Performance of Superimposed Training-Based Channel Estimation in MIMO-CDMA Systems

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***Abstract*— In this paper, we investigate the performance of superimposed training-based channel estimation techniques considering an asynchronous code-division multiple-access (CDMA) uplink transmission over frequency-selective fading channels. In that, we analyze the performance of a channel estimation and data detection scheme based on superimposed training for space-time spreading (STS) systems. Our results show that the scheme enhances the performance of the space-time system by eliminating the interference effect from both the channel and data estimates using two decorrelators; channel and data decorrelators. Compared with other conventional estimation techniques, the underlying system is more robust to channel estimation errors. Furthermore, both simulations and analytical results indicate that full system diversity is achieved.**

I. Introduction

In multiple-input multiple-output (MIMO) systems, channel estimation plays a crucial role in determining the system performance [1]. While the majority of the works in MIMO assume perfect channel estimation, relatively few researchers have investigated the effect of channel estimation errors and possible estimation techniques. For instance, Chong and Milstein in [2], employed the superimposed training-based technique on a STS system with dual-transmit and dual-receive diversity. In their work, the channel estimation was based on employing distinct pilot spreading codes on STS signals transmitted from different antennas. With the help of these pilot signals, the channel coefficients are estimated using a conventional Rake receiver. These channel estimates are then used to combine the received signals in a Rake-like space-time combiner for final data estimates. It is noted that, both data and channel estimates suffer from intersymbol interference (ISI) and multi-access interference (MAI). In light of this, in [3] an enhanced channel estimation and data detection scheme based on the superimposed training-based approach was proposed. In this paper, we analyze the performance of the superimposed training-based channel estimation introduced in [3] over asynchronous CDMA systems. Both analytical and simulation results show that different from joint channel estimation and data detection, where full system diversity is achieved, conventional superimposed training-based channel estimation techniques perform poorly due to the effect of multiuser interference on channel estimates.

The remainder of this paper is organized as follows. The following section describes the space-time system model used in our analysis. The channel estimation and detection scheme is discussed in Sections III and IV. In Section V, the performance of the proposed estimation technique is analyzed in-terms of the average BER. In Section VI, simulations and analytical results are presented. Finally, conclusions are drawn in Section VII.

II. System Model

The transmit diversity scheme considered in our work consists of two transmit antennas at the mobile station and V receive antennas at the base station. The system block diagram for the k^{th} transmitting user is shown in Fig. 1, where real valued data symbols using binary-phase-shift keying (BPSK) baseband modulation and real valued spreading are assumed. We consider the original STS scheme proposed in [4] with two spreading codes per user. As seen in Fig. 1, following the STS, two pilot spreading codes are assigned to each user for the purpose of channel estimation. Each pilot sequence is added (superimposed) on the STS signal before transmission from the corresponding antenna. We also consider an uplink asynchronous transmission from K users over frequency-selective fading channel, where the fading coefficients are fixed for the duration of M-symbol data block but change independently from one block to another. Given the space-time scheme in [4], the duration of the space-time codeword is $T_s = 2T_b$, where T_b is the bit duration. The received complex low-pass equivalent signal at the v^{th} receive antenna is given by

$$r^v(t) = \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{m=0}^{M-1} h_{1l}^{k,v}\left[\sqrt{\frac{\rho_p}{2}}P_{k1}(t-mT_s-\tau_k-\tilde{\tau}_l) + \sqrt{\frac{\rho_d}{2}}(b_{k1}[m]c_{k1}(t-mT_s-\tau_k-\tilde{\tau}_l)+b_{k2}[m]c_{k2}(t-mT_s-\tau_k-\tilde{\tau}_l))\right] + h_{2l}^{k,v}\left[\sqrt{\frac{\rho_p}{2}}P_{k2}(t-mT_s-\tau_k-\tilde{\tau}_l)+\sqrt{\frac{\rho_d}{2}}(b_{k2}[m] \times c_{k1}(t-mT_s-\tau_k-\tilde{\tau}_l)-b_{k1}[m]c_{k2}(t-mT_s-\tau_k-\tilde{\tau}_l))\right]+n^v(t), \quad (1)$$

where ρ_p and ρ_d represent the pilot and data signal-to-noise ratios (SNRs), respectively. The data bits, $b_{k1}[m]$ and $b_{k2}[m]$, represent the odd and even data streams of the k^{th} user within the m^{th} codeword interval. In (1), $c_{k1}(t)$ and $c_{k2}(t)$ are the k^{th} user's data spreading sequences with processing gain $2N$,

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where $N = T_b/T_c$ represents the number of chips per bit, and T_c is the chip duration. The two pilot spreading sequences, $P_{k1}(t)$ and $P_{k2}(t)$, assigned to the k^{th} user have a period of $2T_b$. In our analysis, we assume that the pilot and data spreading sequences assigned to the K users are mutually orthogonal at the transmitter side. However due to asynchronous transmission, this orthogonality condition between codes is no longer valid at the receiver side. τ_k represents the transmit delay of the k^{th} user signal which is assumed to be multiple of chip periods within T_s. $\tilde{\tau}_l$ is the l^{th} path delay ($\tilde{\tau}_l = lT_c$), $h_{il}^{k,v}$, $i = 1, 2$, is the channel coefficient corresponding to the k^{th} user, l^{th} path from the i^{th} transmit antenna to v^{th} receive antenna, and L is the total number of resolvable paths. These fading coefficients are modeled as independent complex Gaussian random variables with zero mean and unit variance. We also consider a time-invariant channel over the duration of an M-symbol data block. The noise $n^v(t)$ is Gaussian with zero mean and unit variance. At the receiver, the received signal is first sent to a channel estimator, where the path gain estimates $\{W_{il}^{k,v}, W_{il}^{k,v}\}$ of the l^{th} path between transmit antenna i=1,2, and receive antenna v are obtained. Then, the STS signals are detected using the estimated path gains. The receiver structure is further illustrated in the following sections.

