Performance of Space-Time Diversity in CDMA over Frequency-Selective Fading Channels

Ayman Assra, Walaa Hamouda, and Amr Youssef
Concordia University
Montreal, Quebec, H3G 1M8, Canada
e-mail: {a_assra,hamouda,youssef}@ece.concordia.ca

***Abstract*— Asynchronous direct-sequence code-division multiple-access (DS-CDMA) using space-time spreading system is investigated over frequency-selective fast-fading channels. The underlaying transmit diversity scheme, previously introduced in the literature, is based on two transmit and one receive antenna. It was shown that when employed in flat fast-fading channels, the received signal quality can be improved by utilizing the spatial and temporal diversities at the receiver side. Here, the bit-error-rate (BER) performance of the underlying system is investigated for an uplink transmission where a decorrelator detector is used at the base station receiver. In particular, we derive a closed form expression for the probability of error over frequency-selective fast-fading channels. The analytical BER is derived as a function of both the number of multipath and users. The BER results show that the space-time spreading scheme exploits both the temporal and spatial diversity in a time-varying multipath channel. The results also show the accuracy of the derived expression when compared with simulation results for different number of users.**

I. Introduction

Multi-input multi-output (MIMO) systems allow the receiver to see independent versions of the information which yields to spatial diversity and/or coding gain compared to single antenna systems. One approach that uses multiple transmit antennas and, if possible, multiple receive antennas to provide reliable and high data rate communication is space-time coding (STC) [1]. It has been shown that STCs can offer these gains by introducing both temporal and spatial correlation into the transmitted signals from different antennas without increasing the total transmitted power or transmission bandwidth [1]. Depending on the structure of the STC used, one can achieve a coding gain and/or diversity gain [1],[2]. There are two major schemes: space time trellis codes (STTC) [1] and space time block codes (STBC) [2],[3]. In [4], a space time transmit diversity scheme suitable for DS-CDMA systems was proposed where it was examined in flat fading environment. Compared to other space-time spreading schemes, the proposed scheme achieves two-fold of the diversity order over fast-fading channels. Recently significant research efforts have aimed at the integration of STC with DS-CDMA systems over frequency-selective fading channels (e.g.,[5],[6]). In this paper, different from [4], we derive the probability of error of the space-time spreading (STS) scheme proposed in [4] for asynchronous DS-CDMA system over frequency-selective fast-fading channels. The proposed receiver is a Rake-type receiver that exploits the path diversity inherent in multipath propagation. Then, a decorrelator detector is used to mitigate the multiple access interference (MAI) and the known near-far problem [7].

II. System model

We consider an uplink transmission for asynchronous DS-CDMA system with K users. The system employs two transmit antennas at the transmitter side and one receive antenna at the receiver side. Since we consider an uplink scenario, setting the number of transmit antennas to two is a realistic assumption. We consider the space-time spreading system proposed in [4]. The proposed scheme can be summarized as follows. Assuming x_1,x_2 are data symbols assigned to each user in two consecutive symbol intervals, the space-time block coded signals transmitted during the first transmission period from antenna 1 and 2 are $x_1^* s_1 + x_2^* s_2$ and $x_1 s_2 - x_2 s_1$ respectively, where s_1 and s_2 are the spreading codes. These space-time coded signals are switched with respect to the antenna order during the second transmission period. We also consider frequency-selective fast-fading channel and binary-phase-shift keying (BPSK) transmission. Consider a multipath channel with L paths for each transmit antenna during each transmission period, the low pass equivalent of the received signal at the base station (assuming asynchronous transmission from the K users) can be expressed as

$$\begin{aligned} r(t) = \sum_{k=1}^{K}\sum_{l=1}^{L}\sqrt{E_s}(h_{1l}^{k,t}(x_1^{k*}s_1^k(t-\tau_k-\tilde{\tau}_l)+ \\ x_2^{k*}s_2^k(t-\tau_k-\tilde{\tau}_l)) + h_{2l}^{k,t}(x_1^k s_2^k(t-\tau_k-\tilde{\tau}_l) \\ - x_2^k s_1^k(t-\tau_k-\tilde{\tau}_l)) + h_{1l}^{k,t+T_b}(x_1^k s_2^k(t-T_b-\tau_k-\tilde{\tau}_l) \\ - x_2^k s_1^k(t-T_b-\tau_k-\tilde{\tau}_l)) + h_{2l}^{k,t+T_b}(x_1^{k*}s_1^k(t-T_b-\tau_k-\tilde{\tau}_l) \\ + x_2^{k*}s_2^k(t-T_b-\tau_k-\tilde{\tau}_l))) + n(t). \quad (1) \end{aligned}$$

