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BER analysis of space–time diversity in CDMA systems over frequency-selective fading channels

A. Assra[1] *W. Hamouda*[1] *A.M. Youssef*[2]

[1]*Department of Electrical and Computer Engineering, Concordia University, Montreal, Quebec H3G 1M8, Canada*
[2]*Concordia Institute for Information Systems Engineering, Concordia University, Montreal, Quebec H3G 1M8, Canada*
E-mail: hamouda@ece.concordia.ca

Abstract: The performance of direct-sequence code division multiple access (DS-CDMA) using space–time spreading system, over frequency-selective fading channels, is investigated. The underlying transmit diversity scheme, previously introduced in the literature, is based on two transmit and one receive antenna. It was shown that when employed in flat fast-fading channels, the received signal quality can be improved by utilising the spatial and temporal diversities at the receiver side. We study the problem of multiuser interference in asynchronous CDMA systems that employ transmit/receive diversity using space–time spreading. To overcome the effects of interference, a decorrelator detector is used at the base station. Considering binary phase-shift keying transmission, we analyse the system performance in terms of its probability of bit error. In particular, we derive the probability of error over frequency-selective Rayleigh fading channels for both fast and slow-fading channels. For the fast-fading channel, both simulations and analytical results show that the full system diversity is achieved. On the other hand, when considering a slow-fading channel, we show that the scheme reduces to conventional space–time spreading schemes where the diversity order is half of that of fast-fading.

1 Introduction

Multi-input multi-output (MIMO) systems allow the receiver to see independent versions of the information yielding to spatial diversity and/or coding gain when compared with single antenna systems. One approach that uses multiple transmit antennas and, if possible, multiple receive antennas to provide reliable and high data rate communication is space–time coding (STC) [1]. It has been shown that STCs can offer these gains by introducing both temporal and spatial correlation into the transmitted signals from different antennas without increasing the total transmitted power or transmission bandwidth [1]. Depending on the structure of the used STC, one can achieve a coding gain and/or diversity gain [1, 2]. There are two major STC techniques: space–time trellis codes (STTC) [1] and space–time block codes (STBC) [2, 3]. When compared with STTC, STBC has the advantage of less complexity while achieving the same diversity gain.

Code division multiple access (CDMA) is seen as one of the generic multiple access schemes in the second and third generations of wireless communication systems. On the other hand, despite its promises, CDMA systems have fundamental difficulties when utilised in wideband wireless communications. As the system bandwidth increases, there are more resolvable paths with different delays. Hence, the received CDMA signals suffer from interchip interference (ICI), causing significant cross correlation between users' signature waveforms. Multiuser detection is considered as a promising solution to the mutual interference problem in wireless communications [4]. Multiuser receivers such as the decorrelator and the minimum mean square error (MMSE) detectors provide performance enhancement by suppressing the multiple access interference (MAI) and resisting the near-far problem [4, 5]. The performance of DS-CDMA systems of asynchronous multipath fading channels has been previously investigated in [6–8] (and references therein). For instance, the authors in [6] have proposed a modified waveform for signal spreading as opposed to the single-chip waveform. In asynchronous DS-CDMA systems over Ricean channels, the authors in [7] have introduced an accurate bit-error analysis for binary phase-shift keying (BPSK) transmission. On the other hand, the performance of DS-CDMA over flat Hoyt fading channels using random spreading sequences has been investigated in [8].

In [9], the authors have proposed a STTC for fast-fading channels where the fading coefficients change independently from one symbol to another. In their work, an exhaustive search is used to find codes that satisfy Tarokh *et al.* [3] design guidelines. Similarly, the authors in [10] have proposed a space–time spreading scheme for direct-sequence CDMA systems over flat fast-fading channels. The proposed scheme was shown to satisfy its orthogonality condition using two codes per user. The performance of this scheme was later investigated in a multiuser system for the case of two transmit and one receive antenna configuration. Other related works on the performance of STC in DS-CDMA systems include [11–18].

In a MIMO CDMA with large levels of multiuser interference due to the non-orthogonality of users' spreading codes, one has to rely on multiuser detection techniques to compensate for the loss because of signals correlations. As the use of the optimum maximum-likelihood (ML) detector is impractical because of the computational complexity that grows exponentially with the number of user and antennas, here employ the decorrelator multiuser detector. This detector is known to achieve a performance close to the ML detector but with lower computational complexity. The receiver in this case is a rake-type receiver which exploits the path diversity inherent in multipath propagation. Different from [12], here we consider a wideband CDMA transmission. The channel is modelled as a frequency-selective fading where a RAKE receiver is incorporated. Previous works on MIMO CDMA systems either consider a single user with no multiple-access interference [11], or focus on the design of the receiver side [13, 15, 16, 18]. Here we focus on the performance analysis of MIMO CDMA systems over both slow and fast frequency-selective fading channels.

In this paper, we derive the probability of error for the space–time spreading (STS) scheme introduced in [12] in DS-CDMA system over frequency-selective fading channels. In our analysis, we obtain the probability density function (pdf) of the signal-to-noise ratio (SNR) at the decorrelator output and after signal combining. This pdf is then used to evaluate the probability of bit error as a function of the system parameters for the two transmit and M receive antenna configuration and a multipath channel with L resolvable paths. Simulation results, for different system loads and number of paths, confirm the accuracy of the derived BER. Both the simulation and analytical results confirm that the full diversity order, of *4ML* for the fast-fading channel and *2ML* for the slowly-fading channel, is achieved.

The remainder of this paper is organised as follows. The following section describes the DS-CDMA system model over frequency-selective fast-fading channels. In Section 3, the performance analysis is developed for the multiuser system where we obtain the pdf of the SNR at the decorrelator output and after signal combining. In Section 4, we derive the probability of bit error. Both simulations and analytical results are presented in Section 5. Finally, conclusions are drawn in Section 6.

2 Multiuser system model

Throughout our analysis, we consider an uplink transmission for a DS-CDMA system with K users. The system employs two transmit antennas at the transmitter side and M receive antennas at the receiver side. We consider the STS system proposed in [10]. This scheme can be summarised as follows. Assuming x_1 and x_2 are data symbols assigned to each user in two consecutive symbol intervals, the STC signals transmitted during the first transmission period from antenna 1 and 2 are $x_1^* s_1 + x_2^* s_2$ and $x_1 s_2 - x_2 s_1$, respectively, where s_1 and s_2 are the spreading codes. These STC signals are switched with respect to the antenna order during the second transmission period. We also consider a frequency-selective fast-fading channel and BPSK transmission. The channel in this case is fixed for the duration of one symbol period and change independently from one symbol to another. Later, we consider the case of slow-fading channel where the fading coefficients are fixed for the duration of at least two symbol periods. For sake of simplicity, in what follows, we assume each user's signal travels through a multipath channel with L paths per transmit antenna. The low pass equivalent of the received signal at the mth receive antenna can be expressed as (see Fig. 1)