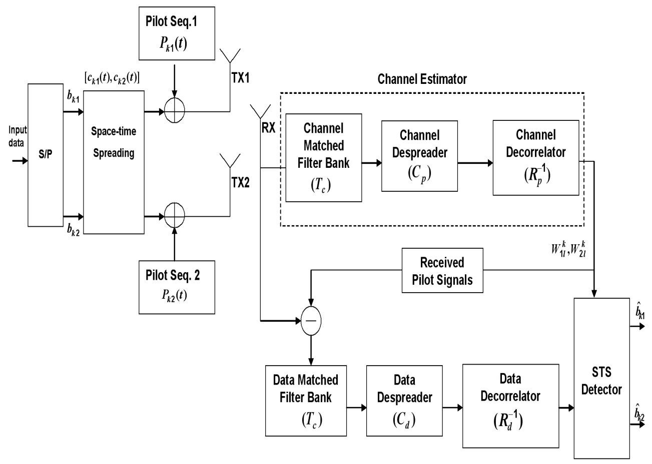

Fig. 1. Block diagram of pilot-sequence-assisted STS transmission system corresponding to a single receive antenna.

III. Channel Estimation

At the receiver side, the received signal at each receive antenna is chip-matched filtered, sampled at a rate $1/T_c$, and accumulated over an observation interval of $(2N + \tau_{max} + L - 1)$ chips corresponding to the first symbol of the received data block for the K-user system. The $(\tau_{max} + L - 1)$ samples are due to the maximum multipath delay (i.e., delay of the L^{th} path) corresponding to the user with the maximum transmit delay, τ_{max}. We have chosen to employ the first symbol observation interval in channel estimation since it is the symbol with the least MAI and ISI contributions. Let $\mathbf{y}^v[0]$ denote the observation vector at the v^{th} receive antenna containing all samples related to the STS symbols transmitted by the K users within the observation interval. Then

$$\mathbf{y}^v[0] = \mathbf{C}[0]\mathbf{H}^v\mathbf{b}[0] + \mathbf{C}[1]\mathbf{H}^v\mathbf{b}[1] + \mathbf{n}^v[0] \qquad (2)$$

where $\mathbf{C}[0] = [\mathbf{C}_1[0], \mathbf{C}_2[0], ..., \mathbf{C}_K[0]]$ represents the code matrix corresponding to the current received symbols, $\mathbf{b}[0]$, within the observation interval. The sub-matrix $\mathbf{C}_k[0]$, $(k = 1, 2, ..., K)$ is a $[(2N + L - 1 + \tau_{max}) \times 4L]$ matrix containing the pilot and data sequences of user k associated with the L resolvable paths. In the same way, $\mathbf{C}[1] = [\mathbf{C}_1[1], \mathbf{C}_2[1], ..., \mathbf{C}_K[1]]$ represents the code matrix of the following received symbols, $\mathbf{b}[1]$, within the observation interval. $\mathbf{C}_k[1]$, $(k = 1, 2, ..., K)$ is a $[(2N + L - 1 + \tau_{max}) \times 4L]$ matrix consisting of the pilot and data code sequences of user k associated with the following STS symbol within the current observation window. The second term in the right-hand side of (2) represents the interference due to the following symbols, $\mathbf{b}[1]$, of the K-user system. In (2), $\mathbf{H}^v$ is the channel impulse response of the K-user system at the v^{th} receive antenna, and is defined by $\mathbf{H}^v = diag\{\mathbf{H}_1^v, \mathbf{H}_2^v, ..., \mathbf{H}_K^v\}$ where $\mathbf{H}_k^v = [\mathbf{H}_k^{vT}(1), \mathbf{H}_k^{vT}(2), ..., \mathbf{H}_k^{vT}(L)]^T, k = 1, 2, ...K$. The superscript T denotes transpose. $\mathbf{H}_k^v(l)$, $(l = 1, 2, ..., L)$ is determined according to the STS scheme in [4]. Here, $\mathbf{H}_k^v(l)$ is modified to include the effect of pilot transmission as follows

$$\mathbf{H}_k^v(l) = \begin{bmatrix} h_{1l}^{k,v} & 0 & 0 & 0 \\ 0 & h_{2l}^{k,v} & 0 & 0 \\ 0 & 0 & h_{1l}^{k,v} & h_{2l}^{k,v} \\ 0 & 0 & -h_{2l}^{k,v} & h_{1l}^{k,v} \end{bmatrix}.$$

