On Reducing Blocking Probability in Cooperative Ad-hoc Networks

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Abstract—**In this paper, we propose a new channel access protocol for transmission over cooperative ad-hoc networks. The proposed protocol, which aims at minimizing the number of blocked nodes and consequently improving the system throughput, employs directional antennas at both source and relay stations. The network performance under the proposed settings is modelled using continues Markov chains. Steady state transmission blocking probability and average network throughput are obtained by analyzing the Markov model. Both analytical and simulation results show the improvement in performance compared to carrier sense multiple access with collision avoidance protocol that employs omnidirectional antennas.**

I. INTRODUCTION

Mobile ad-hoc networks are distributed and infrastructureless networks where mobile stations can communicate with each other without the help of any centralized control or access point which makes them robust to failures, easier to deploy, and more flexible to reconfigure. In mobile ad-hoc networks, channel fading is considered to be a seriously damaging phenomenon that can degrade the overall network performance. Throughout the literature, diversity techniques have been proposed to mitigate the effect of channel fading [1]. Nowadays, multiple-input multiple-output (MIMO) technology has been introduced as a means for improving the reliability of the received signal and supporting high data rate applications [2]-[4]. The notion of diversity is further expanded to include cooperative networks, where the concept of user cooperation diversity is applied [5]. In cooperative networks, a node at any given time can act as a sender, destination or relay depending on the network traffic and topology. Neighboring stations to the transmitter and/or receiver can act as relay nodes to transfer the source data to the desired destination node through an independent relay channel, i.e., independent from the source-destination channel [6]. The function of the relay node can be as simple as to amplify and forward the received source data or to decode and regenerate an estimate of this data.

In ad-hoc networks, carrier sense multiple access with collision avoidance (CSMA/CA) is a commonly used channel access technique. When omnidirectional antennas are used with CSMA/CD, each of the sender, receiver and relay block neighboring stations, within its transmission range, from possible transmissions. This criterion results in one of the main problems associated with cooperative networks, namely increased transmission blocking probability. In [7] an analytical model for evaluating the transmission blocking probability for ad-hoc networks as a function of total number of nodes was developed. In [8] and [9] this work was extended to cooperative ad-hoc networks.

The diversity gain achieved in cooperative ad hoc networks is attained at the extra cost of wasting some wireless resources (i.e., available bandwidth). On the other hand, the use of directional antennas has shown to offer an effective way for efficient bandwidth utilization.

To enhance the performance of cooperative networks, we propose a new channel access protocol for transmission over cooperative ad-hoc networks. The proposed protocol, which aims at minimizing the number of blocked nodes employs directional antennas at both source and relay stations. The network performance under the proposed settings is modelled using continues Markov chains. Steady state transmission blocking probability and average network throughput are obtained by analyzing the above Markov model. Our results show the improvement in performance compared to CSMA/CA protocol that employs omnidirectional antennas.

The rest of the paper is organized as follows. In the following section we describe the proposed protocol and determine the optimum value for the beamwidth of the directional antennas of the source and relay nodes. In section III, the expected value of the average blocked area associated with a given number of transmissions is determined. A Markov chain model for the network performance under the proposed setting is developed in section IV. The simulation results the validate the proposed model are presented in section V.

II. PROPOSED PROTOCOL

In cooperative systems, each time frame is divided into two slots. The source transmits the packet to both the destination and the chosen relay in the first slot. In the second slot, the relay amplifies and forwards the received packet to destination. When using CSMA/CA, in order to avoid interference from other ongoing transmissions, the neighboring nodes of any transmitter, relay or destination, must remain silent, i.e., they are blocked, during the duration of the transmission. In other words, as shown in Fig. 1, each of the source, relay and destination blocks a full circle with radius d_{max}, where d_{max} denotes the maximum transmission range, which we assume to be the same for all nodes.

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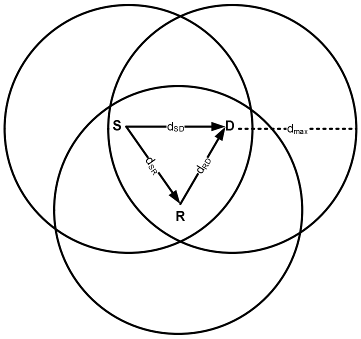

Fig. 1. Blocked area for one transmission in CSMA/CA protocol

Further, we assume that all nodes are also equipped with directional antennas. To evade collisions between ongoing transmissions and newly initiated ones, a narrow band channel is dedicated for channel reservation protocol and the remaining spectrum is dedicated for data transmission. The channel reservation protocol works as follows:

- The relay is chosen so that it lays within the area covered by the source directional antenna.
- A request to send (RTS) is broadcasted by both the source and the relay using their omnidirectional antennas to inform all nodes, within their transmission range, with their direction of transmission. The nodes laying within the directional beam of the source or relay will be blocked. While other nodes within their transmission range could act as a relay or source with their directional antennas facing other directions, but are blocked from being destinations.
- A clear to send to send (CTS) is broadcasted by the destination using its omnidirectional antenna. Consequently, the destination node blocks a full circle with a radius of d_{max}.