$$\begin{aligned} r_m(t) = \sum_{k=1}^{K}\sum_{l=1}^{L} & s_1^k(t-\tau_k-\tilde{\tau}_l)u_{1l,m}^{k,t} + s_2^k(t-\tau_k-\tilde{\tau}_l)u_{2l,m}^{k,t} \\ & + s_1^k(t-T_b-\tau_k-\tilde{\tau}_l)u_{1l,m}^{k,t+T_b} \\ & + s_2^k(t-T_b-\tau_k-\tilde{\tau}_l)u_{2l,m}^{k,t+T_b} + n_m(t) \end{aligned} \quad (1)$$

where $u_{1l,m}^{k,t} = \sqrt{E_s}(h_{1l,m}^{k,t} x_1^{k*} - h_{2l,m}^{k,t} x_2^k)$, $u_{2l,m}^{k,t} = \sqrt{E_s}(h_{1l,m}^{k,t} x_2^{k*} + h_{2l,m}^{k,t} x_1^k)$, $u_{1l,m}^{k,t+T_b} = \sqrt{E_s}(-h_{1l,m}^{k,t+T_b} x_2^k + h_{2l,m}^{k,t+T_b} x_1^{k*})$ and $u_{2l,m}^{k,t+T_b} = \sqrt{E_s}(h_{1l,m}^{k,t+T_b} x_1^k + h_{2l,m}^{k,t+T_b} x_2^{k*})$. E_s is the received signal energy for the single user, x_1^k and x_2^k are the even and odd kth user data symbols, $s_1^k(t)$ and $s_2^k(t)$ are the two spreading codes assigned to the kth user with processing gain T_b/T_c, where T_b is the bit period, T_c is the chip period, and τ_k represents the transmit delay of the kth user signal which is assumed to be multiple of chip periods. $\tilde{\tau}_l$ represents the delay of each path during each transmission period which is modelled as an integer number of chips assumed to be much smaller than the symbol period, and hence we can neglect the effect of intersymbol interference (ISI). The channel coefficients $h_{ql,m}^{k,t}$ and $h_{ql,m}^{k,t+T_b}$, $(q = 1, 2)$ model the fading channel corresponding to the kth user, lth path from the qth

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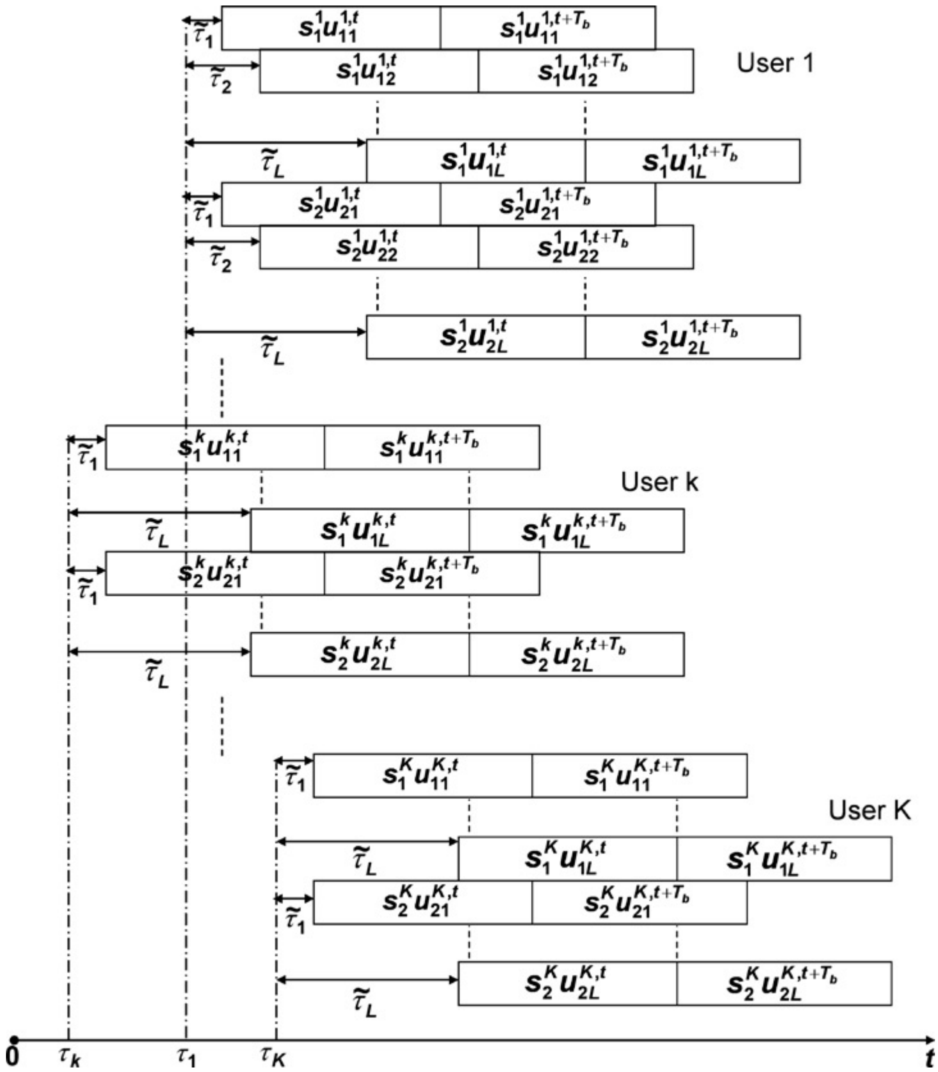

Figure 1 *Received signal for K-user system*

transmit antenna to the mth receive antenna at time t and $t + T_b$, respectively. These fading coefficients are modelled as independent complex Gaussian random variables with zero mean and unity variance. The noise $n_m(t)$ is assumed to be complex Gaussian with zero mean and variance $\sigma_n^2 = N_o/2$ per dimension. As shown in Fig. 2, the mth receiver structure consists of a bank of $2LK$ filters matched to the delayed versions of the signature waveforms of each user. Let P denote the space–time block code interval ($P = 2$ symbols in our case). The output of the mth filter bank, sampled at the chip rate during one ST-block interval is given, in a vector form, by

$$\boldsymbol{Y}_m = \boldsymbol{R}\boldsymbol{U}_m + \boldsymbol{N}_m \qquad (2)$$

The $2LPK \times 1$ vector $\boldsymbol{Y}_m$, in (2), includes the output of the matched filter bank at time t and $t + T_b$ and is given by

$$\boldsymbol{Y}_m = [y_{1,1,1}^{t,m} y_{1,2,1}^{t,m} \cdots y_{1,2,L}^{t,m}\, y_{1,1,1}^{t+T_b,m} \cdots y_{K,1,1}^{t,m} \cdots y_{K,2,L}^{t+T_{b,m}}]^{\mathrm{T}}$$

where the superscript T denotes vector transpose and $y_{k,p,l}^{t,m}$, $y_{k,p,l}^{t+T_{b,m}}$, $p = 1, 2$, represent the outputs at the mth receive antenna of the filter matched to the lth path of the pth sequence for user k at times t and $t + T_b$, respectively. The vector $\boldsymbol{U}_m$ represents the faded data transmitted to the mth receive antenna and is given by

$$\boldsymbol{U}_m = [\boldsymbol{U}_{1,m}^{\mathrm{T}} \boldsymbol{U}_{2,m}^{\mathrm{T}} \cdots \boldsymbol{U}_{k,m}^{\mathrm{T}} \cdots \boldsymbol{U}_{K,m}^{\mathrm{T}}]^{\mathrm{T}}$$

where the $2LP \times 1$ vector $\boldsymbol{U}_{k,m}$ represents the faded data transmitted by the kth user to the mth receive antenna over two successive symbols, is defined as

$$\boldsymbol{U}_{k,m} = [u_{11,m}^{k,t} u_{21,m}^{k,t} u_{12,m}^{k,t} \cdots u_{2L,m}^{k,t} u_{11,m}^{k,t+T_b} u_{21,m}^{k,t+T_b} \cdots u_{2L,m}^{k,t+T_b}]^{\mathrm{T}}$$

