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FIGURE 7.1
An image and its local histogram variations.
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**FIGURE 7.2**
(a) An image pyramid. (b) A simple system for creating approximation and prediction residual pyramids.
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FIGURE 7.3
Two image pyramids and their histograms:
(a) an approximation pyramid;
(b) a prediction residual pyramid.
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\[ f(n) \xrightarrow{\text{Unit delay}} f(n-1) \xrightarrow{\text{Unit delay}} f(n-2) \cdots \xrightarrow{\text{Unit delay}} f(n-K+1) \]

\[ h(0)f(n) \xrightarrow{+} h(1)f(n-1) \xrightarrow{+} h(2)f(n-2) \cdots \xrightarrow{+} h(K-1)f(n-K+1) \]

\[ \hat{f}(n) = f(n) \star h(n) \]

\[ h(0)f(n) + h(1)f(n-1) + h(2)f(n-2) + \cdots + h(K-1)f(n-K+1) = f(n) \star h(n) \]

**Figure 7.4** (a) A digital filter; (b) a unit discrete impulse sequence; and (c) the impulse response of the filter.
FIGURE 7.5 Six functionally related filter impulse responses: (a) reference response; (b) sign reversal; (c) and (d) order reversal (differing by the delay introduced); (e) modulation; and (f) order reversal and modulation.
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**FIGURE 7.6**
(a) A two-band subband coding and decoding system, and (b) its spectrum splitting properties.
FIGURE 7.7
A two-dimensional, four-band filter bank for subband image coding.
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#### Table 7.1

Daubechies 8-tap orthonormal filter coefficients for $g_0(n)$ (Daubechies [1992]).

<table>
<thead>
<tr>
<th>$n$</th>
<th>$g_0(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.23037781</td>
</tr>
<tr>
<td>1</td>
<td>0.71484657</td>
</tr>
<tr>
<td>2</td>
<td>0.63088076</td>
</tr>
<tr>
<td>3</td>
<td>-0.02798376</td>
</tr>
<tr>
<td>4</td>
<td>-0.18703481</td>
</tr>
<tr>
<td>5</td>
<td>0.03084138</td>
</tr>
<tr>
<td>6</td>
<td>0.03288301</td>
</tr>
<tr>
<td>7</td>
<td>-0.01059740</td>
</tr>
</tbody>
</table>
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FIGURE 7.8
The impulse responses of four 8-tap Daubechies orthonormal filters. See Table 7.1 for the values of $g_0(n)$ for $0 \leq n \leq 7$. 
FIGURE 7.9
A four-band split of the vase in Fig. 7.1 using the subband coding system of Fig. 7.7. The four subbands that result are the (a) approximation, (b) horizontal detail, (c) vertical detail, and (d) diagonal detail subbands.
FIGURE 7.10
(a) A discrete wavelet transform using Haar $H_2$ basis functions. Its local histogram variations are also shown. (b)–(d) Several different approximations ($64 \times 64$, $128 \times 128$, and $256 \times 256$) that can be obtained from (a).
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**FIGURE 7.11**
Some Haar scaling functions.
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**Figure 7.12**
The nested function spaces spanned by a scaling function.
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\[ V_2 = V_1 \oplus W_1 = V_0 \oplus W_0 \oplus W_1 \]