Fig. 2 shows the areas blocked by both the source and relay transmissions where the destination uses an omnidirectional antenna (i.e., it always blocks a full circle with radius d_{max}). Assume that the fixed beamwidth of the deployed directional antenna is θ. In what follows, we determine the optimum choice for θ that minimizes the blocking area.

Consider two intersected circles, each with radius $r = d_{max}$, and centers C and B respectively, where B represents the destination, and C represents either the source or the relay. When the source (or relay) is located very close to the destination, i.e., $CB \approx 0$, any choice for $\theta = \angle ACF \leq 180^o$ would satisfy the above constraint (i.e., total blocking area is a circle with radius d_{max} around the destination). In the extreme case, depicted in Fig. 3, the source (or relay) is located at distance d_{max} from the destination, i.e., at the perimeter of the destination's blocked circle. In this case, $\overline{CB} = d_{max}$. Satisfying the above constraint implies that we need to have $\angle ACB = 60^o$, therefore $\theta = \angle ACF = 120^o$.

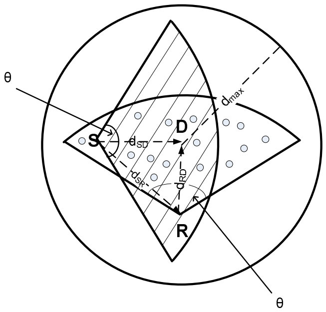

Fig. 2. Blocked area for one transmission (Proposed protocol)

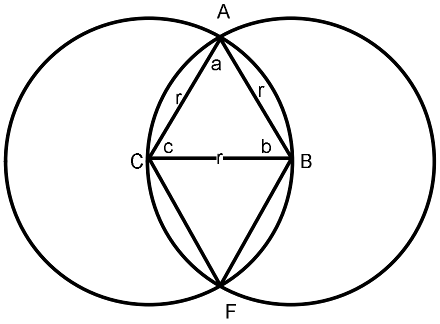

Fig. 3. Restriction on beamwidth ($CB = r$)

Fig. 4 represents the case where the source (or relay) are at a distance $0 < CB < d_{max}$ away from the destination. In this case, keeping $\theta = \angle ACF \leq 120^o$ would also satisfy the above constraints. Using the above argument, it is clear that restricting the maximum beamwidth to less than or equal to 120^o optimizes the blocking area by ensuring that it is reduced to one circle with radius d_{max} around the destination. On the other hand, maximizing the probability of finding a relay within the transmission beamwidth of the source requires that θ is maximized. Thus, throughout the rest of this work, we set $\theta = 120^o$.

III. AVERAGE BLOCKED AREA

Let $A_c = \pi d_{max}^2$ denote the area of the circle blocked due to any ongoing single transmission. It is clear that, at any point of time, nodes can be blocked due to more than one transmission, i.e., circles corresponding to areas blocked by different transmissions may overlap. Assuming a relatively dense network, then estimating the average number of nodes involved (i.e., either being a source, destination, relay or blocked) in a specific number of transmissions is equivalent to determining the average area blocked due to these transmissions.

Let $A_I(l) = A_1 \bigcap A_2$ denote the area of intersection between two circles, A_1 and A_2, with radius d_{max} and centers

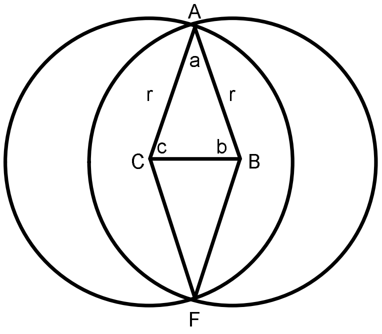

Fig. 4. Restriction on beamwidth ($CB < r$)

separated by $l \leq 2d_{max}$. Then we have

$$A_I(l) = 2d_{max}^2 \cos^{-1}(\frac{l}{2d_{max}}) - \frac{1}{l}\sqrt{4d_{max}^2 - l^2}. \quad (1)$$