The $2LPK \times 2LPK$ cross correlation matrix $\boldsymbol{R}$ is given by [4]

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{R}_{11} & \boldsymbol{R}_{12} & \cdots & \boldsymbol{R}_{1K} \\ \vdots & \cdots & \cdots & \vdots \\ \boldsymbol{R}_{K1} & \cdots & \cdots & \boldsymbol{R}_{KK} \end{bmatrix}$$

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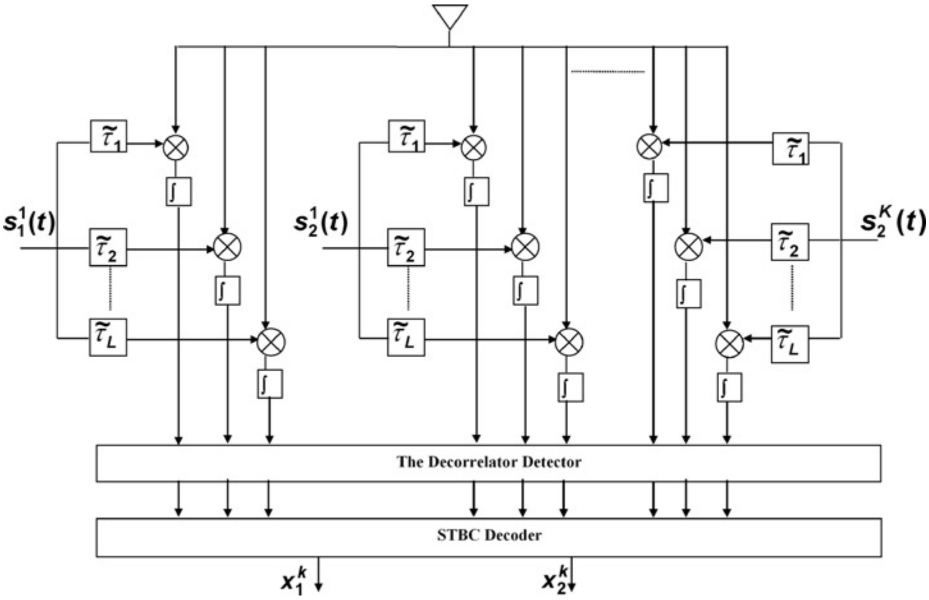

Figure 2 *Multiuser receiver structure in case of single receive antenna*

where $\boldsymbol{R}_{kw}$, $w = 1, \cdots, K$, is $2LP \times 2LP$ matrix with elements [14]

$$\boldsymbol{R}_{kw} = \int_{\tau_k}^{\tau_k+PT} \boldsymbol{S}_k(t)\boldsymbol{S}_w^{\mathrm{H}}(t)\,\mathrm{d}t$$

H denotes Hermitian transpose, and $\boldsymbol{S}_k(t)$ represents all the delayed versions of the two codes assigned to the kth user during the two symbol periods, described as

$$\boldsymbol{S}_k(t) = \begin{bmatrix} s_1^k(t-\tau_k-\tilde{\tau}_1) \\ s_2^k(t-\tau_k-\tilde{\tau}_1) \\ s_1^k(t-\tau_k-\tilde{\tau}_L) \\ s_1^k(t-T_\mathrm{b}-\tau_k-\tilde{\tau}_1) \\ s_2^k(t-T_\mathrm{b}-\tau_k-\tilde{\tau}_L) \end{bmatrix}$$

The $2LPK \times 1$ noise vector $\boldsymbol{N}_m$, in (2), is given by

$$\boldsymbol{N}_m = [\boldsymbol{N}_{1,m}^{\mathrm{T}}\boldsymbol{N}_{2,m}^{\mathrm{T}}\ldots\boldsymbol{N}_{k,m}^{\mathrm{T}}\ldots\boldsymbol{N}_{K,m}^{\mathrm{T}}]^{\mathrm{T}}$$

with

$$\boldsymbol{N}_{k,m} = [n_{11,m}^{k,t}n_{21,m}^{k,t}n_{12,m}^{k,t}\ldots n_{2L,m}^{k,t}n_{11,m}^{k,t+T_\mathrm{b}}n_{21,m}^{k,t+T_\mathrm{b}}\ldots n_{2L,m}^{k,t+T_\mathrm{b}}]^{\mathrm{T}}$$

and each of the elements $n_{pl,m}^{k,t}$, $n_{pl,m}^{k,t+T_\mathrm{b}}$ ($p = 1,2$ and $l = 1, \ldots, L$) are modelled as complex Gaussian random variables, each with variance $\sigma_n^2 = N_o/2$ per dimension. As will be shown later, this scheme yields to $D = 2PML$ diversity order in fast-fading channels.

Note that the output of the matched filter bank suffers from MAI which can be eliminated using the decorrelator detector. In this case, the output of the mth matched filter bank, $\boldsymbol{Y}_m$, is applied to a linear mapper $\boldsymbol{Z}_m = \boldsymbol{R}^{-1}\boldsymbol{Y}_m$ [5], where $\boldsymbol{R}^{-1}$ is the inverse of the cross correlation matrix. The $2LPK \times 1$ vector $\boldsymbol{Z}_m$ represents the output of the mth decorrelator during two successive symbol periods. It includes the L replicas of the signals from the two transmit antennas for each user during one ST-block interval, which can be expressed as follows

$$\boldsymbol{Z}_m = [\boldsymbol{Z}_{1,m}^{\mathrm{T}}\boldsymbol{Z}_{2,m}^{\mathrm{T}}\ldots\boldsymbol{Z}_{k,m}^{\mathrm{T}}\ldots\boldsymbol{Z}_{K,m}^{\mathrm{T}}]^{\mathrm{T}}$$

where the $2LP \times 1$ vector $\boldsymbol{Z}_{k,m}$ is defined by

$$\boldsymbol{Z}_{k,m} = [z_{11,m}^{k,t}z_{21,m}^{k,t}z_{12,m}^{k,t}\ldots z_{2L,m}^{k,t}z_{11,m}^{k,t+T_\mathrm{b}}z_{21,m}^{k,t+T_\mathrm{b}}\ldots z_{2L,m}^{k,t+T_\mathrm{b}}]^{\mathrm{T}}$$

and $z_{pl,m}^{k,t}$, $z_{pl,m}^{k,t+T_\mathrm{b}}$ represent the output of the mth decorrelator corresponding to the lth path of the pth sequence for user k at times t and $t + T_\mathrm{b}$, respectively.