The transmitted data vector from the K users during the m^{th} symbol duration, $\mathbf{b}[m]$, is given by $\mathbf{b}[m] = [\mathbf{b}_1^T[m], \mathbf{b}_2^T[m], ..., \mathbf{b}_K^T[m]]^T, m = 0, 1, \ldots, M - 1$ where $\mathbf{b}_k[m] = \left[\sqrt{\frac{\rho_p}{2}}, \sqrt{\frac{\rho_p}{2}}, \sqrt{\frac{\rho_d}{2}}b_{k1}[m], \sqrt{\frac{\rho_d}{2}}b_{k2}[m]\right]^T, k = 1, 2, ..., K$. Finally, in (2), $\mathbf{n}^v[0]$ is a $[(2N+L-1+\tau_{max})\times 1]$ vector representing the additive white Gaussian noise (AWGN) samples at the v^{th} receive antenna, each with zero mean and unit variance. From (2), the received signal can be represented in a more compact form as

$$\mathbf{y}^v[0] = \mathbf{C}_p\mathbf{H}_p^v\mathbf{b} + \mathbf{n}^v[0] \qquad (3)$$

where $\mathbf{C}_p = [\mathbf{C}[0], \mathbf{C}[1]]$, $\mathbf{H}_p^v = \mathbf{I}_2 \otimes \mathbf{H}^v$, $\mathbf{b} = [\mathbf{b}[0], \mathbf{b}[1]]^T$ and $\otimes$ denotes *Kronecker product* operation [5]. After sampling and despearding of the received signal, $\mathbf{y}^v[0]$, with the pilot and data code matrix $\mathbf{C}_p$, the output is given by

$$\mathbf{y}_c^v[0] = \mathbf{R}_p\mathbf{H}_p^v\mathbf{b} + \mathbf{N}_{cc}^v \qquad (4)$$

where $\mathbf{R}_p = \mathbf{C}_p^H\mathbf{C}_p$, is the pilot and data cross-correlation matrix, $\mathbf{N}_{cc}^v$ is modeled as $N_c(\mathbf{0}, \mathbf{R}_p)$ (zero mean complex Gaussian vector with covariance $\mathbf{R}_p$), and H denotes Hermitian transpose. Note that, the authors in [2] have based their channel estimation on the channel despreader output, $\mathbf{y}_c^v[0]$. That is the channel estimation in [2] treats the multiuser interference and ISI as background noise. Hence, the error signal in the channel estimates will be affected by the presence of ISI, MAI and thermal noise. The self interference between the two pilot signals of each user was also neglected in [2]. In light of this, we employ after the despreader a channel decorrelator detector at each receive antenna to overcome the effects of multiuser interference. In this case, the output of the v^{th} channel decorrelator is given by

$$\mathbf{y}_d^v[0] = \mathbf{H}_p^v \mathbf{b} + \mathbf{N}_{cd}^v \quad (5)$$

where $\mathbf{N}_{cd}$ is $N_c(\mathbf{0}, \mathbf{R}_p^{-H})$. Note that the cross-correlation matrix inversion is based on the pseudo-inverse or the *Moore-Penrose* generalized inverse [6]. The first $4LK$ elements are then chosen from the v^{th} decorrelator output vector, $\mathbf{y}_d^v[0]$, for estimating the channel coefficients at the v^{th} receive antenna, yielding to

$$\begin{aligned} W_{1l}^{k,v} &= Bh_{1l}^{k,v} + w_{1l}^{k,v}, \\ W_{2l}^{k,v} &= Bh_{2l}^{k,v} + w_{2l}^{k,v}, k = 1,2,...,K; l = 1,2,...,L \end{aligned} \quad (6)$$

where $B = \sqrt{\frac{\rho_p}{2}}$,$w_{1l}^{k,v}$ and $w_{2l}^{k,v}$ represent the errors in the channel estimates corresponding to the l^{th} path between the i^{th} transmit antenna (i=1,2) and the v^{th} receive antenna. From (6), we obtain the corresponding channel estimates as

$$\begin{aligned} \hat{h}_{1l}^{k,v} &= h_{1l}^{k,v} + e_{1l}^{k,v}, \\ \hat{h}_{2l}^{k,v} &= h_{2l}^{k,v} + e_{2l}^{k,v}, k = 1,2,...,K; l = 1,2,...,L \end{aligned} \quad (7)$$

where $e_{il}^{k,v} = \frac{w_{il}^{k,v}}{B}$, $(i = 1,2)$.

IV. Data Detection

The effect of the pilot sequences at each receive antenna is eliminated from the received signal defined in (1) by subtracting the pilot signals of the K users, which are defined using the channel estimates obtained in (7), from (1). Then, similar to the channel estimation procedure, $r^{v'}(t)$ is filtered, sampled at a rate $1/T_c$, and accumulated over an observation interval of $(2N+\tau_{max}+L-1)$ chips for the m^{th} data symbol of the received data block.