In (1), E_s is the received signal energy for the single user, x_1^k and x_2^k are the even and odd k^{th} user data symbols, $s_1^k(t)$ and $s_2^k(t)$ are the two spreading codes assigned to the k^{th} user with processing gain (T_b/T_c) where T_b is the bit period, T_c is the chip period, and τ_k represents the transmit delay of the k^{th} user, assumed to be multiple of chip periods. $\tilde{\tau}_l$ represents the delay of each path during each transmission period which is modeled as an integer number of chips and taken small compared to the symbol period to neglect the effect of intersymbol interference (ISI). We also assume that each

1930-529X/07/$25.00 © 2007 IEEE
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2007 proceedings.

pair of paths from the two transmitter antennas of any user arrives with the same set of delays at the receiver antenna. The channel coefficients $h_{il}^{k,t}$ and $h_{il}^{k,t+T_b}$, $(i = 1, 2)$ model the fading channel corresponding to the k^{th} user l^{th} path from the i^{th} transmit antenna to the base station at time t and $t + T_b$ respectively. The fading coefficients are modeled as independent complex Gaussian random variables with zero mean and unity variance. The noise $n(t)$ is Gaussian with zero mean and variance $\sigma_n^2 = N_o/2$. From (1), the received signal can be represented in a more compact form as

$$r(t) = \sum_{k=1}^{K}\sum_{l=1}^{L} s_1^k(t-\tau_k-\tilde{\tau}_l)u_{1l}^{k,t} + s_2^k(t-\tau_k-\tilde{\tau}_l)u_{2l}^{k,t} + s_1^k(t-T_b-\tau_k-\tilde{\tau}_l)u_{1l}^{k,t+T_b} + s_2^k(t-T_b-\tau_k-\tilde{\tau}_l)u_{2l}^{k,t+T_b} + n(t) \quad (2)$$

where $u_{1l}^{k,t} = \sqrt{E_s}(h_{1l}^{k,t}x_1^{k*} - h_{2l}^{k,t}x_2^k)$, $u_{2l}^{k,t} = \sqrt{E_s}(h_{1l}^{k,t}x_2^{k*} + h_{2l}^{k,t}x_1^k)$, $u_{1l}^{k,t+T_b} = \sqrt{E_s}(-h_{1l}^{k,t+T_b}x_2^k + h_{2l}^{k,t+T_b}x_1^{k*})$ and $u_{2l}^{k,t+T_b} = \sqrt{E_s}(h_{1l}^{k,t+T_b}x_1^k + h_{2l}^{k,t+T_b}x_2^{k*})$. The multiuser detector consists of $2LK$ filters matched to the delayed versions of the normalized signature waveforms of each user. The output of this filter bank, sampled at the chip rate during one ST-block code interval which is equivalent to two symbol intervals (i.e., the length of one block interval, P =2), is given in a vector form by $\boldsymbol{Y} = \boldsymbol{RU} + \boldsymbol{N}$. The $(2LPK \times 1)$ vector $\boldsymbol{Y}$ includes the output of the matched filter bank at time t and $t + T_b$, given by

$$\boldsymbol{Y} = [y_{1,1,1}^t y_{1,2,1}^t \dots y_{1,2,L}^t y_{1,1,1}^{t+T_b} \dots y_{K,1,1}^t \dots y_{K,2,L}^{t+T_b}]^T \quad (3)$$

where the superscript T denotes vector transpose and $y_{k,p,l}^t$, $y_{k,p,l}^{t+T_b}$, $p = 1, 2$, represent the outputs of the filter matched to the l^{th} path of the p^{th} sequence for user k at times t and $t+T_b$ respectively. The vector $\boldsymbol{U}$ which represents the faded data is given by $\boldsymbol{U} = [\boldsymbol{U}_1^T\boldsymbol{U}_2^T \dots \boldsymbol{U}_k^T \dots \boldsymbol{U}_K^T]^T$ where the $(2LP \times 1)$ vector $\boldsymbol{U}_k$ represents the faded data transmitted by the k^{th} user over two successive symbols, and is defined as

$$\boldsymbol{U}_k = [u_{11}^{k,t}u_{21}^{k,t}u_{12}^{k,t}\dots u_{2L}^{k,t}u_{11}^{k,t+T_b}u_{21}^{k,t+T_b}\dots u_{2L}^{k,t+T_b}]^T. \quad (4)$$