The two transmitted symbols of the kth user can be extracted by combining the M decorrelators outputs as follows

$$\hat{x}_1^k = \sum_{m=1}^{M}\sum_{l=1}^{L} h_{1l,m}^{k,t}z_{1l,m}^{k,t*} + h_{2l,m}^{k,t*}z_{2l,m}^{k,t} + h_{2l,m}^{k,t+T_\mathrm{b}}z_{1l,m}^{k,t+T_\mathrm{b}*} + h_{1l,m}^{k,t+T_\mathrm{b}*}z_{2l,m}^{k,t+T_\mathrm{b}} \quad (3)$$

$$\hat{x}_2^k = \sum_{m=1}^{M}\sum_{l=1}^{L} h_{1l,m}^{k,t}z_{2l,m}^{k,t*} - h_{2l,m}^{k,t*}z_{1l,m}^{k,t} - h_{1l,m}^{k,t+T_\mathrm{b}*}z_{1l,m}^{k,t+T_\mathrm{b}} + h_{2l,m}^{k,t+T_\mathrm{b}}z_{2l,m}^{k,t+T_\mathrm{b}*} \quad (4)$$

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Considering the first symbol of the kth user and defining the variable $v_k = 2PL(k-1)$, we have

$$\begin{aligned}\hat{x}_1^k &= \sum_{m=1}^{M}\sum_{l=1}^{L}\sqrt{E_s}(|h_{1l,m}^{k,t}|^2 + |h_{2l,m}^{k,t}|^2 \\ &\quad + |h_{1l,m}^{k,t+T_b}|^2 + |h_{2l,m}^{k,t+T_b}|^2)x_1^k \\ &\quad + \sum_{m=1}^{M}\sum_{l=1}^{L} h_{1l,m}^{k,t}(\boldsymbol{R}^{-1}\boldsymbol{N}_m)^*_{2l+v_k-1,1} \\ &\quad + h_{2l,m}^{k,t*}(\boldsymbol{R}^{-1}\boldsymbol{N}_m)_{2l+v_k,1} \\ &\quad + h_{2l,m}^{k,t+T_b}(\boldsymbol{R}^{-1}\boldsymbol{N}_m)^*_{2(L+l)+v_k-1,1} \\ &\quad + h_{1l,m}^{k,t+T_b*}(\boldsymbol{R}^{-1}\boldsymbol{N}_m)_{2(L+l)+v_k,1}\end{aligned} \tag{5}$$

From (5), one can easily see that a diversity order of $2LPM$ is achieved for the single-user system with no MAI. In the following sections, we derive the probability of bit errors for the multiuser system when employing the decorrelator detector after signal combining.

3 Performance analysis

In what follows, and for the sack of simplicity, we consider BPSK transmission. To evaluate the average BER at the decorrelator output, we first obtain the pdf of the output SNR of the decorrelator detector. Using this pdf, the probability of error for both the fast and slow-fading channels can be evaluated. Without loss of generality, consider the case of finding the probability of error for the first symbol of user 1. To avoid complex notation, we drop its corresponding superscript from the fading coefficients.

3.1 Fast fading

In this case, we consider the first $2LP$ elements from each of the M-decorrelator output vectors ($\boldsymbol{Z}_m$, $m = 1, \ldots, M$). Assuming fixed fading gains and perfect estimation of the cross correlation matrix, the Gaussian approximation [19] can be used to find the conditional probability of bit error as

$$\begin{aligned}&P_b(\hat{x}_1 = 1|h_{11,1}^t, h_{21,1}^t, \ldots, h_{1L,M}^{t+T_b}, h_{2L,M}^{t+T_b}) \\ &\quad = Q\left(\frac{\sum_{m=1}^{M}\sum_{l=1}^{L}\sqrt{E_s}(a_{1l,m}^t + a_{2l,m}^t + a_{1l,m}^{t+T_b} + a_{2l,m}^{t+T_b})}{\sqrt{\sigma_{\hat{x}}^2}}\right)\end{aligned} \tag{6}$$

where $Q(\cdot)$ is the Gaussian Q-function, $a_{1l,m}^t = |h_{1l,m}^t|^2$, $a_{2l,m}^t = |h_{2l,m}^t|^2$, $a_{1l,m}^{t+T_b} = |h_{1l,m}^{t+T_b}|^2$, $a_{2l,m}^{t+T_b} = |h_{2l,m}^{t+T_b}|^2$ and $\sigma_{\hat{x}}^2$ is the variance of the noise term in (5) when $k = 1$. It is easy to show that

$$\begin{aligned}\sigma_{\hat{x}}^2 &= \left(\frac{N_o}{2}\right)\sum_{m=1}^{M}\sum_{l=1}^{L} c_{2l-1}a_{1l,m}^t + c_{2l}a_{2l,m}^t + c_{2(L+l)-1}a_{2l,m}^{t+T_b} \\ &\quad + c_{2(L+l)}a_{1l,m}^{t+T_b}\end{aligned} \tag{7}$$

where c_{2l-1}, c_{2l}, $c_{2(L+l)-1}$ and $c_{2(L+l)}$ define the following terms $R^{-1}_{2l-1,2l-1}$, $R^{-1}_{2l,2l}$, $R^{-1}_{2(L+l)-1,2(L+l)-1}$ and $R^{-1}_{2(L+l),2(L+l)}$, respectively, and $R^{-1}_{i,i}$ is the ith diagonal element of the inverse of the cross correlation matrix. The variables $a_{ql,m}^t$ and $a_{ql,m}^{t+T_b}$ $(q = 1,2)$ are chi-square distributed with two degrees of freedom and characteristic function [20]

$$\phi(j\omega) = \frac{1}{1 - j2\omega} \tag{8}$$

Define the variable α as

$$\alpha = \frac{A}{\sqrt{B}} \tag{9}$$

where

$$A = \sum_{m=1}^{M}\sum_{l=1}^{L} a_{1l,m}^t + a_{2l,m}^t + a_{1l,m}^{t+T_b} + a_{2l,m}^{t+T_b}$$

and

$$B = \sum_{m=1}^{M}\sum_{l=1}^{L} c_{2l-1}a_{1l,m}^t + c_{2l}a_{2l,m}^t + c_{2(L+l)-1}a_{2l,m}^{t+T_b} + c_{2(L+l)}a_{1l,m}^{t+T_b}$$

Hence, the joint characteristic function of A and B is given by [20]

$$\begin{aligned}\phi_{A,B}(\omega_1,\omega_2) &= E[\exp j(\omega_1 A + \omega_2 B)] \\ &= E\Bigg[\exp j\sum_{m=1}^{M}\sum_{l=1}^{L} a_{1l,m}^t(\omega_1 + c_{2l-1}\omega_2) \\ &\quad + a_{2l,m}^t(\omega_1 + c_{2l}\omega_2) + a_{2l,m}^{t+T_b}(\omega_1 + c_{2(L+l)-1}\omega_2) \\ &\quad + a_{1l,m}^{t+T_b}(\omega_1 + c_{2(L+l)}\omega_2)\Bigg]\end{aligned} \tag{10}$$

where $E[\cdot]$ denotes the expected value of the enclosed argument. Defining $y = 1/2 - j\omega_1$ and assuming independent fading channels, one can show that

$$\phi_{A,B}(\omega_1,\omega_2) = \frac{1}{(2)^{4LM}}\prod_{u=1}^{4L}\frac{1}{(y - jc_u\omega_2)^M} \tag{11}$$