Using vector notation, the data chip-matched filter output at the v^{th} receive antenna, $\mathbf{g}^v[m]$, can be expressed as

$$\begin{aligned} \mathbf{g}^v[m] = &\mathbf{C}'[0]\mathbf{H}^{v'}\mathbf{b}'[m] + \mathbf{C}'[-1]\mathbf{H}^{v'}\mathbf{b}'[m-1] \\ &+ \mathbf{C}'[1]\mathbf{H}^{v'}\mathbf{b}'[m+1] - \mathbf{P}[0]\mathbf{E}^{v'} - \mathbf{P}[-1]\mathbf{E}^{v'} \\ &- \mathbf{P}[1]\mathbf{E}^{v'} + \mathbf{n}^v[m], m = 1,...,M-2 \end{aligned} \quad (8)$$

where $\mathbf{C}^{'}[0]$, $\mathbf{C}^{'}[-1]$ and $\mathbf{C}'[1]$ are square matrices of the same dimension of $[(2N+L-1+\tau_{max}) \times 2LK]$. These matrices include the data sequences corresponding to the current, previous and following STS symbols of the K-user system within the observation interval. Similarly, $\mathbf{P}^{'}[0]$, $\mathbf{P}^{'}[-1]$ and $\mathbf{P}'[1]$ have the same definition of the corresponding data matrices except that the data sequences are replaced by the pilot sequences. In (8), the terms corresponding to the received pilot sequences are due to the errors in the channel estimates. The channel impulse response of the K users, $\mathbf{H}^{v'}$, is defined by $\mathbf{H}^{v'} = diag\{\mathbf{H}_1^{v'}, \mathbf{H}_2^{v'}, ..., \mathbf{H}_K^{v'}\}$ where $\mathbf{H}_k^{v'} = [\mathbf{H}_k^{v'^T}(1), \mathbf{H}_k^{v'^T}(2), ..., \mathbf{H}_k^{v'^T}(L)]^T, k = 1,2,...K$. $\mathbf{H}_k^{v'}(l), (l = 1,\ldots,L)$ is defined according to the employed STS scheme in [4] as

$$\mathbf{H}_k^{v'}(l) = \begin{bmatrix} h_{1l}^{k,v} & h_{2l}^{k,v} \\ -h_{2l}^{k,v} & h_{1l}^{k,v} \end{bmatrix}.$$

In (8), $\mathbf{E}^{v'}$ represents the channel estimation error vector of the K users which is given by $\mathbf{E}^{v'} = [\mathbf{E}_1^{v'^T}, \mathbf{E}_2^{v'^T}, ..., \mathbf{E}_K^{v'^T}]^T$ where $\mathbf{E}_k^{v'} = \left[e_{11}^{k,v}, e_{21}^{k,v}, e_{12}^{k,v}, \ldots, e_{1L}^{k,v}, e_{2L}^{k,v}\right]^T, k = 1,2,...,K$. Finally, $\mathbf{b}'[m]$ is given by $\mathbf{b}^{'}[m] = [\mathbf{b}_1^{'T}[m], \mathbf{b}_2^{'T}[m], ..., \mathbf{b}_K^{'T}[m]]^T$ where $\mathbf{b}_k{'}[m] = [Ab_{k1}[m], Ab_{k2}[m]]^T, k = 1,2,\ldots,K$ and $A = \sqrt{\frac{\rho_d}{2}}$. Note that in the case of $m = 0$, one can exclude from (8), the effect of previous STS symbols on the data chip-matched filter output, $\mathbf{g}^v[m]$. Also, when $m = M-1$, the effect of following symbols are excluded. From (8), the received signal can be represented in a more compact form as follows

$$\mathbf{g}^v[m] = \mathbf{C}_d\mathbf{H}_d^v\mathbf{b}_d - B\mathbf{P}_p\mathbf{E}^v + \mathbf{n}^v[m], m = 1,...,M-2 \quad (9)$$

where $\mathbf{C}_d = [\mathbf{C}^{'}[0], \mathbf{C}^{'}[-1], \mathbf{C}^{'}[1]]$, $\mathbf{P}_p = [\mathbf{P}[0], \mathbf{P}[-1], \mathbf{P}[1]]$, $\mathbf{H}_d^v = \mathbf{I}_3 \otimes \mathbf{H}^{v'}$, $\mathbf{E}^v = [\mathbf{E}^{v'^T}, \mathbf{E}^{v'^T}, \mathbf{E}^{v'^T}]^T$ and $\mathbf{b}_d = [\mathbf{b}^{'}[m]^T, \mathbf{b}^{'}[m-1]^T, \mathbf{b}^{'}[m+1]^T]^T$. After sampling the received signal, the data matched filter output, $\mathbf{g}^v[m]$ $(v = 1,\ldots,V)$, is correlated with the data code matrix, $\mathbf{C}_d$, as follows

$$\mathbf{g}_c^v[m] = \mathbf{R}_d\mathbf{H}_d^v\mathbf{b}_d - B\mathbf{C}_d^H\mathbf{P}_p\mathbf{E}^v + \mathbf{N}_{dc}^v \quad (10)$$

where $\mathbf{R}_d = \mathbf{C}_d^H\mathbf{C}_d$ represents the data cross-correlation matrix, and $\mathbf{N}_{dc}^v$ is modeled as $N_c(\mathbf{0}, \mathbf{R}_d)$. It is clear from (10) that the data correlator output, $\mathbf{g}_c^v[m]$, suffers from MAI and ISI. Afterwards, the output of the data correlator (despreader) at each receive antenna is applied to a linear mapper defined by the inverse of the cross-correlation matrix, $\mathbf{R}_d^{-1}$, corresponding to the data code sequences to give

