Optimized Scheduling of Linear Projects

O. Moselhi, F.ASCE,1 and A. Hassanein2

Abstract: This paper presents a model designed to optimize scheduling of linear projects. The model employs a two-state-variable, N-stage, dynamic programming formulation, coupled with a set of heuristic rules. The model is resource-driven, and incorporates both repetitive and nonrepetitive activities in the optimization process to generate practical and near-optimal schedules. The model optimizes either project construction duration, total cost, or their combined impact for what is known as cost-plus-time bidding, also referred to as A+B bidding. The model has a number of interesting and practical features. It supports multiple crews to work simultaneously on any activity, while accounting for: (1) multiple successors and predecessors with specified lead and lag times; (2) the impact of transverse obstructions, such as rivers and creeks, on crew assignments and associated time and cost; (3) the effect of inclement weather and learning curve on crew productivity; and (4) variations in quantities of work in repetitive activities from one unit to another. The model is implemented in a prototype software that operates in Windows® environment. It is developed utilizing object-oriented programming, and provides for automated data entry. Several graphical and tabular output reports can be generated. An example project, drawn from the literature, is analyzed to demonstrate the features of the developed model.

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Introduction

Repetitive construction projects can be divided into two main groups: linear, such as highways, railroads, and pipelines; and nonlinear, such as high-rise buildings and multiple housing construction. In nonlinear projects, work crews repeat certain tasks in each unit throughout the project. Linear projects, unlike nonlinear ones, are typically divided into sections, to which crews are assigned. Fig. 1 shows two possible crew assignment scenarios for a crew formation composed of two identical crews assigned to a linear activity. In the first scenario the activity is divided into two sections, where one crew is assigned to each. In Fig. 1(b), crews are assigned to nonadjacent units, resulting in a longer duration. This increase in duration is due to the additional travel time "T" (see Fig. 1), needed to relocate each crew from one unit to another. The applicability of network scheduling techniques, such as the critical path method and program evaluation and review technique, to repetitive projects has long been questioned (O’Brien 1975; Adeli and Karim 1997). As a result, several models were developed to schedule this class of projects, such as the vertical production method (O’Brien 1975), linear scheduling method (Johnston 1981), time space scheduling method (Stradal and Cacha 1982), and linear scheduling model (Harmelink 2001). Different methods were also introduced to optimize the generated schedules using: linear programming (Reda 1990), dynamic programming (Selinger 1980; Russell and Caselton 1988; Moselhi and El-Rayes 1993; Eldin and Senouci 1994; El-Rayes and Moselhi 1998, 2001), optimal control theory (Handa and Barcia 1986), neural networks (Adeli and Karim 1997), and genetic algorithms (Hegazy and Wassell 2001; Leu and Hwang 2001). Except for the model developed by Adeli and Karim (1997), these models do not assign crews to adjacent units when multiple crews are employed. The Adeli and Karim (1997) model requires sections to be defined at the outset. Only the models developed by El-Rayes and Moselhi (1998, 2001) and Leu and Hwang (2001) account for crew availability in the optimization process.

This paper presents a model designed to aid construction personnel in developing practical and near-optimal schedules for linear projects. The model employs a two-state-variable, N-stage dynamic programming formulation, coupled with a set of heuristic rules, to minimize either: (1) total construction cost; (2) construction duration; or (3) their combined impact for what is known as cost-plus-time bidding (Herbsman 1995). The proposed model: (1) accounts for the presence of transverse obstructions, such as rivers and creeks; (2) utilizes resource-driven scheduling; (3) incorporates repetitive and nonrepetitive activities in the optimization procedure; (4) enables the consideration of multiple predecessors and successors for each activity; and (5) accounts for variations in quantity of work and unit length of repetitive activities. The model employs a relational database to store data related to: (1) owned equipment and labor force; (2) assignment dates of equipment and labor force; and (3) equipment rental firms. The database aids in determining the availability status of both equipment and labor associated with each crew. The model accounts for the impact of inclement weather, either by considering reduced crew productivity or complete stoppage of operations. A prototype software is developed utilizing object-oriented programming. A numerical example, drawn from the literature, is analyzed to demonstrate the main features of the proposed model.