In order to simplify our analysis, we use a partial fraction expansion method of a rational function with high order poles. For further details regarding this method, the reader is referred to [21]. Furthermore, we consider the special case where the rational function has no zeros. Thus, the

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characteristic function in (11) is reduced to

$$\phi_{A,B}(\omega_1,\omega_2)=\frac{\prod_{u=1}^{4L}(1/c_u^M)}{(2)^{4LM}}\left(\sum_{u=1}^{4L}\sum_{i=0}^{M-1}\frac{C_{ui}}{[\omega_2-(y/jc_u)]^{M-i}}\right) \tag{12}$$

where,

$$C_{ui}=\frac{K_{ui}}{y^{4ML-M+i}},\quad u=1,\ldots,4L,\quad i=0,\ldots,M-1$$

represents the residue terms obtained from the partial fraction expansion [21]. An exact expression for each of K_{ui} ($u=1,\ldots,4L$ and $i=0,\ldots,M-1$) can also be obtained in terms of the cross correlation coefficients between the users' signature waveforms [21]. From (12), the joint pdf, $f_{A,B}$, can be obtained as [20]

$$\begin{aligned}f_{A,B}&=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi_{A,B}(\omega_1,\omega_2)\\&\quad\times\exp(-j(\omega_1A+\omega_2B))\mathrm{d}\omega_1\mathrm{d}\omega_2\\&=\frac{\prod_{u=1}^{4L}(1/c_u^M)}{4\pi^2(2)^{4LM}}\sum_{u=1}^{4L}\sum_{i=0}^{M-1}\lambda_{ui}B^{M-i-1}\left(A-\frac{B}{c_u}\right)^{4ML-M+i-1}\\&\quad\times\exp-\left(\frac{A}{2}\right)\end{aligned} \tag{13}$$

where $\Gamma(\cdot)$ is the Gamma function and $\lambda_{ui}=(4\pi^2K_{ui})/[j^{M-i}\Gamma(M-i)\Gamma(4ML-M+i)]$. One way to obtain the pdf of the SNR in (9) is through variable transformation. From (9) and by assuming that $W=B$, the joint pdf of α and W can be determined through the following relation [20]

$$f_{\alpha,W}=f_{A,B}|\Omega(\alpha,W)| \tag{14}$$

where $|\Omega(\alpha,W)|=\sqrt{W}$ is the Jacobian of the transformation. Finally with the substitution of (9) in (14), and after some algebraic manipulations, we get

$$\begin{aligned}f_{\alpha,W}&=\frac{\prod_{u=1}^{4L}(1/c_u^M)}{4\pi^2(2)^{4LM}}\sum_{u=1}^{4L}\sum_{i=0}^{M-1}\lambda_{ui}W^{M-i-\frac{1}{2}}\\&\quad\times\left(\alpha\sqrt{W}-\frac{W}{c_u}\right)^{4ML-M+i-1}\exp\left(-\frac{\alpha\sqrt{W}}{2}\right)\end{aligned} \tag{15}$$

From (15), the pdf of the SNR can be expressed as

$$f_\alpha=\frac{\prod_{u=1}^{4L}(1/c_u^M)}{4\pi^2(2)^{4LM}}\sum_{u=1}^{4L}\sum_{i=0}^{M-1}\lambda_{ui}P_{ui} \tag{16}$$

where

$$\begin{aligned}P_{ui}&=\int_0^{c_u^2\alpha^2}W^{M-i-\frac{1}{2}}\left(\alpha\sqrt{W}-\frac{W}{c_u}\right)^{4ML-M+i-1}\\&\quad\times\exp\left(-\frac{\alpha\sqrt{W}}{2}\right)\mathrm{d}W\end{aligned} \tag{17}$$

Using the binomial series expansion, the integration in (17) can be reduced to

$$\begin{aligned}P_{ui}&=\sum_{d=0}^{4ML-M+i-1}\binom{4ML-M+i-1}{d}\frac{(-1)^{4ML-M+i-1-d}\alpha^d}{c_u^{4ML-M+i-1-d}}\\&\quad\times\int_0^{c_u^2\alpha^2}(\sqrt{W})^{8ML-3-d}\exp\left(-\frac{\alpha\sqrt{W}}{2}\right)\mathrm{d}W\end{aligned} \tag{18}$$

In what follows, we denote the integration in (18) by II_{ui} and use

$$\begin{aligned}\int_{c_1}^{c_2}x^ne^{-ax}\,\mathrm{d}x&=\frac{1}{a(n+1)}\left[c_2^n(ac_2)^{-\frac{n}{2}}e^{-\frac{ac_2}{2}}M\left(\frac{n}{2},\frac{n+1}{2},ac_2\right)\right.\\&\quad\left.-c_1^n(ac_1)^{-\frac{n}{2}}e^{-\frac{ac_1}{2}}M\left(\frac{n}{2},\frac{n+1}{2},ac_1\right)\right]\end{aligned} \tag{19}$$

where $M(k,m,z)$ represents the WhittakerM function [22]. Using the substitution $t=\sqrt{W}$, we get

$$\begin{aligned}II_{ui}&=2\int_0^{c_u\alpha}t^{8ML-2-d}\exp\left(-\frac{\alpha t}{2}\right)\mathrm{d}t\\&=\frac{2^{(8ML-d+2)/2}\alpha^{8ML-d}c_u^{(8ML-2-d)/2}}{(8ML-1-d)\alpha}\exp\left(-\frac{c_u\alpha^2}{4}\right)\\&\quad\times M\left(\frac{8ML-2-d}{2},\frac{8ML-1-d}{2},\frac{c_u\alpha^2}{2}\right)\end{aligned} \tag{20}$$

In terms of the confluent hypergeometric function ([22], Eq. (13.1.32))

$$II_{ui}=\frac{2(\alpha c_u)^{8ML-1-d}}{8ML-1-d}\exp\left(-\frac{c_u\alpha^2}{2}\right){}_1F_1\left(1;8ML-d;\frac{c_u\alpha^2}{2}\right) \tag{21}$$

Substituting (21) in (18), we obtain

$$\begin{aligned}P_{ui}&=2\alpha^{8ML-1}c_u^{4ML+M-i}\exp\left(-\frac{c_u\alpha^2}{2}\right)\\&\quad\times\sum_{d=0}^{4ML-M+i-1}\binom{4ML-M+i-1}{d}\frac{(-1)^{4ML-M+i-1-d}}{8ML-d-1}\\&\quad\times{}_1F_1\left(1;8ML-d;\frac{c_u\alpha^2}{2}\right)\end{aligned} \tag{22}$$

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Finally, the probability function of the SNR in (16) can be obtained using (22).