$$\mathbf{g}_d^v[m] = \mathbf{H}_d^v\mathbf{b}_d - B\mathbf{Q}_d\mathbf{E}^v + \mathbf{N}_{dd}^v \quad (11)$$

where $\mathbf{Q}_d = \mathbf{R}_d^{-1}\mathbf{C}_d^H\mathbf{P}_p$ and $\mathbf{N}_{dd}^v$ is modeled as $N_c(\mathbf{0}, \mathbf{R}_d^{-H})$. Finally, the first $2LK$ elements of each decorrelator output vector, $\mathbf{g}_d^v[m]$ $(v = 1,\ldots,V)$, are combined with the corresponding channel estimates defined in (7) for final data estimates. Without loss of generality, we consider the first user as the desired user. Thus the decision variables for the odd and even data bits are given by

$$\hat{b}_{11}[m] = \sum_{v=1}^{V}\sum_{l=1}^{L} Re\left\{\hat{h}_{1l}^{1,v*}(\mathbf{g}_d^v[m])_{2l-1,1} - \hat{h}_{2l}^{1,v*}(\mathbf{g}_d^v[m])_{2l,1}\right\}, \quad (12)$$

$$\hat{b}_{12}[m] = \sum_{v=1}^{V}\sum_{l=1}^{L} Re\left\{\hat{h}_{2l}^{1,v*}(\mathbf{g}_d^v[m])_{2l-1,1} + \hat{h}_{1l}^{1,v*}(\mathbf{g}_d^v[m])_{2l,1}\right\} \quad (13)$$

where $(\mathbf{g}_d^v[m])_{\delta,1}$ $(\delta = 1,2,...,2L)$, is the δ^{th} element of the v^{th} decorrelator output vector, and $Re\{\cdot\}$ denotes real part operation.

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V. Performance Analysis

In this section, we evaluate the performance of the proposed estimation technique in terms of its probability of bit error. We start by finding the decision variables corresponding to the data estimates at the decorrelator output and after signal combining. Then, we obtain the PDF of these decision variables which will facilitate the evaluation of the average probability of error.

A. BER Analysis

From (11), $(\mathbf{g}_d^v[m])_{2l-1,1}$ and $(\mathbf{g}_d^v[m])_{2l,1}$ in (12) and (13) are given by,

$$(\mathbf{g}_d^v[m])_{2l-1,1} = Ah_{1l}^{1,v}b_{11}[m] + Ah_{2l}^{1,v}b_{12}[m] - B(\mathbf{Q}_d\mathbf{E}^v)_{2l-1,1} + (\mathbf{N}_{dd}^v)_{2l-1,1}, \quad (14)$$

$$(\mathbf{g}_d^v[m])_{2l,1} = -Ah_{2l}^{1,v}b_{11}[m] + Ah_{1l}^{1,v}b_{12}[m] - B(\mathbf{Q}_d\mathbf{E}^v)_{2l,1} + (\mathbf{N}_{dd}^v)_{2l,1} \quad (15)$$

where $(\mathbf{Q}_d\mathbf{E}^v)_{2l-1,1}$ and $(\mathbf{Q}_d\mathbf{E}^v)_{2l,1}$ are defined in terms of the channel estimation errors corresponding to the K users at the v^{th} receive antenna. By partitioning $\mathbf{Q}_d$ into three groups: $\mathbf{Q}_{d1}$, $\mathbf{Q}_{d2}$ and $\mathbf{Q}_{d3}$ where each group has the same dimensions of $6LK \times 2LK$, $\mathbf{Q}_d\mathbf{E}^v$ is defined as $\mathbf{Q}_{ds}\mathbf{E}^{v'}$ where $\mathbf{Q}_{ds} = \mathbf{Q}_{d1}+\mathbf{Q}_{d2}+\mathbf{Q}_{d3}$. Consequently,$(\mathbf{Q}_d\mathbf{E}^v)_{2l-1,1}$ and $(\mathbf{Q}_d\mathbf{E}^v)_{2l,1}$ are derived as $(\mathbf{Q}_d\mathbf{E}^v)_{2l-1,1} = \mathbf{X}_{2l-1}\mathbf{E}^{v'}$ and $(\mathbf{Q}_d\mathbf{E}^v)_{2l,1} = \mathbf{X}_{2l}\mathbf{E}^{v'}$ where $\mathbf{X}_f$ $(f = 2l-1, 2l)$, is a $[1 \times 2LK]$ vector consisting of the elements of the f^{th} raw of $\mathbf{Q}_{ds}$. Using (7) and (14)-(15), (12) can be written as