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Proposed Model

A flexible model employing resource-driven and traditional network scheduling techniques is presented. Unlike earlier models (El-Rayes and Moselhi 1998; Hegazy and Wassef 2001), the proposed model imposes no limit on the number of preceding or succeeding activities. The model accounts for the types of activities shown in Fig. 2, and supports finish-to-start, finish-to-finish, start-to-start, and start-to-finish activity relations, with lead and lag times. Each repetitive activity is divided into units along its length. While the definition of what constitutes a unit is physically defined in nonlinear construction, it is not so clearly identifiable in linear construction, where a unit is typically considered to be of a certain length (e.g., 500 m in highway construction) (Johnston 1981). Previous studies (Harris and Evans 1977) proposed that the unit length be based on the minimum buffer required by a crew to work without interference from other crews. Maintaining crew work continuity has been recommended to minimize disruptions and maximize the beneficial effect of the learning curve (Selinger 1980; Reda 1990). Allowing for crew work interruptions has, however, been proposed to reduce project duration (Russell and Caselton 1988; Russell and Wong 1993; El-Rayes and Moselhi 2001). While this might be a valid option for residential construction, the same argument does not stand in the case of linear projects which generally have high mobilization/demobilization costs. Equipment used in this class of projects is heavy, expensive, and their idle time is costly. Accordingly, work interruptions are not permitted in the proposed model. Accounting for resource availability while optimizing cost and/or duration of repetitive construction projects has been addressed (Moselhi and El-Rayes 1993; El-Rayes and Moselhi 2001). These models treat a crew as an entity that has a single availability period. In order to account for multiple availability periods for each crew, alias crews are created, each with its own availability period. The proposed model overcomes this limitation by enabling the consideration of multiple availability periods for crews.

Transverse Obstructions

During the scope definition stage, large projects can be divided into a number of work zones and segments. Transverse obstructions can be either natural, such as rivers and creeks, or man-made, such as canals and existing highways. Access across transverse obstructions can either be (1) granted at an overhead (additional travel time and cost) or (2) not permitted. The latter class of obstructions can either be (1) physical, such as a river or gas pipeline with no overpasses across or (2) legislative, such as interstate and national borders. In the case of phased highway construction, insurmountable transverse obstructions can be used to define borders separating various project phases. Fig. 3 shows sample linear project, intercepted by a number of transverse obstructions. Generally, a project is divided into a number of work zones, defined essentially by insurmountable obstructions. In turn, each work zone is divided into a number of segments, defined by surmountable obstructions. Crews, on the other hand, are assigned to sections (Fig. 3). The relative locations of transverse obstructions can significantly impact crew assignments (definition of sections and assigning crews to them). Clearly, the size of a crew formation (number of crews) assigned to any repetitive activity must allow for at least one crew working at each work zone. For example, at least two crews should be assigned to any repetitive activity in the project shown in Fig. 3.

A square matrix of the order \((n+1)\) (where \(n\) is the total number of transverse obstructions), referred to, hereinafter, as the mobilization matrix, is designed to calculate crew travel time and cost. In this matrix, the travel distance between Segments \(i\) and \(j\) is represented by \(M_{ij} (j>i)\), and travel conditions are represented by \(M_{ij} (i>j)\) \((M_{ij} = \text{unity and zero for paved and off-highway conditions, respectively})\). Diagonal elements \(M_{ii}\), where \(i=j\), store data related to the travel distance from the contractor’s staging and maintenance shop to each of the project segments. For insurmountable obstructions \(M_{i+1,j}\) is set to a negative value, indicating that no access is granted between Sections \(i\) and
Unless otherwise specified, all elements $M_{i,j}$, where $i > j + 1$, default to the total distance traveled along the project route, and can be expressed as:

$$M_{i,j} = \begin{cases} 
\sum_{k=i}^{j-1} M_{k+1,k} + \sum_{k=i}^{j-1} L_k, & \text{if } M_{k+1,k} \neq 0 \\
-1, & \text{otherwise}
\end{cases}$$

(1)

where $L_k$ is the length of Segment $k$, and $M_{k+1,k}$, for $k = i, i + 1, \ldots, j - 1$, are greater than zero. Otherwise $M_{i,j} = -1$.