3.2 Slow fading

For the slow-fading channel, the fading coefficients are assumed to be fixed for the duration of at least two consecutive symbol intervals. Hence (6) reduces to

$$P_b(\hat{x}_1 = 1 | h_{11,1}, h_{21,1}, \ldots, h_{1L,M}, h_{2L,M}) = Q\left(\frac{2\sqrt{E_s}\sum_{m=1}^{M}\sum_{l=1}^{L}(a_{1l,m}+a_{2l,m})}{\sqrt{\sigma_{\hat{x}}^2}}\right) \quad (23)$$

where $a_{1l,m} = |h_{1l,m}^t|^2 = |h_{1l,m}^{t+T_b}|^2$, $a_{2l,m}^t = |h_{2l,m}^t|^2 = |h_{2l,m}^{t+T_b}|^2$ and

$$\sigma_{\hat{x}}^2 = \sigma_n^2 \sum_{m=1}^{M}\sum_{l=1}^{L} |h_{1l,m}|^2 (R_{2l-1,2l-1}^{-1} + R_{2(L+l),2(L+l)}^{-1}) + |h_{2l,m}|^2 (R_{2l,2l}^{-1} + R_{2(L+l)-1,2(L+l)-1}^{-1})$$

Following the same procedure as in the fast-fading case, one can show that the joint characteristic function in (12) reduces to

$$\phi_{A,B}(\omega_1, \omega_2) = \frac{\prod_{u=1}^{2L}(1/c_u^M)}{(2)^{2LM}}\left(\sum_{u=1}^{2L}\sum_{i=0}^{M-1}\frac{C_{ui}}{(\omega_2 - (y/jc_u))^{M-i}}\right) \quad (24)$$

where

$$C_{ui} = \frac{K_{ui}}{y^{2ML-M+i}}$$

Similar to the fast-fading channel, it is straightforward to show that

$$f_{A,B} = \frac{\prod_{u=1}^{2L}(1/c_u^M)}{4\pi^2(2\sigma^2)^{2LM}}\sum_{u=1}^{2L}\sum_{i=0}^{M-1}\lambda_{ui}B^{M-i-1} \times \left(A - \frac{B}{c_u}\right)^{2ML-M+i-1}\exp\left(-\frac{A}{2\sigma^2}\right) \quad (25)$$

where $\lambda_{ui} = \frac{4\pi^2 K_{ui}}{jM-i\Gamma(M-i)\Gamma(2ML-M+i)}$. Using the transformation in (9),

$$f_\alpha = \frac{\prod_{u=1}^{2L}(1/c_u^M)}{4\pi^2(2)^{2LM}}\sum_{u=1}^{2L}\sum_{i=0}^{M-1}\lambda_{ui}P_{ui} \quad (26)$$

with

$$P_{ui} = 2\alpha^{4ML-1}c_u^{2ML+M-i}\exp\left(-\frac{c_u\alpha^2}{2}\right) \times \sum_{d=0}^{2ML-M+i-1}\binom{2ML-M+i-1}{d}\frac{(-1)^{2ML-M+i-1-d}}{4ML-d-1} \times 1F_1\left(1; 4ML-d; \frac{c_u\alpha^2}{2}\right)$$

4 Average probability of bit error

For the fast-fading channel, the probability of error can be obtained by averaging the conditional bit error in (6) over the pdf in (16)

$$P_b = \int_0^\infty Q\left(\sqrt{\gamma\alpha^2}\right) f_\alpha \, d\alpha, \quad (27)$$

where $\gamma = E_s/\sigma_n^2$. To simplify the analysis, we use the *preferred* form of the Gaussian Q-function [23]

$$Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp^{-\frac{x^2}{2\sin^2\theta}} \, d\theta \quad (28)$$

Substituting (16) and (28) in (27), we get

$$P_b = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\int_0^\infty \exp^{-\frac{\gamma\alpha^2}{2\sin^2\theta}} f_\alpha \, d\alpha \, d\theta = \frac{\prod_{u=1}^{4L}(1/c_u^M)}{4\pi^3}\sum_{u=1}^{4L}\sum_{i=0}^{M-1}\lambda_{ui}F_{ui} \quad (29)$$

where

$$F_{ui} = \frac{1}{(2)^{4ML}}\int_0^{\frac{\pi}{2}}\int_0^\infty \exp^{-\frac{\gamma\alpha^2}{2\sin^2\theta}} P_{ui} \, d\alpha \, d\theta$$

Substituting P_{ui} from (22), we get

$$F_{ui} = 2c_u^{4ML+M-i}\sum_{d=0}^{4ML-M+i-1}\binom{4ML-M+i-1}{d} \times \frac{(-1)^{4ML-M+i-1-d}}{8ML-1-d}G_d \quad (30)$$

where by using ([24], Eq. (7.621.4))

$$G_d = \frac{\Gamma(4ML)}{2\bar{\gamma}^{4ML}}\int_0^{\frac{\pi}{2}}(\sin^2\theta)^{4ML} \times {}_2F_1\left(8ML-d-1, 4ML; 8ML-d; -\frac{c_u \sin^2\theta}{\bar{\gamma}}\right)d\theta \quad (31)$$

where $\bar{\gamma} = E\left[\left|h_{ql,m}\right|^2\right]E_s/\sigma_n^2$ is the average SNR per channel, and ${}_2F_1(.,.;.;.)$ is a special case of the generalised hypergeometric function ([24], Eq. (9.14.1)). Substituting $V = \sin^2\theta$ in (31) and by using the integral in ([24], Eq. (7.512.12)), one can show that

$$G_d = \frac{\Gamma(1/2)\Gamma(8ML+1/2)}{16ML\bar{\gamma}^{4ML}}3F_2\left(\frac{8ML+1}{2}, \times 8ML-d-1, 4ML; 4ML+1, 8ML-d; -\frac{c_u}{\bar{\gamma}}\right) \quad (32)$$

Finally, by substituting (32) in (30), we can evaluate the average probability of error in (29). From (32) we can examine the asymptotic BER performance as $\bar{\gamma}$ gets large. In this case, in the limit, the hypergeometric function

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${}_3F_2(\cdot,\cdot,\cdot) \to 1$ and hence

$$P_b(\bar{\gamma} \to \infty) \equiv W\left(\frac{1}{\bar{\gamma}}\right)^{4ML}, \quad W \in \mathbb{R}$$

That is, our system achieves the full system diversity of *4ML*. The same argument applies for the slow-fading channel discussed below, where the full system diversity of *2ML* is also achieved.

