Comparison of Linear Scheduling Model and Repetitive Scheduling Method

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Abstract: Various linear scheduling techniques have been proposed over the years. The limitations on these techniques have been the inability to determine critical activities. Contractors and departments of transportation have identified this need. Recently two different methods have been proposed—the linear scheduling model and the repetitive scheduling method. This paper discusses basic linear scheduling techniques and then the calculation of critical activities of basic linear scheduling elements using the two methods. The results of the two techniques are compared.

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Introduction

Victor Hugo said “He who every morning plans the transactions of the day and follows out that plan, carries a thread that will guide him through the most busy life. But where no plan is laid, where the disposal of time is surrendered merely to the chance of incidence, chaos will soon reign.” As with life, a plan is a necessary part of any successful construction project. Without a plan, there is no way to schedule the required work, no way to track progress, and no way of deciding on corrective action when unexpected events occur (Stella and Glavinich 1994). Scheduling is taking the construction plan one step further, through “determination of the timing of the operations comprising the project and their assembling to give the overall completion time” (Antill and Woodhead 1990). The need for proper planning and scheduling of construction projects is crucial for all parties involved. By proper planning and scheduling, contractors increase the possibility that a project will be completed within budget, will be within the time constraints, and will have better quality, and they can reduce the chances of chaos breaking out.

In construction, the process of planning and scheduling has typically been accomplished using network scheduling techniques. The most common is the critical path method (CPM). Because of the widespread use in the construction industry and on industrial engineering applications, CPM has gone through an evolution over the last 40 years, from arrow diagrams, the program evaluation and review technique, and precedence diagrams to the current sophisticated commercial software used today. Excellent summaries of the evolution of network scheduling can be found in O’Brien (1969) and Moder et al. (1983). It is important to note that the development of CPM took time and different approaches to arrive at what is used today. While different forms of linear scheduling have been around as long as network scheduling, the development has been much slower and the acceptance by industry has been less. The work of Harmelink and Rowings (1998) and Harris and Ioannou (1998) in determining the critical activities of linear schedules is an important step in obtaining acceptance by the industry. The comparison of the results of that work is natural and is what is covered in this paper.

While CPM has been used on countless projects, it has been found inadequate when scheduling linear projects. Linear projects are those that have repetitive activities. Road projects, tunnels, and pipelines are physically linear projects that have the same operations repeated at each station. High-rise buildings have the same activities occurring on each floor. Multiunit housing developments will use the same resources and crews for each building that is repeated. The inability of CPM to accurately model the repetitive nature of linear construction has been identified by Stradel and Cacha (1982), Chrzanowski and Johnston (1986), Reda (1990), Cole (1991), Rahbar and Rowings (1992), Suhail and Neale (1994), Harris (1996), Harris and Ioannou (1998), Harmelink (1995), and Harmelink and Rowings (1998). Briefly, these limitations include the arbitrary division of repetitive activities from location to location, the inability to schedule the continuity of resources, the large number of activities necessary to represent a repetitive or linear project in a CPM schedule, activity rates of progress not indicated, a CPM schedule not accurately reflecting actual conditions, and a CPM schedule not providing any information on where on the project site the work is currently being performed. The discussion of those limitations is beyond the scope of this paper, and the reader is encouraged to explore the references cited.

Linear scheduling has been offered as a solution to these problems by Chrzanowski and Johnston (1986), Harmelink (1995), Harmelink and Rowings (1998), and Harris and Ioannou (1998). Numerous linear scheduling methods have been proposed, including the line-of-balance (Lumsden 1968; O’Brien 1969; Carr and Meyer 1974; Al Sarraj 1990; Halpin and Riggs 1992), the linear scheduling method (Johnston 1981; Chrzanowski and Johnston 1986; Bafna 1991; Vorster and Bafna 1992; Vorster et al. 1992;
activities on an easily interpreted graphical schedule that is more straightforward to understand than a network schedule. These schedules plot activities on an X-Y-coordinate graph, with location on one axis and duration on the other. When this type of schedule is used, continuous, or repetitive, activities need not be arbitrarily interrupted, resource continuity can be maintained, and the location of the work in progress on any given day is easily determined from the schedule.

Linear scheduling has some limitations. Nonrepetitive activities, such as a box culvert or a bridge on a road job, must be scheduled using network techniques and then incorporated into the graphical linear schedule in some manner. The same sophisticated techniques like incorporating budget information or leveling resources cannot be performed using a linear schedule at this time. This may be a drawback for management that relies on these techniques for monitoring and control. The most significant problem with the linear schedule is that it is not a familiar scheduling method, whereas there have been books written, courses taught, software programs developed, and extensive research done on network scheduling.