The first term of the right-hand-side of Eq. (1) is the total distance traveled to overcome all obstructions between Segments $i$ and $j$, while the second part is the total travel distance between the same segments, measured along the project route. If the value of any element $M_{k+1,k}$ for $k = i, i + 1, \ldots, j - 1$ is negative, this indicates the presence of an insurmountable obstruction between the two segments, and access between them is accordingly denied. To explain how the matrix referred to above can be constructed, a numerical example is considered. The example involves the linear project shown in Fig. 3 and the data listed in Table 1. Assuming that the contractor’s staging and maintenance shop is 100 km away from the project site, the mobilization matrix can be expressed as:

$$M_{(4 \times 4)} = \begin{bmatrix}
100 & 3.4 & -1 & -1 \\
0 & 100 & -1 & -1 \\
-1 & -1 & 100 & 5.7 \\
-1 & -1 & 1 & 100
\end{bmatrix}$$

The mobilization matrix is employed to determine the mobilization/demobilization time of crews according to Eq. (2):

$$t_{c_{lj-k}} = \frac{1}{wh} \times \left( \max_{i=1 \ldots n_{eq}} \left( T_{\text{demob}_i} + \frac{M_{ij-k}}{v_{lj-k}} + T_{\text{mob}_i} \right) \right)$$

(2)

where $t_{c_{lj-k}}$ is the time (working days) required to relocate Crew $c$ from Segment $j$ to Segment $k$; $T_{\text{demob}_i}$ and $T_{\text{mob}_i}$ are demobilization and mobilization times (h) required by Equipment $i$, respectively; $wh$ is the number of working hours per day; $M_{ij-k}$ is the distance (km) between Segments $j$ and $k$; $v_{lj-k}$ is the travel speed (km/h) of Equipment $i$ corresponding to the road conditions as given by $M_{ij-k}$; and $n_{eq}$ is the number of pieces of equipment comprising the crew.

Travel costs incurred by Crew $c$ to travel between Segments $j$ and $k$, $C_{c_{lj-k}}$, is expressed as:

$$C_{c_{lj-k}} = \sum_{i=1}^{n_{eq}} (t_{c_{lj-k}} \times d_{ci} + C_{\text{demob}_i} + C_{\text{mob}_i})$$

(3)

where $C_{\text{mob}_i}$ and $C_{\text{demob}_i}$ are mobilization and demobilization costs of Equipment $i$, respectively; $d_{ci}$ is the direct daily cost of Equipment $i$; $C_{\text{mob}_i}$ is the cost of transporting Equipment $i$ from Segment $j$ to Segment $k$; and $wh$, $t_{c_{lj-k}}$, and $n_{eq}$ are as defined in Eq. (2).

While mobilization/demobilization costs and duration, $C_{\text{mob}_i}$, $C_{\text{demob}_i}$, $C_{\text{mob}_i}$, $C_{\text{mob}_i}$, and $T_{\text{mob}_i}$, are zero for certain types of equipment (such as haul and dump trucks), they could be substantial for others (such as cranes and pavers). Transportation cost, $C_{\text{mob}_i}$, would only be incurred if a float is required, and can be calculated as follows:

$$C_{\text{mob}_i} = DFC \times t'_{c_{lj-k}}$$

(4)

where $DFC$ is the daily float cost (operating or rental), and $t'_{c_{lj-k}}$ is as defined in Eq. (2), rounded to the next whole number and $C_{\text{mob}_i}$ is as defined in Eq. (3).

Should a bypass for a transverse obstruction be planned, its completion date marks the elimination of the impact of that obstruction on crew movement across project segments. The completion date of a bypass is represented by the finish date of its surface preparation. The mobilization matrix, in this case, can be updated accordingly.

### Table 1. Obstruction Details

<table>
<thead>
<tr>
<th>Obstruction number</th>
<th>Location (station)</th>
<th>Distance to overcome (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.5</td>
<td>3.4</td>
</tr>
<tr>
<td>2</td>
<td>9.1</td>
<td>a</td>
</tr>
<tr>
<td>3</td>
<td>13.4</td>
<td>5.7</td>
</tr>
</tbody>
</table>

*Obstruction is insurmountable*
Cost Estimating

The direct cost of executing a repetitive activity (DAC), can be expressed as

\[
DAC = \sum_{c=1}^{n} \left( d_c \times C_c + 2 \times MC_c + \sum_{i=1}^{p} C_{elj-k} \right) + MCU \times \sum_{j=1}^{m} Q_j
\]

where \(d_c\) = duration (working days) that Crew \(c\) spends on the job; \(C_c\) = daily cost of Crew \(c\); \(MC_c\) = mobilization and demobilization cost of Crew \(c\); \(C_{elj-k}\) = cost of Relocation \(r\) of Crew \(c\), as computed in Eq. (3); MCU is the material cost per unit; \(Q_j\) = quantity of work in Unit \(j\); \(p\) = number of times Crew \(c\) is relocated; and \(m\) = number of units in the activity.