The BER for the slow-fading channel can be found in a similar way by averaging the conditional BER in (23) over the SNR pdf in (26). That is,

$$P_{\rm b} = \frac{\prod_{u=1}^{2L}(1/c_u^M)}{4\pi^3} \sum_{u=1}^{2L} \sum_{i=0}^{M-1} \lambda_{ui} F_{ui} \tag{33}$$

where

$$F_{ui} = 2c_u^{2ML+M-i} \sum_{d=0}^{2ML-M+i-1} \binom{2ML-M+i-1}{d} \times \frac{(-1)^{2ML-M+i-1-d}}{4ML-1-d} G_d$$

and

$$G_d = \frac{\Gamma(1/2)\Gamma\left(\frac{4ML+1}{2}\right)}{8ML\bar{\gamma}^{2ML}} {}_3F_2\left(\frac{4ML+1}{2}, \times 4ML-d-1, 2ML; 2ML+1, 4ML-d; -\frac{c_u}{\bar{\gamma}}\right)$$

5 Numerical and simulation results

In this section we examine the BER performance of the space–time system discussed in the previous sections using both Monte Carlo simulations and the analytical results in (29) and (33). In all cases, we consider a DS-CDMA system with BPSK transmission where the user data is spread using Gold codes of length 31 chips. The delay between users, τ_k, is assumed to be multiple of chip periods within the symbol interval. To neglect the effect of ISI, the delay of each path, $\bar{\tau}_{\rm p}$, is taken as a multiple of chip periods of length less than 10% of the symbol period. In cases where ISI is dominant, one can resort to pulse shaping/equalisation techniques to overcome the degradation in the system performance. Furthermore, we assume perfect knowledge of the channel coefficients at the receiver. Also, in all the results, we assume that all the channels are independent. Our results and analysis are based on two transmit antennas at the user side and M receive antennas at the base station. However, one can generalise these results to $N > 2$ transmit antennas. In this case to ensure full diversity using simple decoding, one has to search for a spreading code matrix that satisfies the full rank criterion using orthogonal designs as discussed in [3].

Fig. 3 presents the error performance for different number of users in the frequency-selective fast-fading channel. For

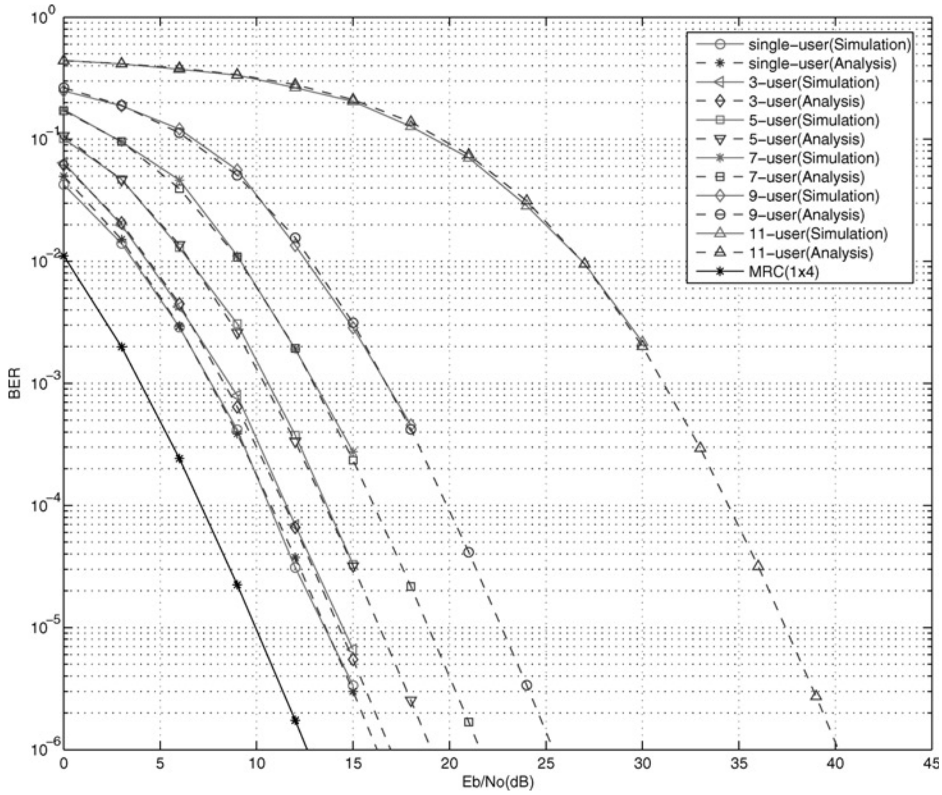

Figure 3 *BER performance for asynchronous DS-CDMA systems with two transmit and one receive antenna over frequency-selective fast-fading channels with L = 2 paths*

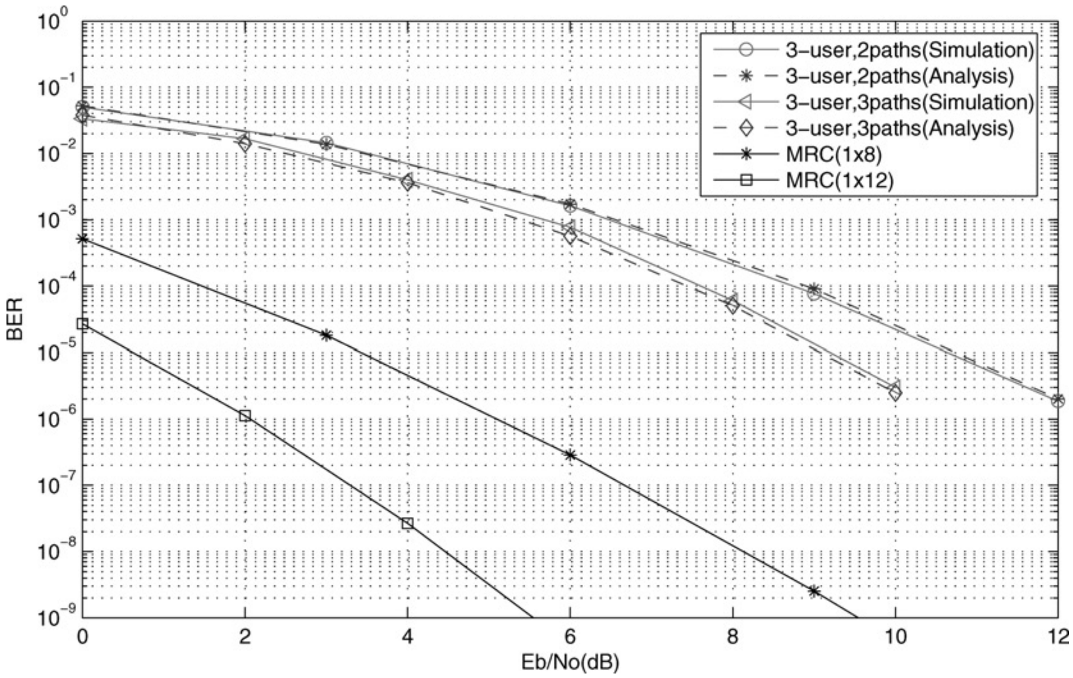

Figure 4 *BER performance for a 3-user system as a function of the number of paths, L = 2,3, over frequency-selective fast-fading channels*