$$\begin{aligned}\hat{b}_{11}[m] = \sum_{v=1}^{V}\sum_{l=1}^{L} Re\{(A|h_{1l}^{1,v}|^2 + A|h_{2l}^{1,v}|^2 + Ae_{1l}^{1v*}h_{1l}^{1,v} \\ + Ae_{2l}^{1,v*}h_{2l}^{1,v})b_{11}[m] + (Ae_{1l}^{1,v*}h_{2l}^{1,v} - Ae_{2l}^{1,v*}h_{1l}^{1,v})b_{12}[m] \\ - Bh_{1l}^{1,v*}\mathbf{X}_{2l-1}\mathbf{E}^{v'} - Be_{1l}^{1,v*}\mathbf{X}_{2l-1}\mathbf{E}^{v'} + Bh_{2l}^{1,v*}\mathbf{X}_{2l}\mathbf{E}^{v'} \\ + Be_{2l}^{1,v*}\mathbf{X}_{2l}\mathbf{E}^{v'} + h_{1l}^{1,v*}(\mathbf{N}_{dd}^v)_{2l-1,1} + e_{1l}^{1,v*}(\mathbf{N}_{dd}^v)_{2l-1,1} \\ - h_{2l}^{1,v*}(\mathbf{N}_{dd}^v)_{2l,1} - e_{2l}^{1,v*}(\mathbf{N}_{dd}^v)_{2l,1}\}. \quad (16)\end{aligned}$$

Now consider the case of $b_{11}[m] = +1$, then the probability of error is given by $P_b = \frac{1}{2}P(\hat{b}_{11}[m] < 0|b_{12}[m] = +1) + \frac{1}{2}P(\hat{b}_{11}[m] < 0|b_{12}[m] = -1)$. Let the estimate $\hat{b}_{11}[m]$ equivalent to, Z_1 when $b_{12}[m] = +1$, and Z_2 when $b_{12}[m] = -1$. Then P_b can be written as

$$P_b = \frac{1}{2}P_b(Z_1 < 0) + \frac{1}{2}P_b(Z_2 < 0). \quad (17)$$

Given the complex variables x and y, we have $Re\{xy^*\} = \frac{1}{2}(xy^* + yx^*)$. Then Z_1 and Z_2 can be expressed as a sum of independent symmetric quadratic forms as follows:

$$Z_i = \sum_{v=1}^{V} \mathbf{X}^{vH}\mathbf{S}_i\mathbf{X}^v = \sum_{v=1}^{V} Z_{iv}, \ i = 1, 2, \quad (18)$$

where $\mathbf{X}^v = [h_{11}^{1,v}, h_{21}^{1,v}, h_{12}^{1,v}, \ldots, h_{2L}^{1,v}, e_{11}^{1,v}, e_{21}^{1,v}, \ldots, e_{2L}^{1,v}, \ldots, e_{2L}^{K,v}, (\mathbf{N}_{dd}^v)_{1,1}, (\mathbf{N}_{dd}^v)_{2,1}, \ldots, (\mathbf{N}_{dd}^v)_{2L-1,1}, (\mathbf{N}_{dd}^v)_{2L,1}]^T$ and $\mathbf{S}_1$ and $\mathbf{S}_2$ are coefficient matrices. It should be noted that $\mathbf{X}^v$ is a $[4L + 2LK]$ complex normal vector with zero mean and covariance matrix $\mathbf{R}$. Note that the vectors $\mathbf{X}^v$ $(v = 1, \ldots, V)$, are statistically independent with the same covariance matrix $\mathbf{R}$. Also, the coefficient matrices, $\mathbf{S}_1$ and $\mathbf{S}_2$ are identical for each receive antenna. From (18), we can find the characteristic function of the decision statistic Z_1 as [7],[8]

$$\Phi_{Z_1}(\omega) = \prod_{v=1}^{V} \phi_{Z_{1v}}(\omega) \quad (19)$$

where

$$\phi_{Z_{1v}}(\omega) = E\left[\exp\left(j\omega Z_{1v}\right)\right] \quad (20)$$

and $E[\cdot]$ denotes statistical expectation. Note that the characteristic function of the quadratic form in (20) can be derived in terms of the eigenvalues of the matrix $\mathbf{S_1R}$ as [8],

$$\phi_{Z_{1v}}(\omega) = \prod_{n=1}^{N} \frac{1}{(1 - j\omega\lambda_n)} \quad (21)$$

where λ_n $(n = 1, \ldots, N)$, are the N eigenvalues of the $\mathbf{S_1R}$ matrix. Based on the matrix structure of $\mathbf{S_1}$ and $\mathbf{R}$, both $\mathbf{S_1}$ and $\mathbf{R}$ are symmetric matrices. Also, we can notice that $\mathbf{R}$ is positive definite while $\mathbf{S_1}$ is generally singular matrix. Accordingly, the eigenvalues, λ_n $(n = 1, \ldots, N)$, are real valued but may be positive or negative. Substituting (21) in (19), the characteristic function $\Phi_{Z_1}(\omega)$ is given by

$$\Phi_{Z_1}(\omega) = \prod_{n=1}^{N} \frac{1}{(1 - j\omega\lambda_n)^V}. \quad (22)$$

From (22), we can find the PDF of Z_1, f_{Z_1}, [9]. Using this PDF, one can evaluate the probability $P_b(Z_1 < 0)$ as follows [10]:

$$P_b(Z_1 < 0) = \frac{1}{2} - \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\prod_{n=1}^{N}\frac{1}{(1 - j\omega\lambda_n)^V}\right](j\omega)^{-1}d\omega. \quad (23)$$