The $(2LPK \times 2LPK)$ cross correlation matrix $\boldsymbol{R}$ is given by

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{R_{11}} & \boldsymbol{R_{12}} & \cdots & \boldsymbol{R_{1K}} \\ \vdots & \cdots & \cdots & \vdots \\ \boldsymbol{R_{K1}} & \cdots & \cdots & \boldsymbol{R_{KK}} \end{bmatrix} \quad (5)$$

where $\boldsymbol{R}_{kw}(w = 1, \cdots, K)$ is $(2LP \times 2LP)$ matrix defined as $\boldsymbol{R}_{kw} = \int_{\tau_k}^{\tau_k+PT} \boldsymbol{S}_k(t)\boldsymbol{S}_w^H(t)\,dt$ where the superscript H denotes Hermitian transpose. $\boldsymbol{S}_k(t)$ represents all the delayed versions of the two codes assigned to the k^{th} user during the two symbol periods, described as

$$\boldsymbol{S}_k(t) = [s_1^k(t-\tau_k-\tilde{\tau}_1)s_2^k(t-\tau_k-\tilde{\tau}_1)\cdots s_1^k(t-\tau_k-\tilde{\tau}_L) \cdots s_1^k(t-T_b-\tau_k-\tilde{\tau}_1)\cdots s_2^k(t-T_b-\tau_k-\tilde{\tau}_L)]^T.$$

The $(2LPK \times 1)$ noise vector $\boldsymbol{N}$ is given by

$$\boldsymbol{N} = [\boldsymbol{N}_1^T\boldsymbol{N}_2^T\dots\boldsymbol{N}_k^T\dots\boldsymbol{N}_K^T]^T \quad (6)$$

with $\boldsymbol{N}_k = [n_{11}^{k,t}n_{21}^{k,t}n_{12}^{k,t}\dots n_{2L}^{k,t}n_{11}^{k,t+T_b}n_{21}^{k,t+T_b}\dots n_{2L}^{k,t+T_b}]^T$ and each of the elements $n_{pl}^{k,t}$, $n_{pl}^{k,t+T_b}$ $(p = 1, 2$ and $l = 1, .., L)$ are modeled as complex Gaussian random variables, each with variance $N_o/2$ per dimension. As will be shown later, this scheme will yield to D-fold diversity order where $D = 2PL$. Note that the output of the matched filter bank suffers from MAI which can be eliminated using the decorrelator detector. In this case, the output of the matched filter bank $\boldsymbol{Y}$ is applied to a linear mapper $\boldsymbol{Z} = \boldsymbol{R^{-1}Y}$, where $\boldsymbol{R^{-1}}$ is the inverse of the cross correlation matrix. The $(2LPK \times 1)$ vector $\boldsymbol{Z}$ represents the output of the decorrelator during two successive symbol periods. It includes the L replicas of the signals from the two antennas for each user during one ST-block interval as follows $\boldsymbol{Z} = [\boldsymbol{Z}_1^T\boldsymbol{Z}_2^T\dots\boldsymbol{Z}_k^T\dots\boldsymbol{Z}_K^T]^T$ where the $(2LP \times 1)$ vector $\boldsymbol{Z}_k$ is defined by

$$\boldsymbol{Z}_k = [z_{11}^{k,t}z_{21}^{k,t}z_{12}^{k,t}\dots z_{2L}^{k,t}z_{11}^{k,t+T_b}z_{21}^{k,t+T_b}\dots z_{2L}^{k,t+T_b}]^T \quad (7)$$

with the elements $z_{pl}^{k,t}$, $z_{pl}^{k,t+T_b}$ represent the output of the decorrelator corresponding to the l^{th} path of the p^{th} sequence for user k at times t and $t + T_b$, respectively. One can extract the two transmitted symbols of the k^{th} user as follows

$$\hat{x}_1^k = \sum_{l=1}^{L} h_{1l}^{k,t}z_{1l}^{k,t*} + h_{2l}^{k,t*}z_{2l}^{k,t} + h_{2l}^{k,t+T_b}z_{1l}^{k,t+T_b*} + h_{1l}^{k,t+T_b*}z_{2l}^{k,t+T_b}$$

$$\hat{x}_2^k = \sum_{l=1}^{L} h_{1l}^{k,t}z_{2l}^{k,t*} - h_{2l}^{k,t*}z_{1l}^{k,t} - h_{1l}^{k,t+T_b*}z_{1l}^{k,t+T_b} + h_{2l}^{k,t+T_b}z_{2l}^{k,t+T_b*}.$$