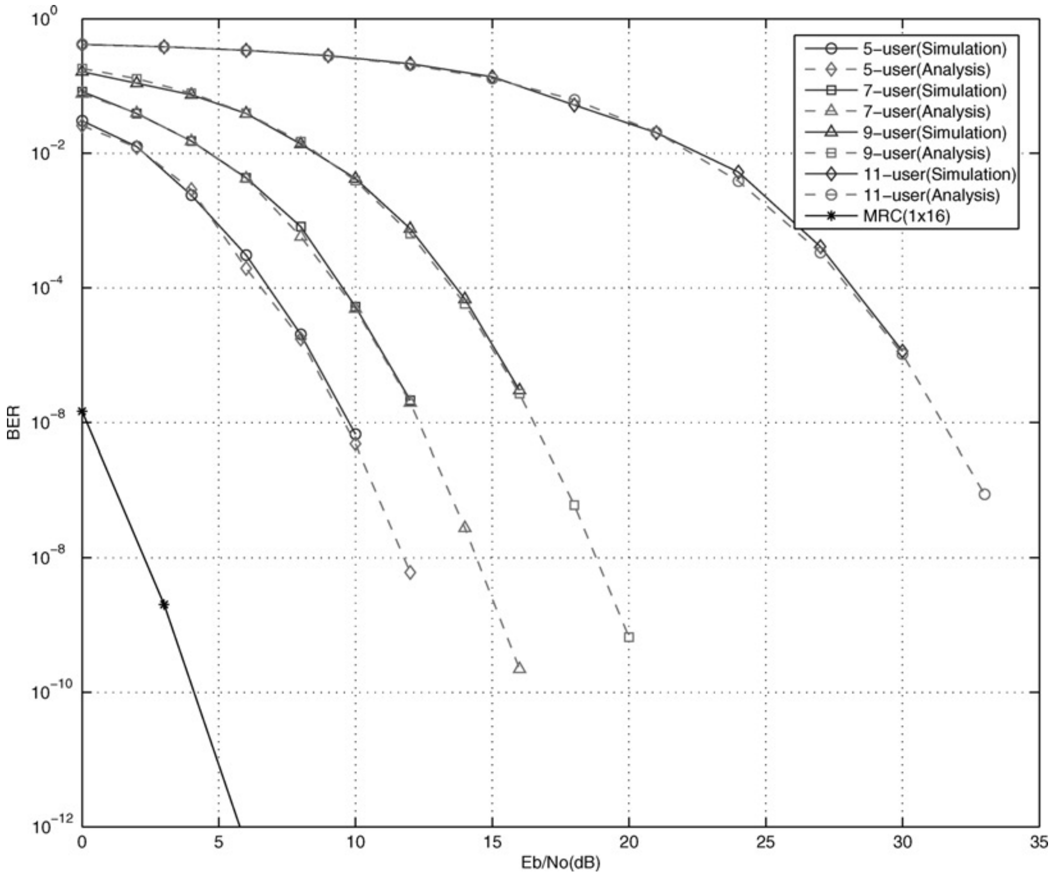

Figure 5 *BER performance for a multiuser system with two transmit and two receive antennas over frequency-selective fast-fading channels with L = 2 paths*

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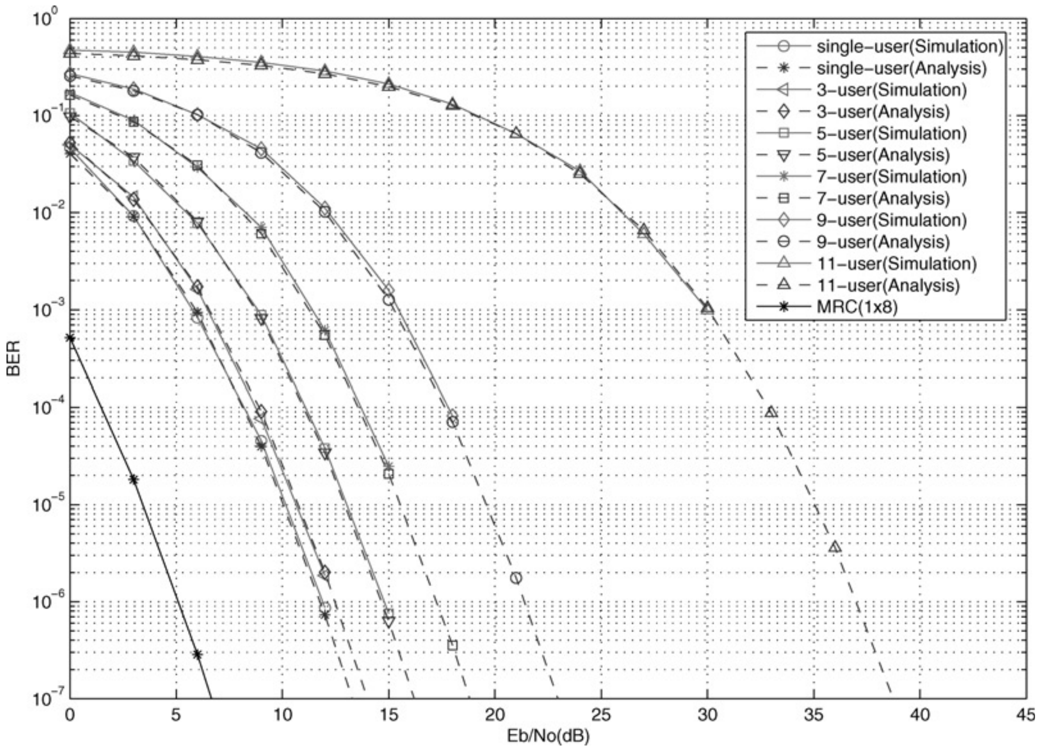

Figure 6 *BER performance for asynchronous DS-CDMA systems with two transmit and one receive antenna over frequency-selective slow-fading channels with $L = 2$ paths*

reference, we include the BER performance of the maximal ratio combiner (MRC) with eight receive diversity branches. Note that the performance of the MRC is merely used for diversity order comparisons, and the SNR gap is because of the fixed transmit power constraint and noise enhancement of the decorrelator. The results in Fig. 3 demonstrate the accuracy of the derived bit-error rate expression in (29) when compared with simulation results. Furthermore, it is evident that a diversity order of eight is achieved for different number of users. This diversity is delivered by the $Q = 2$ transmit antennas, $L = 2$ paths, $M = 1$ receive antenna and $P = 2$ length of the space–time block interval.

Fig. 4 shows the BER performance of the STS scheme for a three-user system considering two and three paths per transmit antenna. The results clearly show the multipath diversity gain delivered by the RAKE receiver when the number of resolvable paths increases for 2×1 antenna configuration. In this case, the transmit diversity scheme with $L = 3$ paths achieves diversity order $QLP = 12$ when compared with the MRC with the same number of diversity branches.

Fig. 5 examines the BER performance for 2×2 antenna configuration where we consider transmission over frequency-selective fast-fading channel with two resolvable paths. The accuracy of the derived BER as function of the number of users (K), the number of resolvable paths (L) and the number of receive antennas (M) is evident for different number of users. It should also be noticed that the diversity gain is improved when doubling the number of receive antennas

Finally, Fig. 6 shows both simulations and analytical results as a function of the number of users for the slow-fading channel. The results show that the proposed system is able to deliver the same diversity order as the MRC with four diversity branches. Note that the diversity order of four is because of $Q = 2$ transmit antennas and $L = 2$ paths.

6 Conclusions

The performance of transmit diversity using space–time spreading in DS-CDMA systems has been examined through simulations and mathematical analysis. Our results show that the full system diversity can be maintained when that a decorrelator detector is employed at the base station. These results are valid for both fast and slow-fading channels, where the resulting SNR loss from the optimal single-user system is only a function of the number of active users. Throughout our work, we assumed a perfect channel state information and independent fading across the antennas. Future works should address these problems and investigate their effect on the achieved diversity order.

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