Total activity cost (TAC) is expressed as

\[
TAC = DAC + \Delta t \times IDC
\]

where \(\Delta t\) = increase in project duration and IDC = project daily indirect cost.

For a nonrepetitive activity, a crew formation can only be composed of a single crew \((n = 1)\). Accordingly, the direct cost of that activity, DAC, can be expressed as

\[
DAC = d_1 \times C_1 + 2 \times MC_1
\]

Dynamic Programming Formulation

The objectives of the developed optimization procedure are to (1) identify the optimum or near optimum crew formation for each activity and (2) assign each crew of that formation to its work area(s) (i.e., activity units) in a practical manner that achieves the stated optimization objective. As the number of activities and potential crew formations increases, this problem becomes computationally infeasible if all combinations of crew formations and crew assignments are to be considered. A set of heuristic rules has been developed in conjunction with the two-state variable, \(N\)-stage, dynamic programming formulation, to provide an efficient solution. The heuristic rules are designed to reduce the solution space, while ensuring an optimum (or near-optimum) solution can be obtained. A sample of these rules is presented in Appendix I. For each activity, the model identifies the optimum crew formation from a given pool, and assigns each crew within that formation to a set of units in order to minimize either: (1) project duration; (2) project total cost; or (3) their combined impact in cost-plus-time bidding, also referred to as A+B bidding (Herbsman 1995). The optimization process takes place at the activity and project levels.

Activity Level

For a crew formation assigned to an activity, the objective is to identify the optimum crew assignments (i.e., which crews work on which units). Neither nonrepetitive activities nor repetitive ones with crew formations composed of a single crew are considered at the activity level. The crew assignment scenario that satisfies the stated optimization objective is determined by following the procedure described below, and outlined in Fig. 4.

- **Step 1**
  The project is divided into sections based on: (1) locations of transverse obstructions and (2) possible start dates in all units of the activity being considered, respecting job logic and the required buffers between succeeding activities. Locations of discontinuities are determined by inspecting the possible start dates (PSD) (see Fig. 5), of all activity units, i.e.,

\[
PSD_{j+1} \leq PSD_j \quad (j = 1, \ldots ,m)
\]

The total number of sections \((b)\) is determined, along with the number of insurmountable obstructions \((N_{ins})\).

- **Step 2**
  The quantity of work for the activity under consideration in Section \(k\), \(QW_k\), is determined using Eq. (9)

\[
QW_k = \sum_{j=1}^{u_{kl}} Q_j
\]

where \(QW_k\) = quantity of work in Section \(k\); \(u_{kl}\) and \(u_{kl}\) = numbers of the first and last units in Section \(k\); and \(Q_j\) as defined in Eq. (5).

- **Step 3**
  The total quantity of work in the activity under consideration is initially subdivided among crews in the current crew formation. The quantity of work assigned to Crew \(c\) \((c = 1 \ldots n)\), where \(n\) is the total number of crews in the formation), IAQ, is based on the performance of that crew compared to that of other crews in the formation. If cost is being optimized, IAQ, is expressed as

\[
IAQ_c = \frac{C_c}{P_c} \times \frac{\sum_{j=1}^{n} Q_j}{\sum_{j=1}^{n} Q_j}
\]

or otherwise

\[
IAQ_c = \frac{P_c}{C_c} \times \frac{\sum_{j=1}^{m} Q_j}{\sum_{j=1}^{m} Q_j}
\]

where \(m\) = total number of activity units; \(n\) = total number of crews in the crew formation; \(P_c\) = daily productivity of Crew \(c\); and \(Q_j\) and \(C_c\) are as defined in Eq. (5).