Considering the first symbol estimate of the k^{th} user, and defining the variable $v_k = 2PL(k-1)$,

$$\hat{x}_1^k = \sum_{l=1}^{L} \sqrt{E_s}(|h_{1l}^{k,t}|^2 + |h_{2l}^{k,t}|^2 + |h_{1l}^{k,t+T_b}|^2 + |h_{2l}^{k,t+T_b}|^2)x_1^k + \sum_{l=1}^{L} h_{1l}^{k,t}(\boldsymbol{R^{-1}N})_{2l+v_k-1,1}^* + h_{2l}^{k,t*}(\boldsymbol{R^{-1}N})_{2l+v_k,1} + h_{2l}^{k,t+T_b}(\boldsymbol{R^{-1}N})_{2(L+l)+v_k-1,1}^* + h_{1l}^{k,t+T_b*}(\boldsymbol{R^{-1}N})_{2(L+l)+v_k,1}. \quad (8)$$

III. Performance Analysis

The average BER can be derived by finding the probability density function (PDF) for the signal to interference ratio (SIR). Then, using this PDF, the probability of error using the decorrelator multiuser detector is obtained. Without loss of generality, we consider the case of finding the probability of making error on the first symbol of user 1. To avoid complex notation, we drop its the corresponding superscript from the fading coefficients. In this case, we consider the first $2LP$ elements from the decorrelator output vector $\boldsymbol{Z}$. Hence, the probability of error conditioned on all channel coefficients can be expressed as

$$P_b(\hat{x}_1 = 1|h_{11}^t, h_{21}^t, ..., h_{2L}^t, ..., h_{2L}^{t+T_b}) = Q\left(\frac{\sum_{l=1}^{L}\sqrt{E_s}(a_{1l}^t + a_{2l}^t + a_{1l}^{t+T_b} + a_{2l}^{t+T_b})}{\sqrt{\sigma_{\hat{x}}^2}}\right) \quad (9)$$

where $Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} exp\left(-\frac{\nu^2}{2}\right) d\nu$, $a_{1l}^t = |h_{1l}^t|^2$, $a_{2l}^t = |h_{2l}^t|^2$, $a_{1l}^{t+T_b} = |h_{1l}^{t+T_b}|^2$, $a_{2l}^{t+T_b} = |h_{2l}^{t+T_b}|^2$, and $\sigma_{\hat{x}}^2$ is the variance of the noise term in (8) when $k = 1$. It is easy to show that

$$\sigma_{\hat{x}}^2 = \sigma_n^2 \sum_{l=1}^{L} \Big(|h_{1l}^t|^2 R_{2l-1,2l-1}^{-1} + |h_{2l}^t|^2 R_{2l,2l}^{-1} + |h_{2l}^{t+T_b}|^2 R_{2(L+l)-1,2(L+l)-1}^{-1} + |h_{1l}^{t+T_b}|^2 R_{2(L+l),2(L+l)}^{-1} \Big)$$

where $R_{i,i}^{-1}$ represents the i^{th} diagonal element of the inverse of the cross correlation matrix in (5). The variables a_{il}^t and $a_{il}^{t+T_b}$ $(i = 1, 2)$ are chi-square distributed with two degrees of freedom and characteristic function, $\phi(j\omega) = \frac{1}{1-j2\omega\sigma^2}$. Also, the noise variance $\sigma_{\hat{x}}^2$ can be expressed in terms of a_{il}^t and $a_{il}^{t+T_b}$ $(i = 1, 2)$ as follows

$$\sigma_{\hat{x}}^2 = \sigma_n^2 \sum_{l=1}^{L} \Big(c_{2l-1} a_{1l}^t + c_{2l} a_{2l}^t + c_{2(L+l)-1} a_{2l}^{t+T_b} + c_{2(L+l)} a_{1l}^{t+T_b} \Big)$$

where the parameters c_{2l-1}, c_{2l}, $c_{2(L+l)-1}$ and $c_{2(L+l)}$ define the following terms $R_{2l-1,2l-1}^{-1}$, $R_{2l,2l}^{-1}$, $R_{2(L+l)-1,2(L+l)-1}^{-1}$ and $R_{2(L+l),2(L+l)}^{-1}$, respectively. Let us define the variable α as $\alpha = \frac{A}{\sqrt{B}}$, where $A = \sum_{l=1}^{L} (a_{1l}^t + a_{2l}^t + a_{1l}^{t+T_b} + a_{2l}^{t+T_b})$ and $B = \sum_{l=1}^{L} (c_{2l-1} a_{1l}^t + c_{2l} a_{2l}^t + c_{2(L+l)-1} a_{2l}^{t+T_b} + c_{2(L+l)} a_{1l}^{t+T_b})$. Defining $y = \frac{1}{2\sigma^2} - j\omega_1$ and assuming independent fading channels, one can show that the joint characteristic function of A and B is given by

$$\phi_{A,B}(\omega_1, \omega_2) = \frac{1}{(2\sigma^2)^{4L}} \prod_{l=1}^{L} \frac{1}{(y - jc_{2l-1}\omega_2)(y - jc_{2l}\omega_2)} \times \frac{1}{(y - jc_{2(L+l)-1}\omega_2)(y - jc_{2(L+l)}\omega_2)}. \quad (10)$$

