Distributed Coverage Algorithms for a Network of Nonidentical Mobile Sensors

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Abstract

In this report, efficient deployment algorithms are proposed for a mobile sensor network to improve the coverage area. The proposed algorithms find the target position of each sensor iteratively, based on the existing coverage holes in the network. The multiplicatively weighted Voronoi (MW-Voronoi) diagram is used to discover the coverage holes corresponding to different sensors with different sensing ranges. Three sensor deployment algorithms are provided: Under the proposed procedures, the sensors move in such a way that the coverage holes in the target field are reduced. Simulations confirm the effectiveness of the deployment algorithms proposed in this report.

I. INTRODUCTION

Wireless sensor networks have attracted considerable attention in the literature in recent years, due to their widespread applications in both military and civilian environments [1], [2], [3].

A Voronoi-based approach for network coverage is presented in [4], where no global location assurance condition is required for the sensors. A multi-objective deployment and power assignment algorithm is proposed in [5], where the optimization problem is decomposed into several scalar single-objective problems which need to be solved simultaneously. In [6], an algorithm is presented to add a relatively small number of mobile sensors to a set of static sensors, in order to improve network coverage. The algorithm employs a strategy which aims to optimize the contribution of the mobile sensors to the overall coverage. In [7], location services for mobile ad-hoc networks are studied, as a means for obtaining the local information of the destination. A new coverage model called surface coverage is introduced in [8], and the problem of expected coverage ratio and optimal deployment strategy are addressed accordingly. Three approximation algorithms are also given in [8], with provable approximation ratios.

In [9], [10], [11], it is assumed that the sensing radii of the sensors are equal, and the Voronoi diagram is used to partition the target field to find the coverage holes corresponding to each sensor. In [11], three algorithm, namely VEC, VOR, and Minimax are proposed to determine the final destination of each sensor in the network, to increase the coverage. In [10], [11], two different approaches, namely, basic protocols and virtual movement protocols are introduced to deploy the sensors in proper positions in order to improve network coverage. In all of the above papers and most of the results reported in prior literature, the sensing capabilities of all sensors are assumed to be identical [4], [9], [10], [11]. The Voronoi diagram is subsequently used to discover coverage holes, and move the sensors accordingly.

The objective of this report is to develop sensor deployment algorithms in a network of mobile sensors with different sensing capabilities, for effective network coverage. The multiplicatively weighted Voronoi (MW-Voronoi) diagram is utilized to find the coverage holes, where the weight assigned to each sensor is proportional to its sensing radius [12], [13], [14]. Three algorithms are proposed: Weighted Vector Based (WVB), Minimax-curve and Maximin-curve. The main idea behind these algorithms is to move each sensor iteratively in such a way that its sensing coverage is increased. Once a new location for a sensor is computed, the corresponding local coverage area of the sensor (in the previously constructed MW-Voronoi diagram) is compared to the preceding local coverage area. If the new local coverage area would be larger than the preceding one, the sensor moves to the new location; otherwise, it remains in its current position. If the local coverage area by each sensor in an iteration does not exceed a certain threshold, the algorithm is terminated (to ensure a finite number of iterations).

The rest of the report is organized as follows. In Section II, some preliminaries and important notions and definitions are provided. Section III presents the main results of the report, where new deployment algorithms are introduced. In Section IV, simulation results are given to demonstrate the effectiveness of the proposed strategies. Finally, concluding remarks are drawn in Section V.

A. MW-Voronoi Diagram

Let \( S \) be a set of \( n \) distinct weighted nodes in the plane denoted by \((S_1,w_1),(S_2,w_2),\ldots,(S_n,w_n)\), where \( w_i > 0 \) is the weighting factor associated with \( S_i \), for any \( i \in \mathbb{N} := \{1,2,\ldots,n\} \). It is desired now to partition the plane into \( n \) regions such that:

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• Each region contains only one node, and
• the nearest node, in the sense of weighted distance, to any point inside a region is the node assigned to that region.

The diagram obtained by the partitioning described above is called the *multiplicatively weighted Voronoi* (MW-Voronoi) diagram [13]. Analogous to conventional Voronoi diagram, the mathematical characterization of each region obtained by the partitioning described above is as follows:

\[ \Pi_i = \{ Q \in \mathbb{R}^2 | w_i d(Q, S_i) \leq w_j d(Q, S_j), \forall j \in n - \{i\} \} \]  

for any \( i \in n \), where \( d(Q, S_i) \) is the Euclidean distance between \( Q \) and \( S_i \).