A reason why linear scheduling has not become a more popular method is because, until recently, there has not been a way to establish a critical path for a linear schedule (Harmelink 1995). The critical path determines the minimum duration of a project, and identifies the activities that will extend the project duration if they are delayed. It is an essential part of a construction schedule. It is the characteristic that makes the schedule more than just a chronological list of activities. The information obtained from a critical path analysis is necessary to do accurate scheduling, resource leveling and allocation, schedule optimization, and claims analysis.

This paper uses two methods published in the July/August 1998 issue of the ASCE Journal of Construction Engineering and Management. Harris and Ioannou (1998) present the repetitive scheduling method (RSM) as a tool for producing a linear schedule, and include a method for finding the controlling sequence. Harmelink and Rowings (1998) define the linear scheduling model (LSM), which provides a means to find the controlling activity path. In a linear schedule, the RSM controlling sequence and the LSM controlling activity path are analogous to the CPM critical path. This paper compares the results obtained from the repetitive scheduling method and the linear scheduling model.

Controlling Activities for Linear Schedules

For linear scheduling methods to be more useful in the construction industry, a method for determining the critical path must be available. Harris and Ioannou (1998) present the RSM as a generalized method for linear scheduling and calculating a controlling sequence. The paper focuses on scheduling multiunit projects rather than physically linear projects, although RSM can be used for both. A method for constructing the RSM schedule is proposed, and, using control points, the controlling sequence can be found. Control points are the locations on each activity line where the activity changes from controlling to noncontrolling, or vice versa. They are established based on activity interrelationships like lag times or common resources. The control points in the RSM schedule are used to define the controlling sequence of activities, the critical path in a CPM schedule, which in turn determines the project duration. Methods for altering the project duration are provided. A key point is the resource continuity provided in addition to the technical constraints of the activities.

Harmelink and Rowings (1998) provide an algorithm for determining the controlling activity path in a linear schedule, the LSM. The controlling path is found by performing an upward and downward pass, similar to a forward and backward pass in CPM scheduling. The controlling path is based on the shortest time intervals between activities. Terminology for various activity types is presented and examples of the LSM technique are provided. The paper suggests that the linear scheduling model provides a much more realistic controlling path for linear projects than can be obtained from CPM schedules.

Components of Linear Schedule

A linear schedule is a visual representation for a repetitive project's construction plan. It shows the plan's logic and the relationships between activities. The schedule is displayed as a graph, with time on one axis and location on the other axis. Time and location can be on either axis, depending on which makes more sense. For a high-rise building, putting location on the vertical axis coincides with the building rising from floor to floor. For a highway project, putting location on the horizontal axis coincides with the dimensional nature of the project. Fig. 1 is an example of a linear schedule for a rural highway project (Mattila 1997).
There are three main categories in which all construction activities are classified—linear activities, block activities, and bar activities (Vorster et al. 1992). Harmelink (1995) proposes terminology for variations on these basic activity types. The terminology and the corresponding graphical representation are shown in Fig. 2.

A linear activity is one that occurs along the project from one location to another. This may mean from floor to floor for a building, like the activity “install doors,” or it may mean from station to station on a road job, like the activity “install subbase.” A linear activity is represented on the graph using a line, starting at the beginning location and continuing to the final location. The production rate of an activity will determine the slope of the activity line. Activities A (ditch excavation), C (pavement removal), E (embankment), F (utility work), G (subbase), H (gravel), and I (pave) in Fig. 1 are linear activities.

A block activity is an activity that transpires across several locations, but lasts for an extended duration. Block activities are represented by a rectangle on the schedule, covering the locations and the length of time that the activity will require. Activity D (swamp excavation and backfill) in Fig. 1 is a block activity.

The last type of activity is a bar activity. The bar activity is represented as a vertical line on the linear schedule. Bar activities have significantly less dimensional area than block activities. An example of a bar activity is the installation of a culvert. This is shown as activity B in Fig. 1.

When completed, a linear schedule provides an easy-to-read plot of what will happen on the project from beginning to end. For linear activities, it shows location, production rate, start date, and completion date. A complete discussion of the activities comprising a linear schedule can be found in Vorster and Parvin (1990), Bafna (1991), Vorster et al. (1992), Harmelink (1995), Harmelink and Rowings (1998), and Harris and Ioannou (1998).