Using the partial fraction method, we get

$$\phi_{A,B}(\omega_1, \omega_2) = \frac{1}{(2\sigma^2)^{4L}} \sum_{l=1}^{L} \Big(\frac{b_{2l-1}}{y - jc_{2l-1}\omega_2} + \frac{b_{2l}}{y - jc_{2l}\omega_2} + \frac{b_{2(L+l)-1}}{y - jc_{2(L+l)-1}\omega_2} + \frac{b_{2(L+l)}}{y - jc_{2(L+l)}\omega_2} \Big) \quad (11)$$

where $b_d = \frac{1}{K_d y^{4L-1}}$, $d \in \{\, 2l-1, 2l, 2(L+l)-1, 2(L+l) \}$, denote the residue terms obtained from the partial fraction expansion. An exact expression for K_d can be obtained in terms of the cross correlation coefficients between the users' signature waveforms. Using (11), the joint PDF, $f_{A,B}$, can be obtained as

$$f_{A,B} = \frac{1}{4\pi^2 (2\sigma^2)^{4L}} \sum_{l=1}^{L} I_{2l-1} + I_{2l} + I_{2(L+l)-1} + I_{2(L+l)} \quad (12)$$

where

$$I_d = \int_{-\infty}^{\infty} b_d \exp(-j\omega_1 A)\, d\omega_1 \times \int_{-\infty}^{\infty} \frac{\exp(-j\omega_2 B)}{y - jc_d\omega_2} d\omega_2 \quad (13)$$

and $d = 2l-1, 2l, 2(L+l)-1, 2(L+l)$. By solving (13) first with respect to ω_2 then ω_1, it is easy to show that

$$I_d = \frac{4\pi^2}{\Gamma(4L-1) c_d K_d} \left(A - \frac{B}{c_d} \right)^{4L-2} \exp\left(-\frac{A}{2\sigma^2} \right) \quad (14)$$

where $\Gamma(\cdot)$ is the Gamma function. The second integration in (13) with respect to ω_1 is estimated using the inverse of the characteristic function which corresponds to Gamma distribution conditioned on $(A - \frac{B}{c_d}) \geq 0$ [8]. Substituting $I_d, d = 2l-1, 2l, 2(L+l)-1, 2(L+l)$ in (12), we get

$$f_{A,B} = \frac{1}{\Gamma(4L-1)(2\sigma^2)^{4L}} \exp\left(-\frac{A}{2\sigma^2} \right) \times \sum_{l=1}^{L} \sum_{d \in \{2l-1, 2l, 2(L+l)-1, 2(L+l)\}} \frac{\left(A - \frac{B}{c_d} \right)^{4L-2}}{K_d c_d}. \quad (15)$$

One way to obtain the PDF of α is through variable transformation. From the definition of α and by assuming that $W = B$, the joint PDF of α and W can be determined through the following relation

$$f_{\alpha,W} = f_{A,B} |J(\alpha, W)| \quad (16)$$

where $|J(\alpha, W)| = \sqrt{W}$ is the jacobian of the transformation. After some algebraic manipulations, we have

$$f_{\alpha,W} = \frac{\sqrt{W}}{\Gamma(4L-1)(2\sigma^2)^{4L}} \exp\left(-\frac{\alpha\sqrt{W}}{2\sigma^2} \right) \times \sum_{l=1}^{L} \sum_{d \in \{2l-1, 2l, 2(L+l)-1, 2(L+l)\}} \frac{\left(\alpha\sqrt{W} - \frac{W}{c_d} \right)^{4L-2}}{K_d c_d}. \quad (17)$$