- **Step 4**
  In order to ensure that all crews are employed, the number of Sections \(b\) determined in Step 1 is compared to the number of crews in the crew formation under consideration \(n\). If \(b > n\), then the number of sections is sufficient to employ all crews and the computation proceeds with Step 5. Otherwise, sections need to be further divided until \((b = n)\) enabling the employment of all crews. If \(b = 1\), the project needs to be divided into a number of sections equal to the number of crews in the formation. This division is carried out such that the quantity of work in Segment \(k\) is equal to the initial quantity of work assigned to Crew \(c\) \((QW_c = IAQ_c \times c = 1 \ldots n)\). On the other hand, if \(n > b > 1\), then the quantity of work in the smallest section (Section \(x\)) \([QW_x = \text{Min}(QW_{j})]k = 1 \ldots b\) is compared to the initial quantities assigned to all crews. If \(QW_x \leq IAQ_c\), \(c = 1 \ldots n\), then Crew \(c\) is assigned to the section, and both are discarded from future consideration. This procedure is repeated until no section can be assigned. At this stage, the largest remaining unassigned section (Section \(y\)) \([QW_y = \text{Max}(QW_{j})]k = 1 \ldots b\) is divided into two portions \(y_1\) and \(y_2\). The quantities of work in Sections \(y_1\) and \(y_2\) are set equal to \(IAQ_c\), and \(Q_j - IAQ_c\), respectively, and Crew \(c\) is assigned to Section \(y_1\). Next, the chaining at the boundary between those two sections is determined. This is achieved by inspecting the location of Section \(y\) with respect to the remaining unassigned length of the project. Crew \(c\) and its associated
section $y_1$ are removed from further considerations, and the above procedure is repeated for the remaining sections and crews. This is carried out until only one section remains to which multiple crews are assigned. The assignment process for the last section is similar to that of a project composed of a single section.

- **Step 5**
The time span between start times of the first and last units of immediately preceding activity(ies) in Section $k$, $D_k$ [where $b = 1 \ldots b$, where $b$ is the total number of sections] (see Fig. 5), is expressed as

$$D_k = \text{PSD}_{ak/l} - \text{PSD}_{ak/l}$$

(11)

where $D_k =$ duration of works at Section $k$ for the immediately preceding activity(ies); $u_{k/l}$ and $u_{k/l}$ are as defined in Eq. (9), and PSD$_{ak/l}$ and PSD$_{ak/l}$ = possible start dates for units $u_{k/l}$ and $u_{k/l}$, respectively.

- **Step 6**
A ratio of the quantity of work of the section being assigned and that expressed by the performance of the crew is calculated as

$$r_{ck} = \frac{\text{IAQ}_c}{\text{QW}_k}$$

(12)

where $\text{QW}_k$ and $\text{IAQ}_c$ are as defined in Eqs. (9) and (10), respectively.

Considering crew performance, the closer $r_{ck}$ is to unity, the higher the priority of assigning Crew $c$ to Section $k$.

- **Step 7**
The time required for Crew $c$ to complete the work in Section $k$ can be expressed as

$$T_{ck} = \frac{\text{QW}_k}{P_c}$$

(13)

where $T_{ck} =$ time required by Crew $c$ to complete work at Section $k$ and $\text{QW}_k$ and $P_c$ are as defined in Eqs. (5) and (10), respectively.

Crews are then ranked in ascending order of $Z_{ck}$ ($c = 1 \ldots n$), based on the absolute difference between $T_{ck}$ and $D_k (Z_{ck} = |T_{ck} - D_k|)$ (see Fig. 5).

- **Step 8**
Four performance indices ($\alpha$, $\beta$, $\gamma$, and $\eta$) have been introduced to aid in the crew assignment process. These indices are assigned a value between unity and $n$, where $n$ is the total
number of crews in the crew formation under study. Their application reduces the solution space, while ensuring that an optimum or near-optimum solution can be achieved. The first index $\alpha_{ck}$ provides a measure as to how close the capacity of the assigned Crew $c$ is to the scope of work in Section $k$, i.e., if the scope of work $(QW_k)$ is within the limits of maximum and minimum capacities specified for Crew $c$, then $\alpha_{ck}$ is set equal to unity. Otherwise, it is set equal to $n$. Unity indicates a ranking of first choice, as opposed to $n$, least desirable. The next performance index $\beta_{ck}$ provides a measure as to how close the initially assigned quantity of work (IAQ) to Crew $c$ is to the entire scope of work of Section $k$, i.e., $\beta_{ck}$ represents the order of crews when ranked in ascending order of $r_{ck}$, as computed in Step 6 above. The third index $\gamma_{ck}$ provides a measure as to how close the rate of progress of Crew $c$ is, if assigned the scope of work in Section $k$, i.e., $\gamma_{ck}$ represents the order of crews when ranked in ascending order of $Z_{ck}$, as discussed in Step 7 above. A composite index combining the three indices $\eta_{ck}$ can be expressed as

$$\eta_{ck} = \frac{\alpha_{ck} + \beta_{ck} + \gamma_{ck}}{3} \quad \text{if } (n=b) \quad (14a)$$

$$\eta_{ck} = \gamma_{ck} \quad \text{if } (n\neq b) \quad (14b)$$

where $\eta_{ck}$=composite index for Crew $c$ to be assigned Section $k$ and $\alpha_{ck}$, $\beta_{ck}$, and $\gamma_{ck}$=performance indices of Crew $c$ to be assigned to Section $k$ based on its capacity, relative performance within the crew formation and balancing with preceding activity(ies), respectively.