The remaining integral in (23) can be evaluated using contour integration [11] where the contour integral is given by

$$\oint_C f(z)dz = \oint_C \left[\prod_{n=1}^{N}\frac{1}{(1 - jz\lambda_n)^V}\right](jz)^{-1}dz. \quad (24)$$

Using the residue theorem [11], the contour integral defined in (24) is given by

$$\oint_C f(z)dz = j\pi Res(f(z), z = 0) + j2\pi\sum_{q=1}^{n_1} Res\left(f(z), z = -\frac{j}{\lambda_q}\right) \quad (25)$$

where $Res(f(z), z_o)$ denotes the residue of $f(z)$ at the pole $z = z_o$ and n_1 represents the number of negative eigenvalues of the matrix $\mathbf{S_1R}$. In order to evaluate the residues, we use the partial fraction expansion method of a rational function with high order poles. In (25), for all values of V, we have

$$Res(f(z), z = 0) = -j. \quad (26)$$

For the remaining distinct poles, the residues are found for different values of V according to [12]. For instance, we have

$$Res\left(f(z), z=-\frac{j}{\lambda_q}\right) = \prod_{q'=1,\lambda_{q'}\neq\lambda_q}^{N} \frac{j}{\left(1-\frac{\lambda'_q}{\lambda_q}\right)}, \quad (27)$$

and

$$Res\left(f(z), z=-\frac{j}{\lambda_q}\right) = j\lambda_q. \prod_{n=1,\lambda_n\neq 0}^{N} \lambda_n^{-2}\times$$
$$\prod_{q'=1,\lambda_{q'}\neq\lambda_q}^{N} \left(\frac{1}{(\lambda_{q'}^{-1}-\lambda_q^{-1})^2}.\left[\lambda_q - \sum_{q'=1,\lambda_{q'}\neq\lambda_q}^{N} \frac{2}{\lambda_{q'}^{-1}-\lambda_q^{-1}}\right]\right), \quad (28)$$

for $V = 1, 2$ receive antennas, respectively. Now using the obtained residues, we can evaluate the contour integral in (25). It can be shown that the integration defined in (25) can be evaluated as [[11], theorem (19.5)]

$$\int_{-\infty}^{\infty} f(\omega)d\omega = j\pi Res(f(z), z=0) + j2\pi\sum_{q=1}^{n_1} Res\left(f(z), z=-\frac{j}{\lambda_q}\right) \quad (29)$$

where a represents the radius of the contour C, and ω denotes the real part of the complex variable z. Substituting (29) in (23), we get $P_b(Z_1 < 0)$ for the cases V=1 and 2 antennas respectively as follows:

Case 1 (V=1):
$$P_b(Z_1<0) = \left(\prod_{n=1,\lambda_n\neq 0}^{N} \lambda_n^{-1}\right)\times \sum_{q=1}^{n_1}\left(\prod_{q'=1,\lambda_{q'}\neq\lambda_q}^{N} \frac{1}{\left(\lambda_{q'}^{-1}-\lambda_q^{-1}\right)}\right), \quad (30)$$

Case 2 (V=2):
$$P_b(Z_1<0) = \left(\prod_{n=1,\lambda_n\neq 0}^{N} \lambda_n^{-2}\right).\sum_{q=1}^{n_1}\left(\lambda_q\times \prod_{q'=1,\lambda_{q'}\neq\lambda_q}^{N} \frac{1}{(\lambda_{q'}^{-1}-\lambda_q^{-1})^2}.\left[\lambda_q - \sum_{q'=1,\lambda_{q'}\neq\lambda_q}^{N} \frac{2}{\lambda_{q'}^{-1}-\lambda_q^{-1}}\right]\right). \quad (31)$$

Following the same procedure, we can evaluate the probability, $P_b(Z_2 < 0)$, by replacing λ_n by β_n and n_1 by n_2, where β_n ($n = 1, \ldots, N$), are the eigenvalues of $\mathbf{S}_2\mathbf{R}$ and n_2 is the number of the corresponding negative eigenvalues. Finally, the average BER in (17) is obtained.

VI. Simulation Results

In this section we examine the BER performance of the STS system employing the proposed channel and data estimation technique. Both Monte-Carlo simulations and the analytical results are presented for different system configurations. In all cases, we consider a DS-CDMA system with two transmit and $V = 1, 2$ receive antennas. We also consider an uplink asynchronous transmission of a data block of three symbols (M=3) over a frequency-selective fading channel. Throughout the simulations, we consider a multiuser system where all users are assigned Walsh code sequences of length 64 chips for the pilot and data sequences. The delay among user signals, (τ_k, $k \in \{1, 2, ..., K\}$), are assumed to be multiple of chip periods within T_s. Without loss of generality, we assume that the users' delays satisfy the condition $\tau_1 = 0 \leq \ldots \leq \tau_k \leq \ldots \leq \tau_K \leq T_s$ [13]. The path delay is also assumed to be multiple of chip intervals, $\tilde{\tau}_l = lT_c$ ($l = 1, 2, \ldots, L$). Fig. 2 shows the BER performance for different channel estimation and data detection techniques: (i) perfect knowledge of the channel at the receiver (reference case); (ii) conventional channel estimation and data detection [2] (no MAI removal from both the channel and data estimates); (iii) conventional channel estimation followed by decorrelating data detection (only interference removal from the data estimates); and (iv) the proposed decorrelating channel and data estimation technique. Confirmed by simulations, our analytical results prove that the proposed scheme achieves a performance very close to the perfect channel estimation case for ρ_p=40dB. The results also show the effect of interference removal from both the channel and data estimates on the system performance. The third system renders a slight improvement over the conventional technique [2], due to the MAI removal from the data estimates but still affected by the imperfect channel estimation. With MAI removal from both the channel and data estimates, the proposed receiver outperforms the other estimation techniques. For reference, we included the BER performance of the MRC with four diversity branches. We can notice that the proposed scheme is able to deliver the full system diversity ($2VL$) at the prescribed ρ_p.