**FIGURE 7.13**
The relationship between scaling and wavelet function spaces.
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FIGURE 7.14
Haar wavelet functions in $W_0$ and $W_1$. 
FIGURE 7.15
A wavelet series expansion of $y = x^2$ using Haar wavelets.
Figure 7.16
The continuous wavelet transform (c and d) and Fourier spectrum (b) of a continuous 1-D function (a).
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![Diagram: An FWT analysis bank.](image)
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FIGURE 7.18
(a) A two-stage or two-scale FWT analysis bank and (b) its frequency splitting characteristics.
### Table 7.2
Orthonormal Haar filter coefficients for $h_\varphi(n)$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$h_\varphi(n)$</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>$1/\sqrt{2}$</td>
</tr>
<tr>
<td>1</td>
<td>$1/\sqrt{2}$</td>
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</tbody>
</table>
FIGURE 7.19 Computing a two-scale fast wavelet transform of sequence \{1, 4, -3, 0\} using Haar scaling and wavelet vectors.
FIGURE 7.20
The $\text{FWT}^{-1}$ synthesis filter bank.
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FIGURE 7.21
A two-stage or two-scale FWT⁻¹ synthesis bank.
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**Figure 7.22** Computing a two-scale inverse fast wavelet transform of sequence \( \{1, 4, -1.5 \sqrt{2}, -1.5 \sqrt{2}\} \) with Haar scaling and wavelet functions.
Figure 7.23 Time-frequency tilings for the basis functions associated with (a) sampled data, (b) the FFT, and (c) the FWT. Note that the horizontal strips of equal height rectangles in (c) represent FWT scales.
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**FIGURE 7.24** The 2-D fast wavelet transform: (a) the analysis filter bank; (b) the resulting decomposition; and (c) the synthesis filter bank.
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**FIGURE 7.25**
Computing a 2-D three-scale FWT: (a) the original image; (b) a one-scale FWT; (c) a two-scale FWT; and (d) a three-scale FWT.
Fourth-order symlets: (a)–(b) decomposition filters; (c)–(d) reconstruction filters; (e) the one-dimensional wavelet; (f) the one-dimensional scaling function; and (g) one of three two-dimensional wavelets, $\psi^V(x, y)$. See Table 7.3 for the values of $h_\psi(n)$ for $0 \leq n \leq 7$. 

### TABLE 7.3

Orthonormal fourth-order symlet filter coefficients for $h_\varphi(n)$.
(Daubechies [1992].)

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<thead>
<tr>
<th>$n$</th>
<th>$h_\varphi(n)$</th>
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<tbody>
<tr>
<td>0</td>
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FIGURE 7.27
Modifying a DWT for edge detection: (a) and (c) two-scale decompositions with selected coefficients deleted; (b) and (d) the corresponding reconstructions.
Figure 7.28
Modifying a DWT for noise removal:
(a) a noisy CT of a human head; (b),
(c) and (e) various reconstructions
after thresholding the detail
coefficients;
(d) and (f) the information
removed during the reconstruction
of (c) and (e).
(Original image courtesy
Vanderbilt
University
Medical Center.)
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\[ W_\varphi(J, n) = f(n) \]

\[ W_\varphi(J - 1, n) \quad W_\psi(J - 1, n) \]

\[ W_\varphi(J - 2, n) \quad W_\psi(J - 2, n) \]

\[ V_{J-1} \quad W_{J-1} \]

\[ V_{J-2} \quad W_{J-2} \]

**Figure 7.29**
An (a) coefficient tree and (b) analysis tree for the two-scale FWT analysis bank of Fig. 7.18.
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**FIGURE 7.30**
A three-scale FWT filter bank:
(a) block diagram;
(b) decomposition space tree; and
(c) spectrum splitting characteristics.
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**Figure 7.31**
A three-scale wavelet packet analysis tree.
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**FIGURE 7.32**

The (a) filter bank and (b) spectrum splitting characteristics of a three-scale full wavelet packet analysis tree.
FIGURE 7.33
The spectrum of the decomposition in Eq. (7.6-5).
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**FIGURE 7.34**
The first decomposition of a two-dimensional FWT: (a) the spectrum and (b) the subspace analysis tree.
FIGURE 7.35 A three-scale, full wavelet packet decomposition tree. Only a portion of the tree is provided.
Figure 7.36 (a) A scanned fingerprint and (b) its three-scale, full wavelet packet decomposition. (Original image courtesy of the National Institute of Standards and Technology.)
FIGURE 7.37  
An optimal wavelet packet decomposition for the fingerprint of Fig. 7.36(a).
Figure 7.38 The optimal wavelet packet analysis tree for the decomposition in Fig. 7.37.
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<table>
<thead>
<tr>
<th></th>
<th>$h_0(n)$</th>
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</table>

**TABLE 7.4**

Biorthogonal Cohen-Daubechies-Feauveau filter coefficients (Cohen, Daubechies, and Feauveau [1992]).
Figure 7.39
A member of the Cohen-Daubechies-Feauveau biorthogonal wavelet family: (a) and (b) decomposition filter coefficients; (c) and (d) reconstruction filter coefficients; (e)–(h) dual wavelet and scaling functions. See Table 7.3 for the values of $h_0(n)$ and $h_1(n)$ for $0 \leq n \leq 17$. 