*Step 9*

The optimum assignment recommendations for all sections are investigated to determine whether all crews are employed. The project sections are then ranked in a descending order of their respective quantity of work (QW$_k$). The possible assignment scenarios are generated according to the relative values of $b$ and $n$ as described below.

*Case 1: ($b=n$)*

In this case, each crew can only be assigned to one section. The goal is to generate assignment scenarios such that each section is, at least once, assigned its optimum crew. A maximum of $n$ assignment scenarios is generated, where one section is assigned its optimum crew [crew with lowest $(\eta_{1k})$ ($k = 1 \ldots b$)]. The remaining sections are assigned a priority based on their respective scope of work, and

- **Case 2: ($b>n$)**
  In this case, crews (at least one) will be assigned to work on more than one section. The approach adopted in this case depends on the number of insurmountable obstructions ($N_{ins}$) defined in the project. Three scenarios are considered: (1) $N_{ins} = 0$; (2) $N_{ins} = n-1$; and (3) $n > N_{ins} > 1$.

- **Scenario i: ($N_{ins} = 0$)**
  In this case, crews are free to relocate from any project segment to another. A maximum of $n$ assignment scenarios are generated, where the largest $n$ sections are allocated crews in a manner similar to Case 1 above. In each of these scenarios, the quantities of work assigned to each crew are computed, and compared to the initially assigned quantities (IAQ$_c$). A crew is declared unavailable once it is assigned a quantity of work equal to, or greater than (IAQ$_c$). The optimum crews for the remaining sections are assigned based on crew availability.

- **Scenario ii: ($N_{ins} = n - 1$)**
  In this case, only the largest section in each of the work zones are considered. Crew assignment is carried out in a manner similar to Case 1 above. It should be noted that in each assignment scenario, the crew assigned to the largest section within a work zone executes the remainder of the work within that zone, and

- **Scenario iii: ($n > N_{ins} > 1$)**
  A procedure similar to that in Scenario (ii) above is employed to assign crews to each of the largest sections of the project work zones. A total of $N_{ins} + 1$ assignment scenarios are thus generated. In each generated scenario, the crew assigned to the largest section within a work zone completes the work within that zone (Q$_{Ind\_Seg}$). These quantities are compared with the initially-assigned quantities, IAQ$_c$, and the segment that has the highest value of Q$_{Ind\_Seg}$/IAQ$_c$ is assigned one of the unassigned crews. This process is repeated until all crews have been assigned. Sections that did not receive crews at this stage are queued for crew assignment pending availability and the above stated criteria.

*Step 10*

The assignment scenarios are investigated to remove duplicates. For each assignment scenario, finish times of all units are determined accounting for factors such as weather and the learning curve effect, while maintaining crew work continuity. Periods during which each crew would be required are determined, and the availability of each crew during that period is
determined. If necessary, a revised activity schedule accounting for equipment availability is developed. Total cost is then estimated using Eq. (6).

• Step 11
  The assignment scenario that satisfies the stated optimization objective is identified as the local optimum solution.

Project Level

At the project level, the objective is to identify the optimum crew formation for each activity in the project, and consequently the global optimum solution. This is achieved in two paths; forward and backward. The forward path starts with the first activity, and propagates through all activities, investigating all crew formations and assignments, as described above. The local optimum solution for the last activity is identified as the global optimum solution, and the backward path is initiated. The objective of the backward path is to scan the local optimum solutions that were identified in the forward path, i.e., identify the optimum crew formation and crew assignments for each activity, along with the optimum crew formation and assignment scenario for its preceding activity(ies). The above procedure is repeated for all activities, until the optimum crew formation and assignment scenario for the first activity are identified.