The PDF of the SINR can be obtained from (17) as

$$f_\alpha = \frac{1}{\Gamma(4L-1)(2\sigma^2)^{4L}} \sum_{l=1}^{L} \left(\frac{P_{2l-1}}{K_{2l-1} c_{2l-1}} + \frac{P_{2l}}{K_{2l} c_{2l}} + \frac{P_{2(L+l)-1}}{K_{2(L+l)-1} c_{2(L+l)-1}} + \frac{P_{2(L+l)}}{K_{2(L+l)} c_{2(L+l)}} \right) \quad (18)$$

where the terms P_{2l-1}, P_{2l}, $P_{2(L+l)-1}$ and $P_{2(L+l)}$ are defined through the general form P_d $(d = 2l-1, 2l, 2(L+l)-1, 2(L+l))$ as follows

$$P_d = \int_0^{c_d^2 \alpha^2} \sqrt{W} \left(\alpha\sqrt{W} - \frac{W}{c_d} \right)^{4L-2} \exp\left(-\frac{\alpha\sqrt{W}}{2\sigma^2} \right) dW. \quad (19)$$

Using the binomial series expansion, the integration in (19) can be reduced to

$$P_d = \sum_{q=0}^{4L-2} \binom{4L-2}{q} \frac{(-1)^{4L-2-q} \alpha^q}{c_d^{4L-2-q}} \times \int_0^{c_d^2 \alpha^2} (\sqrt{W})^{8L-3-q} \exp\left(-\frac{\alpha\sqrt{W}}{2\sigma^2} \right) dW. \quad (20)$$

Note that the limit of W in (20) is a consequence of the second integration in (13) as it was conditioned on $(A - \frac{B}{c_d}) \geq 0$. Using $\alpha = \frac{A}{\sqrt{B}}$ and the assumption that $W = B$, this condition can be mapped to $0 \leq W \leq c_d^2\alpha^2$. In the following, we denote the integration in (20) by II_d and use

$$\int_{c_1}^{c_2} x^n e^{-ax} dx = \frac{1}{a(n+1)}\Big[c_2^n(ac_2)^{-\frac{n}{2}} e^{-\frac{ac_2}{2}} \times M(\frac{n}{2}, \frac{n+1}{2}, ac_2) - c_1^n(ac_1)^{-\frac{n}{2}} e^{-\frac{ac_1}{2}} M(\frac{n}{2}, \frac{n+1}{2}, ac_1)\Big] \tag{21}$$

where $M(k, m, z)$ represents the WhittakerM function [9]. Using the substitution $t = \sqrt{W}$, we have

$$II_d = 2\int_0^{c_d\alpha} t^{8L-2-q} \exp\left(-\frac{\alpha t}{2\sigma^2}\right) dt = \beta_d \exp\left(-\frac{c_d\alpha^2}{4\sigma^2}\right) M(\frac{8L-2-q}{2}, \frac{8L-1-q}{2}, \frac{c_d\alpha^2}{2\sigma^2}). \tag{22}$$

where $\beta_d = \frac{2^{\frac{8L-q+2}{2}}\sigma^{8L-q}c_d^{\frac{8L-2-q}{2}}}{(8L-1-q)\alpha}$. The result in (22) can be expressed in terms of the confluent hypergeometric function ([9], eq.(13.1.32))

$$M(k, m, z) = z^{\frac{1+2m}{2}} e^{-\frac{z}{2}} \ {}_1F_1(m - k + \frac{1}{2}; 1 + 2m; z) \tag{23}$$

where ${}_1F_1(a; b; z) = \sum_{n=0}^{\infty} \frac{(a)_n}{(b)_n}\frac{z^n}{n!}$ is the confluent hypergeometric function with $(a)_n$ and $(b)_n$ representing Pochhammer symbols. Using (23),

$$II_d = \gamma_d \exp\left(-\frac{c_d\alpha^2}{2\sigma^2}\right) \ {}_1F_1(1; 8L - q; \frac{c_d\alpha^2}{2\sigma^2}). \tag{24}$$

where $\gamma_d = \frac{2(\alpha c_d)^{8L-1-q}}{8L-1-q}$. Substituting (24) in (20), we obtain

$$P_d = (2\alpha^{8L-1}c_d^{4L+1}) \exp\left(-\frac{c_d\alpha^2}{2\sigma^2}\right) \times \sum_{q=0}^{4L-2} \binom{4L-2}{q} \frac{(-1)^{4L-2-q}}{8L-q-1} \ {}_1F_1(1; 8L - q; \frac{c_d\alpha^2}{2\sigma^2}). \tag{25}$$

Using (25) we can find the forms corresponding to P_{2l-1}, P_{2l}, $P_{2(L+l)-1}$ and $P_{2(L+l)}$, and hence the probability function of the SINR in (18).