The probability density function of the distance l between any two uniformly distributed nodes within a transmission area of z^2 is given by [10]

$$p(l) = \begin{cases} \frac{2l}{z^2}(\frac{l^2}{z^2} - \frac{4l}{z} + \pi), & 0 \leq l \leq z \\ \frac{2l}{z^2}(4\sqrt{\frac{l^2}{z^2} - 1} - (\frac{l^2}{z^2} + 2) & \\ +2(\sin^{-1}(\frac{z}{l}) - \cos^{-1}(\frac{z}{l}))), & z \leq l \leq \sqrt{2}z \\ 0, & Otherwise. \end{cases} \quad (2)$$

To allow two transmissions within two intersected circles, we must have $l \geq d_{max}$, otherwise the two destinations would block each other. Hence, the expected value of the area of intersection between any two transmissions is given by

$$\begin{aligned} \overline{A}_I &= \frac{\int_{d_{max}}^{2d_{max}} p(l)A_I(l)dl}{1-\int_0^{d_{max}} p(l)dl} \\ &= \frac{\pi d_{max}^4(\frac{3d_{max}^2}{4z^4} - \frac{11d_{max}}{5z^3} + \frac{3\sqrt{3}}{4z^2})}{1-(\frac{d_{max}^4}{2z^4} - \frac{8d_{max}^3}{3z^3} + \pi\frac{d_{max}^2}{z^2})}. \end{aligned} \quad (3)$$

Using the inclusion exclusion principle [11], the total area occupied by x transmissions is given by

$$A(x) = \sum_{i=1}^{x} A_i - \sum_{i,j:1\leq i<j\leq x} (A_i \bigcap A_j) + \sum_{i,j,k:1\leq i<j<k\leq x} (A_i \bigcap A_j \bigcap A_k) - \cdots + (-1)^x (A_1 \cdots A_x). \quad (4)$$

Deriving the expected value for the higher order terms in the above equation seems an intractable problem. However, as confirmed by our simulations, at moderate node density, the expected value for $A(x)$ can be well approximated by considering only the first two terms of the above equation, i.e., we have

$$\overline{A(x)} \simeq xA_c - \binom{x}{2}\overline{A}_I. \quad (5)$$

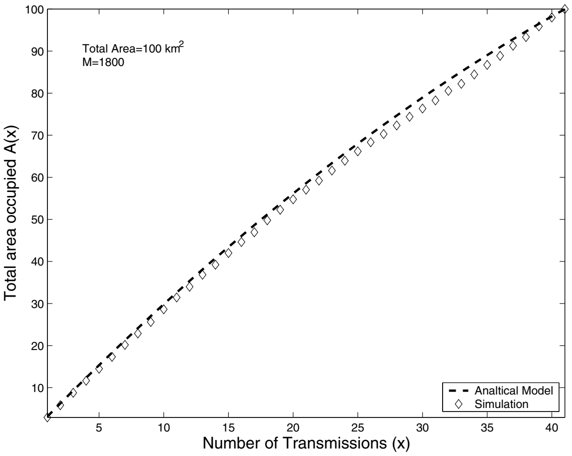

Fig. 5. Expected value of $A(x)$

Fig. 5 confirms the quality of the above approximation. The simulations were conducted for $M = 1800$ nodes and $z = 10$ km.

Let $N(x)$ denote the number of nodes (blocked, source, relay and destination nodes) that are involved in x transmissions. Thus

$$N(x) = \frac{A(x)}{z^2}M \quad (6)$$

where M is the total number of nodes, z^2 is the total area. Therefore, the average number of blocked nodes associated with x transmissions is given by

$$B(x) = N(x) - 3x, \quad (7)$$

where the term $3x$ denotes the number of source, relay and destination nodes.

Let Ω denote the maximum number of transmission that can be conducted simultaneously in area of z^2 km^2. Then $A(\Omega) = z^2$. Thus Ω can be approximately estimated using (5) by setting

$$A_c\Omega - \binom{\Omega}{2}\overline{A}_I = z^2, \quad (8)$$

and solving for Ω as follows

$$\Omega = \frac{(A_c + \frac{\overline{A}_I}{2}) - \sqrt{(A_c + \frac{\overline{A}_I}{2})^2 - 2\overline{A}_I z^2}}{\overline{A}_I}. \quad (9)$$

IV. MODELLING THE NUMBER OF TRANSMISSIONS

Throughout our work, we assume that all nodes are distributed uniformly over the covered area. We also assume that the traffic is distributed uniformly over all nodes. Packets inter-arrival and processing times are exponentially distributed with arrival and service rates of λ and μ, respectively. Each node is assumed to serve only one packet at a time, i.e., if the node is blocked or busy, the incoming data packets are dropped.