Computer Implementation

The proposed model was incorporated in a prototype software, operating in Microsoft Windows® environment. The software has been coded in Visual C++, utilizing object-oriented programming. Microsoft Access® was employed as the database management system. The software provides a user-friendly interface to facilitate data entry, and generates reports at varying levels of detail to suit the requirements of the various members of the project team. Bar (Gantt) charts are generated, depicting both working and calendar days, as well as assignment dates of all crews. The software also generates linear schedules in the format adopted in highway construction (Harmelink 2001), where the vertical and horizontal axes represent time and chainage, respectively. In addition, the developed software has a number of interesting features: (1) enabling the user to specify holidays and designated work weeks; (2) accepting several input file formats (Microsoft Excel® and tab-delimited text files); and (3) facilitating revisions pertaining to activities and crews. The developed prototype also automates the data input process, employing the capabilities of geographic information systems (Hassanein and Moselhi 2001).

Numerical Example

In order to illustrate the use of the proposed scheduling algorithm and demonstrate its capabilities, a highway project drawn from the literature (El-Rayes and Moselhi 1998) is analyzed. Five serial repetitive activities are considered in the construction of the 15-km stretch of the highway. These activities, in their order of precedence, are (1) cut and chip trees; (2) grub and remove stumps; (3) excavation; (4) base; and (5) paving, and all precedence relations are finish to start, with no lag time. The unit size of all activities is set to 1 km. The quantities of work and the crews assigned to each activity are included in Appendix II for easy reference.

<p>| Table 2. Tabular Output for Activity “Grub and Remove Trees” |
|---|---|---|---|---|</p>
<table>
<thead>
<tr>
<th>Unit number</th>
<th>Chainage</th>
<th>Assigned crew</th>
<th>Schedule (day/month/year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0+000 1+000</td>
<td>Crew 1</td>
<td>6/5/02 9/5/02</td>
</tr>
<tr>
<td>2</td>
<td>1+000 2+000</td>
<td>Crew 2</td>
<td>9/5/02 14/5/02</td>
</tr>
<tr>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>9</td>
<td>8+000 91+000</td>
<td>Crew 1</td>
<td>22/7/02 25/7/02</td>
</tr>
<tr>
<td>10</td>
<td>9+000 10+000</td>
<td>Crew 2</td>
<td>20/5/02 24/5/02</td>
</tr>
<tr>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>15</td>
<td>14+000 15+000</td>
<td>Crew 2</td>
<td>5/8/02 8/8/02</td>
</tr>
</tbody>
</table>
The overall project duration was estimated to be 87 working days (Fig. 6), and Table 2 lists a portion of the generated tabular schedule. The estimated duration is slightly longer than that reported in the original example (83 days). This is attributed to the use of interruption vectors, which is not considered in the proposed model. It is worth noting that when work enforcing work continuity in the earlier model (El-Rayes and Moselhi 1998), the overall project duration is estimated to be 87 working days. Of particular importance here is to note that, unlike the earlier model (El-Rayes and Moselhi 1998), the proposed model assigns crews to adjacent units, for assigning crews to alternating units is an impractical assignment scheme in linear projects. This is due to the additional travel costs and times (see Fig. 1), along with the losses in the learning process due to the frequent interruptions.

In order to illustrate the impact of transverse obstructions on crew assignments and overall project duration, three transverse obstructions are introduced at stations 4, 7.2, and 10.3. The obstruction at station 7.2 is assumed insurmountable, while the distances traveled to overcome obstructions at stations 4 and 10.3 are 4.8 and 3.2 km, respectively. The mobilization matrix can be expressed as

\[
M_{(4\times 4)} = \begin{bmatrix}
100 & 4.8 & -1 & -1 \\
0 & 100 & -1 & -1 \\
-1 & -1 & 100 & 3.2 \\
-1 & -1 & 1 & 100
\end{bmatrix}
\] (15)

The project duration considering obstructions was found to be 98 working days, based on the schedule shown in Fig. 7. The figure shows that the crew assignments have been altered to minimize mobilization/demobilization costs and time, and account for the insurmountable obstruction at Station 7.2.

**Concluding Remarks**

A flexible model designed to optimize scheduling of linear construction in general, and highway construction in particular, is presented. The model employs a two-state-variable, N-stage, dynamic programming formulation to obtain practical, near-optimal solutions. Resource-driven scheduling is utilized, and both repetitive and nonrepetitive activities are considered in the optimization procedure. The model assists in optimizing construction time, cost or both under cost-plus-time bidding, and was validated against an earlier model, and found to yield comparable results. The model has a number of interesting and practical features, such as accounting for transverse obstructions, accounting for nonserial activities, assigning multiple crews, and accounting for mobilization/demobilization cost. The model has been implemented in a prototype software operating in Windows® environment, developed utilizing object-oriented programming. The software provides a user-friendly interface and generates several graphical and tabular output reports. A numerical example was analyzed, and the impact of transverse obstructions was assessed.

