.

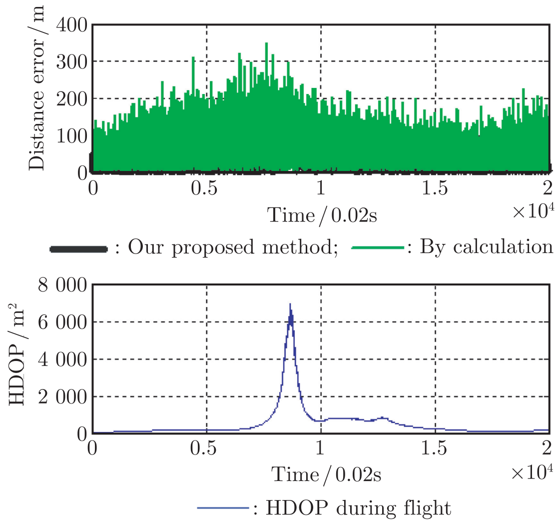

Fig. 13 Localization accuracy of the UAV with malfunction

3.5 Results analysis

From Fig. 6, one can also see that the proposed method can calculate the location of a UAV with malfunction in terms of the minimal HDOP. In other words, the UAV with malfunction can be relocated by adaptively switching other three reference UAVs during the cooperative flight.

Comparing Fig. 7 with Fig. 8, it can be observed that the trajectory distance error by the proposed method is within 50 m and the mean error is actually 10.2 m. However the trajectory distance error using constant reference points is within 100 m and the mean is 12.8 m. Therefore, two conclusions can be drawn. First, the localization accuracy of the UAV is related to the HDOP, and the smaller HDOP is, the better accuracy of the calculated location can get; Second, since the option of reference points minimizes the HDOP, the proposed method is actually optimal.

Fig. 9 shows that the trajectory of the UAV with malfunction using the proposed method is much accurate than the result using the relative ranges calculation directly.

By comparing the two figures in Fig. 13 one can observe that the location error curve goes up with the increase of HDOP. Because the proposed algorithm can choose the reference positioning points automatically, better localization accuracy has been achieved.

4. Conclusion

Based on the principle of GPS, this paper proposes a novel fault-tolerant cooperative localization algorithm for low-cost small-size UAVs in the presence of malfunction in GPS receiver. During the cooperative flight of multiple UAVs, and based on the assumption that the inter-UAV relative range measurements to the malfunctioning UAV are still available, the location of the UAV with GPS signal loss can be re-localized with the effective use of inter-UAV relative range measurements of other healthy UAVs in cooperative flight, with the help of an adaptive Kalman filter.

To show the effectiveness and benefits of the proposed method, the two simulation examples in different cases are given by Matlab/ Simulink simulations.

Although the assumption is of that all the UAVs are flying at a same constant altitude for the purpose to develop the proposed method in 2D plane, the proposed method can also be easily applied into the 3D space, which is one of our future works.

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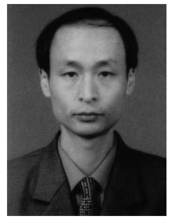
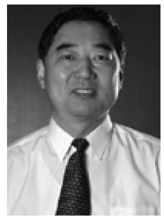
Biographies



Yaohong Qu was born in 1971. He obtained his Ph.D. degree in 2006 and is currently a lecturer in Northwestern Polytechnical University in China. His research interests are navigation & flight control of UAV, multi-UAV cooperative control and wind field estimation in flight.
E-mail: qyh0809@nwpu.edu.cn



Youmin Zhang is an associate professor at the Department of Mechanical and Industrial Engineering of Concordia University in Canada. He obtained his Ph.D. degree in 1995 from Northwestern Polytechnical University. His main research interests are in the areas of avionics and flight control of manned and unmanned aerial vehicles, and fault diagnosis, fault-tolerant control and cooperative control with applications to aerospace systems.
E-mail: ymzhang@encs.concordia.ca