Thus our system resembles the M/M/C/C queueing system where customers have i.i.d exponentially distributed inter-arrival times with mean $1/\lambda$, and i.i.d exponentially distributed service times with mean $1/\mu$. Assuming that the inter-arrival and service times are independent, then the above system can be modelled as continuous-time Markov chain.

The following notation will be used throughout the rest of this section:

- x denotes the number of ongoing transmissions at time t.
- λ_x denotes the transition rate from state x to state $x+1$.
- μ_x denotes the service rate from state x to state $x-1$.
- $p_S(x)$ denotes the probability that a source is not blocked or used when there is an active x transmission.
- p_D denotes the probability that there is a destination available within the transmission range of the destination.
- p_R denotes the probability that there is a relay available within the transmission range of the destination.
- p_{DR} denotes the probability that both the destination and relay are within the beamwidth of the source to support this transmission.

Thus λ_x is given by

$$\lambda_x = \lambda \times (M - N(x)) \times p_S(x) \times p_D \times p_R \times p_{DR}, \quad (10)$$

where

$$p_S(x) = \frac{M - N(x)}{M}, \quad (11)$$

$$p_D = \frac{N(1) - 1}{N(1)}, \quad (12)$$

$$p_R = \frac{N(1) - 2}{N(1)}, \quad (13)$$

where $N(1)$ denotes the average number of nodes located in the transmission range of the destination and is given by $N(1) = \frac{A_c}{z^2} M$ (see (6)).

From section II, $\theta = 120^o$ was shown to be optimum. Thus the beamwidth of the source covers $\frac{1}{3}$ of the circle around the destination. Therefore,

$$p_{DR} = \sum_{2}^{N(1)} \binom{N(1)}{i} \frac{1}{3}^{i} \frac{2}{3}^{N(1)-i}. \quad (14)$$

The service rate μ_x is based on the service rate per existing transmission μ and the number of current transmissions x, hence

$$\mu_x = x\mu. \quad (15)$$

The transmission blocking probability is defined as the probability that a transmission is blocked given that a packet has arrived, therefore

$$P_B(x) = 1 - p_S(x) p_D p_R p_{DR}. \quad (16)$$

In what follows, we present the steady state analysis for our system. As mentioned above, our system resembles the M/M/C/C queuing system for which the steady state probability of any state x can be obtained from the balance equations:

$$\begin{aligned} \lambda_0 \pi(0) &= \mu \pi(1) \\ \lambda_1 \pi(1) &= 2\mu \pi(2) \\ &. \\ &. \\ \lambda_{x-1} \pi(x-1) &= x\mu \pi(x). \end{aligned} \quad (17)$$

By solving the above system of equations we obtain

$$\pi(x) = \frac{\prod_{i=0}^{x-1} a_i}{x!} \pi(0) \quad (18)$$

where $a_i = \frac{\lambda_i}{\mu}$ and λ_i is given by Eq. (10). $\pi(0)$ can be obtained by solving the normalization equation

$$\sum_{j=0}^{\Omega} \pi(j) = 1 \Rightarrow \pi(0) = \frac{1}{\left(\sum_{j=0}^{\Omega} \frac{\prod_{i=0}^{j} a_i}{j!}\right)}.$$

Substituting into (18), we get

$$\pi(x) = \frac{\frac{\prod_{i=0}^{x-1} a_i}{x!}}{\sum_{j=0}^{\Omega} \frac{\prod_{i=0}^{j} a_i}{j!}} \quad (19)$$

The average number of ongoing transmissions is given by

$$\overline{N}_T = \sum_{\forall x} \pi(x) x \quad (20)$$

Let R denote the data rate per time frame. One should realize that for each packet transmission it takes 2 time frames to deliver the packet to destination. Hence the effective data rate is given by $R/2$. Therefor the total average throughput of the network is given by

$$\overline{TH} = \overline{N}_T \frac{R}{2} \quad (21)$$

The steady state blocking probability is given by

$$P_B = \sum_{\forall x} \pi(x) P_B(x) \quad (22)$$

where $P_B(x)$ is given by (16).

V. SIMULATION RESULTS

In what follows we consider a total area of 100km^2, transmission range, d_{max} = 1km, λ=0.1-0.9, μ=1, total number of nodes, M=150-950, frame time of 1 sec, and data rate per transmission of 1Mbps. For the purpose of comparison, we also simulated the cooperative network protocol in [9] which employs only omnidirectional antennas.