**Acknowledgments**

The writers would like to thank Mr. John Mirabelli of SNC Lavalin for his time and valuable input. The financial support provided by the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged.

**Appendix I. Sample of Developed Heuristic Rules**

- If the performance of Crew c is less than the average performance of all crews of the formation, then do not assign it more beyond that initially assigned (IAQc),
- If minimizing time and more than one crew share priority to be assigned to a section, then select the crew that is more balanced with preceding activities,
- If minimizing total cost and more than one crew share priority to be assigned to a section, then select the crew that has the least cost-per-unit, and
- If assigning the optimum crew to a section requires relocation

**Fig. 7. Schedule considering obstructions**
The following symbols are used in this paper:

\[ C_c = \text{daily cost of Crew } c; \]
\[ C_{ctj-k} = \text{mobilization cost of Crew } c \text{ from Segment } j \text{ to Segment } k; \]
\[ C_{\text{demob}_i} = \text{demobilization cost of Equipment } i; \]
\[ C_{\text{mob}_i} = \text{mobilization cost of Equipment } i; \]
\[ CT_{ij-k} = \text{cost of transporting Equipment } i \text{ from Segment } j \text{ to Segment } k; \]
\[ D_k = \text{duration of works at Segment } k; \]
\[ dc_i = \text{direct daily cost of Equipment } i; \]
\[ IAQ^*_c = \text{quantity initially assigned to Crew } c; \]
\[ L_k = \text{length of Segment } k; \]
\[ M_{ij} = \text{element } [i][j] \text{ of mobilization matrix}; \]
\[ N_{\text{ins}} = \text{number of insurmountable obstructions}; \]
\[ n = \text{number of crews comprising crew formation}; \]
\[ n_{eq} = \text{number of pieces of equipment comprising crew}; \]
\[ n_{eq}' = \text{number of pieces of equipment requiring float for travel}; \]
\[ P_c = \text{daily productivity of Crew } c; \]
\[ Q_j = \text{quantity of work in Unit } j; \]
\[ T_{\text{demob}_j} = \text{float demobilization time (h)}; \]
\[ T_{\text{mob}_j} = \text{demobilization time (h) required by Equipment } i; \]
\[ T_{\text{mob}_k} = \text{mobilization time (h) required by Equipment } i; \]
\[ t_{ctj-k} = \text{time (working days) required to relocate Crew } c \text{ from Segment } j \text{ to Segment } k; \]
\[ u_{k/l} \text{ and } u_{k/L} = \text{numbers of first and last units of Section } k, \text{ respectively}; \]
\[ v_{i,j-k} = \text{travel speed (km/h) of Equipment } i \text{ corresponding to road conditions between Segments } j \text{ and } k; \]
\[ wh = \text{number of working hours per day}; \]
\[ \alpha_{ck} = \text{performance indicator of Crew } c \text{ to work on Section } k \text{ based on its capacity}; \]
\[ \beta_{ck} = \text{performance indicator of Crew } c \text{ to work on Section } k \text{ based on its performance}; \]
\[ \gamma_{ck} = \text{rank of Crew } c \text{ to work on Section } k \text{ based on balancing progress with immediately preceding activity(ies)}; \]
\[ \eta_{ck} = \text{composite performance indicator of Crew } c \text{ to work on Section } k. \]

References


Appendix II. Quantities of Work and Crews Assigned for Numerical Example

See Tables 3 and 4.

Notations

Table 3. Work Quantities

<table>
<thead>
<tr>
<th>Unit (km)</th>
<th>Cut and chip trees (m³)</th>
<th>Grub and remove stumps (m³)</th>
<th>Earthmoving (m³)</th>
<th>Base (m³)</th>
<th>Paving (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12,000</td>
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<td>6,000</td>
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<td>32,000</td>
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<td>6,000</td>
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</table>

<table>
<thead>
<tr>
<th>Activity</th>
<th>Crew number</th>
<th>Crew identification (means)</th>
<th>Daily productivity (units/day)</th>
<th>Earliest available date</th>
<th>Latest available date</th>
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<td>a</td>
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</table>

*aCrew is available till project completion.

of that crew and the scope of work of that section is less than (IAQ/c4), then do not relocate that crew.