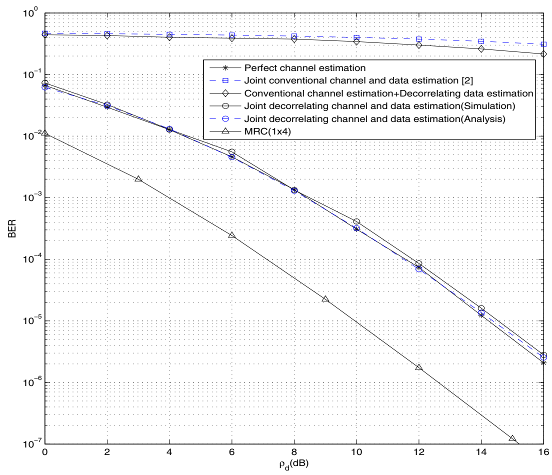

Fig. 2. Comparison between different channel estimation and data detection techniques for the 5-user STS system with L=2 paths, ρ_p=40dB, two antennas at the transmitter and one antenna at the receiver side.

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Fig. 3 also shows the performance of the proposed scheme when the number of resolvable paths is increased to $L = 3$ per transmit antenna. The proposed receiver is shown to offer accurate channel estimates even when the number of resolvable paths increases. Also note that the system in this case offers a diversity order of six $(2VL = 6)$, as evident from the comparison with the equivalent MRC with same number of diversity branches. Finally, in Fig. 4, the BER performance of our system is examined for 2×2 MIMO systems with two resolvable paths per transmit antenna and different number of users. The results conclude that the effect of increased interference only appears as a SNR loss and no diversity loss is incurred. In all the results, our analytical results are shown to be in excellent agreement with the simulated ones, and the full system diversity is maintained.

VII. Conclusion

We have analyzed the performance of channel estimation and data detection technique for CDMA space-time spreading systems in fading channels. The scheme employs channel and data decorrelation prior to channel and data estimation and hence removes the effect of interference from both the channel and data estimates. In particular, we have shown that the underlying scheme is robust to channel interference caused by the multiple-access transmission in asynchronous CDMA uplinks. Through both simulations and analytical results, we have proven that the estimation technique is able to deliver the full system diversity.

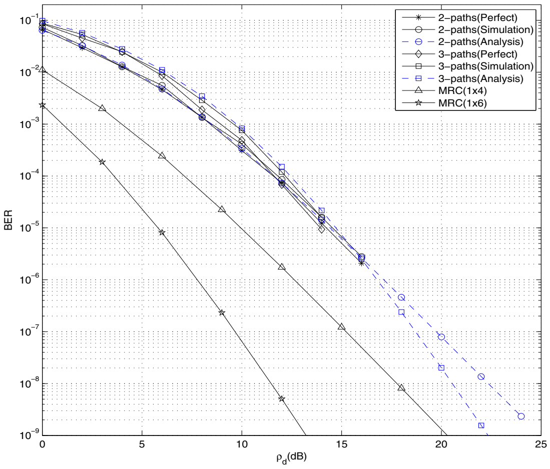

Fig. 3. BER performance of the superimposed estimation technique for a 5-user STS system with L=3 paths and ρ_p=40dB, two antennas at the transmitter and one antenna at the receiver side.

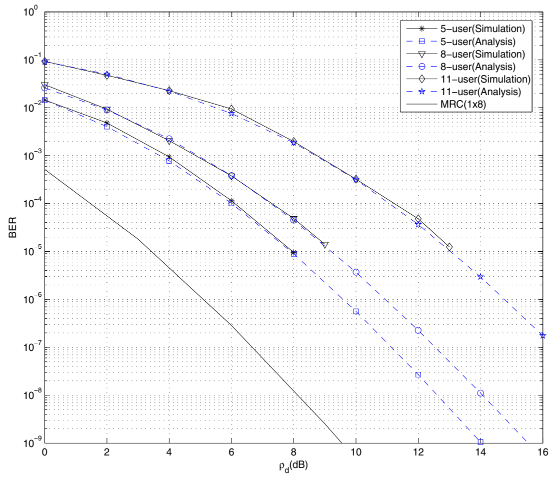

Fig. 4. BER performance of the superimposed estimation technique for the multiuser STS system with L=2 paths and ρ_p=40dB. The STS system employs two transmit antennas and V=2 antennas at the receiver side.

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