IV. THE AVERAGE PROBABILITY OF ERROR

The probability of error can be obtained by averaging the conditional bit error in (9) over the PDF of α

$$P_b = \int_0^{\infty} Q\left(K_s\alpha\right) f_\alpha d\alpha, \tag{26}$$

where $K_s = \frac{\sqrt{E_s}}{\sigma_n}$. To simplify the analysis, we use the *preferred* form of the Gaussian Q-function [10], $Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp^{-\frac{x^2}{2sin^2\theta}} d\theta$. Substituting (18) and the *preferred* form of the Gaussian Q-function in (26), we get

$$P_b = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\int_0^{\infty} \exp^{-\frac{K_s^2\alpha^2}{2sin^2\theta}} f_\alpha d\alpha d\theta = \rho\sum_{l=1}^{L}\sum_{d\in\{2l-1, 2l, 2(L+l)-1, 2(L+l)\}} \frac{F_d}{K_d c_d}. \tag{27}$$

where $\rho = \frac{1}{\pi\Gamma(4L-1)(2\sigma^2)^{4L}}$. Let

$$F_d = \int_0^{\frac{\pi}{2}}\int_0^{\infty} \exp^{-\frac{K_s^2\alpha^2}{2sin^2\theta}} P_d d\alpha d\theta \tag{28}$$

where $d = 2l - 1, 2l, 2(L + l) - 1$ and $2(L + l)$. Substituting P_d from (25), we get

$$F_d = 2c_d^{4L+1}\sum_{q=0}^{4L-2}\binom{4L-2}{q}\frac{(-1)^{4L-2-q}}{8L-1-q} G_q \tag{29}$$

where

$$G_q = \int_0^{\frac{\pi}{2}}\int_0^{\infty} \alpha^{8L-1} \exp^{-\alpha^2(\frac{K_s^2}{2sin^2\theta}+\frac{c_d}{2\sigma^2})} \times \ {}_1F_1(1; 8L - q; \frac{c_d\alpha^2}{2\sigma^2}) d\alpha d\theta. \tag{30}$$

Substituting $r = \alpha^2$ in (30) and using ([11], eq. (7.621.4)), (30) is reduced to

$$G_q = \frac{\Gamma(4L)2^{4L-1}}{K_s^{8L}}\int_0^{\frac{\pi}{2}} (sin^2\theta)^{4L} \times \ {}_2F_1(8L - q - 1, 4L; 8L - q; -\frac{c_d sin^2\theta}{\sigma^2 K_s^2}) d\theta \tag{31}$$

where ${}_2F_1(.,.;.;.)$ is a special case of the generalized hypergeometric function defined by ([11], eq. (9.14.1)) and $(\alpha_p)_k$, $(\beta_q)_k$ are Pochhammer symbols. Using ([11], eq. (7.512.12)), and by introducing the substitution $V = sin^2\theta$ in (31),

$$G_q = \frac{2^{4L-2}\Gamma(1/2)\Gamma(\frac{8L+1}{2})}{4LK_s^{8L}} \times \ {}_3F_2(\frac{8L+1}{2}, 8L - q - 1, 4L; 4L + 1, 8L - q; -\frac{c_d}{\sigma^2 K_s^2}). \tag{32}$$

Substitution of the different values of d in the general form of (29) and (32), we can evaluate the average probability of error (27).

V. SIMULATION RESULTS

We consider an asynchronous DS-CDMA system with BPSK transmission where every user data is spread using Gold codes of length 31 chips is considered. For the asynchronous channel, the delay between users, τ_k, is assumed to be multiple of chip periods within the symbol interval. To neglect the effect of ISI, the delay of each path, $\tilde{\tau}_p$, is taken as a multiple of chip periods of length less than 10% of the symbol period. Furthermore, we assume perfect knowledge of the channel coefficients at the receiver. To simplify the simulations, we consider a multipath channel with $L = 2$ per transmit antenna per transmission interval. Fig. 1 shows the error performance analysis for different number of users in the frequency-selective fast-fading channel. The results demonstrate the accuracy of the derived bit error rate expression in (27) when compared with the simulation results. From the slope of the BER at high SNRs, we find that an 8-fold diversity order is achieved for different number of users. It should be noted that the proposed receiver is only optimal in the high SNR region. This diversity is delivered by the M=2 transmit antennas, L=2 paths and $P = 2$ length of the space-time block interval where the fading coefficients change independently from one symbol to another. Fig. 2 shows the BER performance of the STS scheme for 3-user system over frequency-selective fast-fading channel assuming two and three paths per transmit antenna. The results clearly show the effect of increasing the number of multipath on the diversity gain. The transmit diversity scheme with $L = 3$ paths achieves diversity order of MLP =12 when compared with the maximal ratio combiner (MRC) diversity branches.