### Finding Controlling Path: LSM and RSM Comparison

The critical path is the sequence of activities on a CPM schedule that determines the minimum duration of a project. These critical activities must not be delayed, or the entire project will be delayed. In a linear schedule, the LSM controlling activity path (Harmelink and Rowings 1998) and the RSM controlling sequence (Harris and Ioannou 1998) are analogous to the CPM critical path. For simplicity, the controlling path will be used for both the LSM technique and the RSM technique.

To compare the LSM and RSM controlling paths, the analysis will begin at the simplest stage. Two continuous full-span linear (CFL) activities will be compared—first, when converging; second, when diverging. These two examples will show the similarities or differences between the most basic principles for LSM and RSM using the basic elements of a linear schedule.

#### Converging Activities: Linear Scheduling Model

The converging activities are represented in Fig. 3. Both activity 1 and activity 2 occur along the entire project, and they converge as the station numbering increases because activity 2 has a higher production rate than activity 1. To find the controlling path using the linear scheduling model, first the activity sequence list is derived. The activity sequence list is a list of potential paths through a linear schedule. These paths can be found by drawing a vertical line through the schedule at every location where a unique activity order, a path through the schedule containing different activities, can be found. Fig. 3 shows an example of a simple two activity linear schedule, with the possible activity paths indicated by vertical dashed lines and labeled sequence 1. An activity sequence list is found for each potential activity path. In this case, there is only one activity sequence list, which is activity 1 followed by activity 2.
The least time (LT) interval and least distance (LD) interval are found next. The LT interval is the shortest vertical line that can be drawn between two consecutive activities. The LD interval is the shortest horizontal line between two consecutive activities that intersects the least time interval. The activity sequence list with the shortest least time interval duration will be the one that contains the controlling path. When the activity sequence list is determined, the least time interval and least duration interval are found for those activities and the sequence (Fig. 3).

After the LTs and LDs are plotted, an upward pass is performed (Fig. 4). Starting with activity 1 in the activity sequence list, a potentially controlling line is drawn from the origin of the activity to the first intersecting line, the least distance interval. The intersecting line becomes a potentially controlling link, and is traced back until the next activity is encountered. The potentially controlling link transfers the potentially controlling path from activity to activity on the upward pass. This process is repeated for every activity on the activity sequence list. For activity 2, the potentially controlling line is drawn from the start of the activity to the end of the activity. Since activity 2 is the last activity on the schedule, the potentially controlling path is complete. The upward pass identifies any potentially controlling activities in the project. The bold line of Fig. 4 indicates this.

The downward pass (Fig. 5) is the final step in determining the controlling path. The downward pass starts at the end of the last activity and follows it back to the nearest potentially controlling link. This link now becomes a controlling link, and transfers the actual controlling path from one activity to another. The controlling path continues to the right along the now controlling link to the next activity, then along the activity line to the next intersection, and so on. The controlling path in Fig. 5 shows that activity 1 will be controlling for the entire duration of the activity, and activity 2 will be controlling from station 21 to the end. This means that if activity 1 is delayed at all, the completion of the project will be delayed if the least time must be maintained between activities 1 and 2. It also means that the segment of activity 2 before station 21 could be delayed without altering the completion date of the project if resource continuity is not required.

Converging Activities: Repetitive Scheduling Method

The RSM method for determining the controlling path hinges on control points. The RSM control points are the points on the schedule through which the controlling path will be drawn. To establish the control points, it must first be determined if the consecutive activities are converging or diverging. In the case under consideration, converging activities are present and the first of two basic RSM principles is used (Harris and Ioannou 1998).

When the unit production rate of an activity is greater than the unit production rate of the preceding activity, the two production lines will tend to converge as the number of units increase[s]. Owing to the desired continuous utilization of resources from location to location, this convergence tends to place any dependency control between the activities toward the end of the activity.

The control points are found using RSM earlier. Therefore, for full-span linear activities that have a constant production rate and converge, the LSM and RSM find the same controlling path.

Diverging Activities: Linear Scheduling Model

The second case is of diverging CFL activities, as shown in Fig. 8. The activities diverge because the production rate of activity 2 is slower than the production rate of activity 1. For the LSM
schedule, there is only one activity sequence, 1-2, for the entire schedule. The LD and LT are found for sequence 1-2 (Fig. 8). The upward pass is then performed. A potentially controlling path is drawn along activity 1 until the LD is reached. Then the LD is traced back to activity 2 as a potentially controlling link. Another potentially controlling line is drawn from the start of activity 2 to the end. The results of the upward pass and the identification of the potentially controlling segments are indicated by the bold line in Fig. 8.