Fig. 6 and Fig. 7 show how our protocol improves the transmission blocking probability and throughput relative to the CSMA/CA protocol in [9] for different utilization factors, $\rho = \frac{\lambda}{\mu}$.

The variation of the average network throughput with the node density is shown in Fig. 8. It is clear that for small node density, adding more nodes significantly increases the network throughput since it makes it easier to satisfy the conditions for finding a proper relay. Also, one can realize that the protocol using omnidirectional antennas [9] saturates at lower node

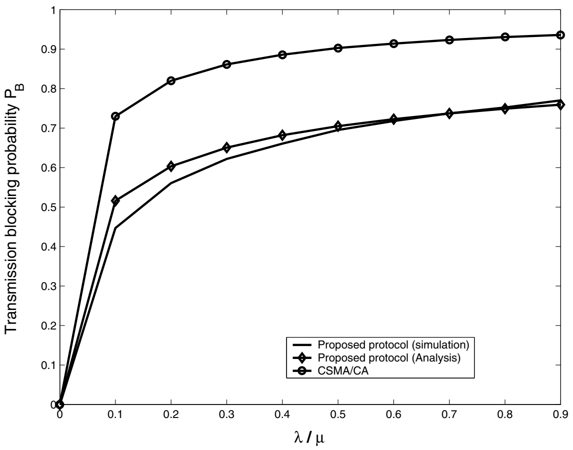

Fig. 6. Transmission blocking probability (10 nodes per km^2)

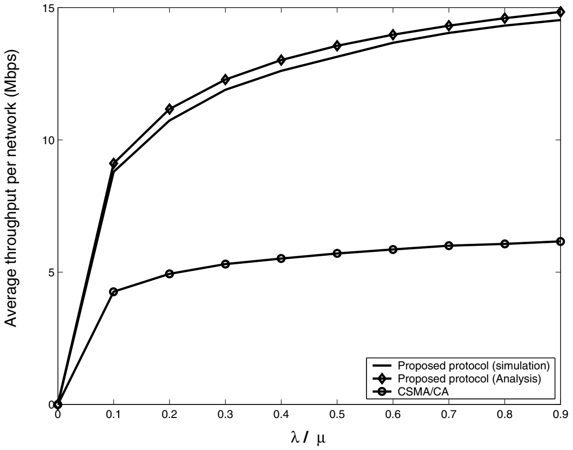

Fig. 7. Average throughput per network (10 nodes per km^2)

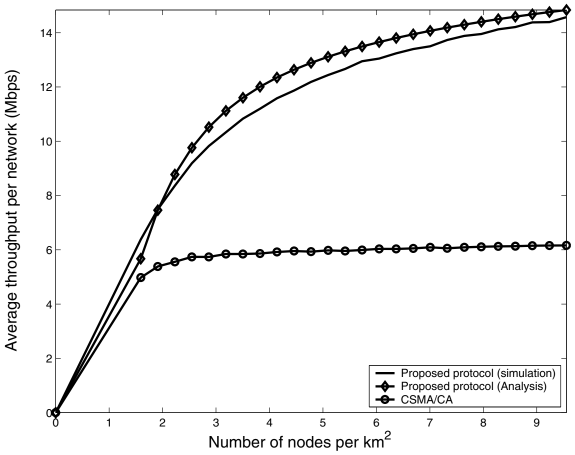

Fig. 8. Average throughput per network ($\lambda/\mu = 0.9$)

density as the criterion of choosing a relay in this case has less constrains than the criterion used in our protocol.

On the other hand, for large node density, the network throughput is more influenced by its utilization factor. It is also clear that the throughput of our protocol saturates at a much higher value compared to the protocol using omnidirectional antennas.

Both the analytical results above, and its matched simulation results, confirm that our proposed protocol outperforms the protocol using omnidirectional antennas [9].

VI. CONCLUSION

In this paper we proposed a new channel access protocol for transmission over cooperative ad-hoc networks. By utilizing directional antennas at both the source and relay stations, the proposed protocol improves the transmission blocking probability and average network throughput.

Markov chain modelling is used to model the network performance under the proposed settings. Steady state results for transmission blocking probability and average network throughput are obtained by analyzing the above Markov model and validated throughout simulations.

It should be noted that equipping the destination node with two directional receive antennas (one directed at the source and one at the relay) may provide further improvement in the network throughput. Providing a channel access protocol that utilizes this additional hardware seems to be an interesting future extension for this work. Incorporating the effect of the physical layer of the transmission channel provides another useful research direction.

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