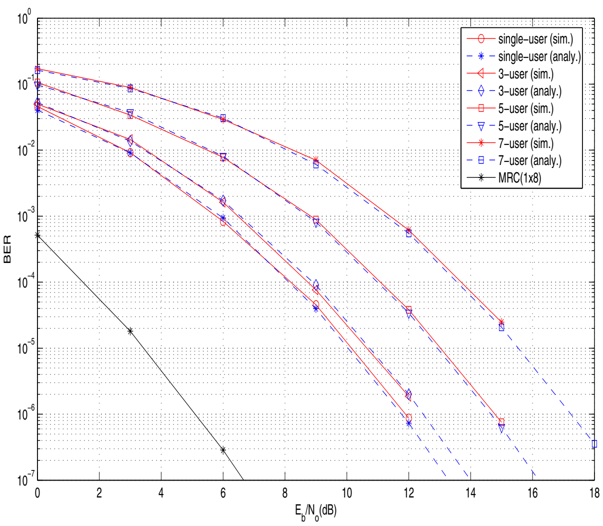

Fig. 1. BER performance for asynchronous DS-CDMA systems with two transmit and one receive antenna over frequency-selective fast-fading channels with L=2 paths.

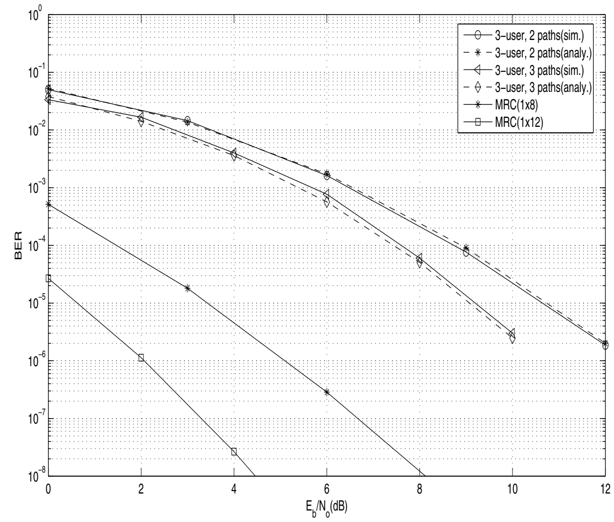

Fig. 2. Performance improvement for a 3-user system as a function of the number of paths, L=2,3, over frequency-selective fast-fading channel.

VI. Conclusion

We examined the performance of transmit diversity using space-time spreading in asynchronous DS-CDMA systems over frequency-selective fast-fading channels. For the multi-path fading channel with L paths, we formulated a space-time combining technique to extract the full system diversity. The performance of the decorrelator detector was examined where we derived analytical expression for the probability of bit error over the fast-fading channel as a function of both the number of users and paths. Our results have shown that the space-time system is able to deliver the full system diversity with different interference levels.

References

[1] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction", *IEEE Trans. Inform. Theory*, vol. 44, pp. 774–765, Mar. 1998.

[2] S. M. Alamouti, "A simple transmit diversity technique for wireless communications", *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, Oct. 1998.

[3] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs", *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456–1467, July 1999.

[4] M. Aljerjawi and W. Hamouda, "Performance of space-time spreading in multiuser DS-CDMA systems over fast fading channels", in *Proc. IEEE Global Telecommunications Conference*, Nov./Dec. 2005, vol. 3, pp. 1525–1529.

[5] B. Hochwald, T. Marzetta, and C. Papadias, "A transmitter diversity scheme for wideband CDMA systems based on space-time spreading", *IEEE J. Select. Areas Commun.*, vol. 19, no. 1, pp. 1451–1458, Jan. 2001.

[6] L. Chong and L. Milstein, "The effects of channel-estimation errors on a space-time spreading CDMA system with dual transmit and dual receive diversity", *IEEE Trans. Commun.*, vol. 52, no. 7, pp. 1145–1151, July 2004.

[7] R. Lupas and S. Verdu, "Near-far resistance of multiuser detectors in asynchronous channels", *IEEE Trans. Commun.*, vol. 38, no. 4, pp. 496–508, April 1990.

[8] A. Papoulis and S.U. Pillai, *Probability, Random Variables and Stochastic Processes*, McGraw Hill, 2002.

[9] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*, New York:Dover, 1964.

[10] M. K. Simon and M. Alouini, "A unified approach to the performance analysis of digital communication over generalized fading channels", *Proc. of the IEEE*, vol. 86, pp. 1860–1877, Sep. 1998.

[11] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, New York:Academic, 1995.

1930-529X/07/$25.00 © 2007 IEEE
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2007 proceedings.