The downward pass follows the potentially controlling segments backward. The downward pass starts at the end of activity 2 and follows back to the beginning, where the controlling LD link is met. The LD is followed back to activity 1 and then back to the origin. The resulting critical activity path, shown in Fig. 9, identifies that activity 1 is only critical from station 0 to station 20, but activity 2 is critical for the entire distance.

Diverging Activities: Repetitive Scheduling Method

In the case under consideration, diverging activities are present and the second of two basic RSM principles is used (Harris and Ioannou 1998)

When the production rate of an activity is smaller than the production rate of the preceding activity, the two production lines will tend to diverge as the number of units increases. Owing to the desired continuous utilization of resources from location to location, this divergence tends to place any dependency control between the activities toward the beginning of the activity.

The control point for the same diverging CFL activities shown in Fig. 10 will be located at the beginning of the second activity, according to the RSM principle. The controlling path (Fig. 11) starts at the end of activity 2, runs to the control point, then moves to activity 1 and back to the origin. The resulting controlling path determined using RSM is the same as the LSM critical path for these two lines, and is indicated by the bold line in Fig. 11. Therefore, for full-span linear activities that have a constant production rate and diverge, the LSM and RSM find the same controlling path.

Activities with Changing Production Rates

Having first considered both possible relationships between two CFL activities with constant production rates, this section examines what will happen when the production rates of the activities change at some point in the project. This situation might arise from the leveling of resources, the arrival of additional labor or equipment, the departure of labor or equipment, a change in the quantity of an activity, and so forth. Harmelink (1995) and later Harmelink and Rowsings (1998) have addressed this situation for LSM schedules, and as was demonstrated for constant production rates, the potentially controlling links will again lie along the LD intervals. Harris (1996) and later Harris and Ioannou (1998), on the other hand, failed to address this issue in the determination of the RSM controlling path. Therefore, in order to continue with the comparison of LSM and RSM controlling path determination methods, the establishment of RSM control points must be refined.
In both the converging CFL and the diverging CFL cases for RSM discussed earlier in this paper (Figs. 6 and 10), the control points were projected along a horizontal path. This path was analogous to the least distance interval in the LSM schedule, and is why both methods produced the same controlling path. It is important to note that using a least distance approach to locate control points would not violate the two basic RSM principles given by Harris and Ioannou (1998); that approach is used in our previous determinations of controlling paths for RSM. In the case of converging activities, the control point tends toward the end of the preceding activity, as outlined in the first basic RSM principle. It is easy to see that this principle is congruent to a “least distance” analysis simply by geometric observation. In the case of diverging activities, it is again verified by geometric observation that the second basic principle of RSM is conserved by a least distance analysis. The fact that both the LSM least distance intervals and the control point line of projection were analogous is why both methods yielded the same critical paths when working with CFL activities with a constant production rate.

Diverging Then Converging Activities

Fig. 12 illustrates a linear schedule where two activities initially diverge, but begin to converge when the first activity experiences a change of production rate. The least distance interval is shown. The left end point becomes an RSM control point, while the interval itself represents an LSM potentially controlling link. The controlling path (Fig. 13) is therefore the same regardless of which method is used. The intermediate steps necessary to complete the analysis have been left out for brevity, but readers are encouraged to demonstrate for themselves using the techniques explained for diverging and converging CFL activities earlier in this paper.

Converging Then Diverging Activities

Now, consider a linear schedule with two converging activities that become diverging when activity 1 experiences an increase in productivity (Fig. 14). The least distance interval is again shown, with the left end point locating an RSM control point and the interval itself locating an LSM potentially controlling link. Once again the two methods produce the same controlling path (Fig. 15). The intermediate steps necessary to complete the analysis have been left out for brevity, but readers are encouraged to demonstrate for themselves using the techniques explained for diverging and converging CFL activities earlier in this paper.

Schedules with Multiple Production Rate Changes

In many instances, activities will have more than one change in production rate. Single changes in the production rate were discussed in the previous two sections. In those cases where activities have multiple changes in production rate, the controlling path for either the LSM or the RSM will be according to the method outlined in the previous section. That is, the controlling path determination will follow the analysis presented above for a single change in production rate and will follow a path that is the least distance between activities.
Schedules with Continuous Partial Span Linear Activities (CPLs)

To accurately represent a construction schedule, some activities may need to be shown in a linear schedule using CPLs. The use of continuous partial span linear activities is discussed earlier in this paper in the “Components of Linear Schedule” section. These activities in many situations may be critical and must be considered to provide an accurate controlling path.

The consideration of partial span linear activities in a linear schedule was addressed by Harmelink (~1995) and Harmelink and Rowings (1998) for the LSM technique. Harris (1996) and Harris and Ioannou (1998) do not include partial span linear activities in the determination of the controlling path in RSM. However, the use of the basic RSM principles will be applied.

Fig. 16 illustrates a linear schedule with three activities. In this schedule, activity 2 is a continuous partial span linear activity. The first step in using LSM is the determination of the activity sequence list. The activity sequence list consists of the following two paths: activities 1-3 and activities 1-2-3. Shown in Fig. 16 are the least times between activities 1-2 and between activities 1-2-3. The least time through activities 1-2 is less than the sum through activities 1-2-3. The results of the upward and downward passes are shown in Fig. 17, with the controlling path indicated by the bold line.

Fig. 18 represents the same schedule with the RSM control points indicated. These were determined between each activity—that is, from activities 1-2, 1-3, and 2-3 using the methods discussed earlier. The controlling path is shown in Fig. 19 and is indicated by the bold line. It is identical to the path found using LSM.

Fig. 20 shows a different configuration of the continuous partial span linear activity. In this instance, activity 2 has a slower production rate than in the example presented in Figs. 16, 17, 18, and 19. Both LSM and RSM will identify the same controlling path, which is indicated by the bold line in Fig. 20.

Only diverging activities were presented here. If activities 1 and 3 were converging, the procedure would be the same. In that case, LSM and RSM would identify identical controlling paths.

Schedules with Block or Bar Activities

Many activities must be presented in a linear schedule using blocks or bars. The use of blocks and bars is discussed earlier in this paper in the “Components of Linear Schedule” section. These activities in many situations may be critical, and must be considered to provide an accurate controlling path.

The consideration of blocks and bars in a linear schedule was addressed by Harmelink (~1995) and Harmelink and Rowings (1998) for the LSM technique. Harris (1996) includes a block activity (bridge scheduled using CPM) in a linear schedule, but no mention is made as to how to include blocks or bars in determining the controlling path using RSM. Blocks and bars were not covered in Harris and Ioannou (1998). Therefore, this discussion will start with the LSM analysis.

Fig. 21 shows two linear activities with a partial block activity between. Also shown in Fig. 21 are the least time and least dis-
tance for the two activity sequences, activity 1-2 and activity 1-2-3. After performing the upward and downward pass, the controlling path is shown in Fig. 22.

In Fig. 23, the RSM control points for each of the activity-to-activity relationships are shown. The control points were identified using the least distance between activities. Following the procedure detailed earlier, the controlling path can be determined. It is identical to that obtained from LSM, and is shown in Fig. 22. Therefore, both LSM and RSM identify the same path.

In this case, diverging activities were considered. It can be shown that for converging activities, LSM and RSM will identify the same controlling path. Additionally, if full-span block activities are considered, LSM and RSM will identify identical critical paths. Bar activities will be treated in the same manner as blocks for both LSM and RSM.

Conclusions

Linear scheduling provides a means to describe the logic of a linear construction project. Beyond that, it is important that a mechanism be provided to identify a controlling path that will determine which activities on the project cannot be delayed without delaying the project completion date. This paper compared two different techniques for determining the controlling path in linear schedules—the LSM (Harmelink and Rowings 1998) and the RSM (Harris and Ioannou 1998).

When activities are considered that have a constant rate of production, it was found that for converging, continuous full-span linear activities LSM and RSM identify the same controlling path. This same result was found for diverging, continuous full-span linear activities.

When activities are considered that have a change in the rate of production, it was found that for diverging then converging, continuous full-span linear activities LSM and RSM identify the same controlling path. In this situation, it was necessary to address the establishment of control points for RSM based upon the least distance between activities. This same result was found for converging then diverging, continuous full-span linear activities that have a change in production rate.

If partial span linear activities are incorporated into the schedule, the controlling path identified by LSM and RSM is the same. For RSM, the identification of control points is made using the least distance between activities.

The controlling path when partial span blocks, full-span blocks, and bars are considered is identical for both LSM and RSM. When using RSM, the identification of control points is made using the least distance between activities.

In summary, both LSM and RSM identify the same controlling activity path for the simple configurations presented in this paper. Additional work on a more complicated schedule is necessary to confirm this.

Both LSM and RSM have the potential to be useful tools for finding the controlling path. However, the techniques must be computerized in order to become useful and to gain acceptance by the industry.
Fig. 23. RSM—diverging activities with partial span block

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