According to (1), any point \( Q \) in the \( i \)-th MW-Voronoi region \( \Pi_i \) has the following property:

\[ \frac{d(Q, S_i)}{d(Q, S_j)} \leq \frac{w_i}{w_j}, \forall i \in n, \forall j \in n - \{i\} \]  

**Definition 1.** Similar to the conventional Voronoi diagram, the nodes \( S_i \) and \( S_j \) \((i, j \in n, i \neq j)\) in an MW-Voronoi diagram are called neighbors if \( \Pi_i \cap \Pi_j \neq \emptyset \) (note that the intersection of the Voronoi regions of two neighboring sensors only includes some boundary points). The set of all neighbors of \( S_i \), \( i \in n \), is denoted by \( N_i \) and is formulated below

\[ N_i = \{ S_j \in S | \Pi_i \cap \Pi_j \neq \emptyset, \forall j \in n \} \]  

**Definition 2.** Consider the sensor \( S_i \) with the sensing radius \( r_i \) and the corresponding MW-Voronoi region \( \Pi_i \), \( i \in n \), and let \( Q \) be an arbitrary point inside \( \Pi_i \). The intersection of the region \( \Pi_i \) and a circle of radius \( r_i \) centered at \( Q \) is referred to as the *\( i \)-th coverage area w.r.t. \( Q \).* The \( i \)-th coverage area w.r.t. the location of the sensor \( S_i \) is called the *local coverage area of that sensor.*

**Definition 3.** Consider an arbitrary point \( Q \) inside the MW-Voronoi region \( \Pi_i \), \( i \in n \). The area inside the MW-Voronoi region \( \Pi_i \) which lies outside the \( i \)-th coverage area w.r.t. \( Q \) is referred to as the *\( i \)-th coverage hole w.r.t. \( Q \).* and is denoted by \( \Theta_i^Q \). The \( i \)-th coverage hole w.r.t. the location of the sensor \( S_i \) is called the *local coverage hole* of that sensor, and is denoted by \( \Theta_i \). Also, the union of all local coverage holes in the sensing field is referred to as the *total coverage hole,* and is denoted by \( \Theta \), i.e., \( \Theta = \sum_{i=1}^{n} \Theta_i \).

**Definition 4.** The *Apollonian circle* of the segment \( AB \), denoted by \( \Omega_{AB,k} \), is the locus of all points \( E \) such that \( \frac{AE}{BE} = k \) [15].

To construct the \( i \)-th MW-Voronoi region, first the Apollonian circles of the neighboring partitions are found for the \( i \)-th sensor. In other words, the Apollonian circles \( \Omega_{S_j,S_i} \) are found for all \( S_j \in N_i \). The smallest region created by these circles which contains the \( i \)-th node is, in fact, the \( i \)-th MW-Voronoi region (as an example, see Fig. 1). An MW-Voronoi diagram with 15 sensors is sketched in Fig. 2.

![Fig. 1. The MW-Voronoi region for a sensor \( S_1 \) with four neighbors \( S_2, \ldots, S_5 \) [16].](image-url)

The MW-Voronoi diagram is used to develop sensor deployment strategies in this report. Each sensor has a sensing area which is a circle whose size can be different for distinct sensors. Consider each sensor in the field as a node with a weight equal to its sensing radius, and sketch the MW-Voronoi diagram. From the characterization of the MW-Voronoi regions provided in (1), it is straightforward to show that if a sensor cannot detect a phenomenon in its corresponding region, other sensors cannot detect it either. This means that in order to find the "so-called" coverage holes (i.e., the undetectable points in...
the network), it would suffice to find the points in the MW-Voronoi region of each node, which lie outside its local coverage area.

**Assumption 1.** In this report, it is assumed that there is no obstacle in the field. Therefore, the sensors can move to any desired location without any obstacle avoidance concern, using existing techniques (e.g., the schemes proposed in [17], [18], [19]).

**Assumption 2.** The sensors are capable of localizing themselves in the field, with sufficient accuracy (using, for instance, the methods proposed in [20], [2]).

**Assumption 3.** The communication range of every sensor is bounded (and not necessarily the same for all sensors). This is a limiting factor for the sensors, potentially preventing them from reaching their neighbors, which in turn results in a wrong Voronoi region around some of the sensors. Such a limitation can negatively affect the accurate detection of coverage holes.

![Fig. 2. An example of the MW-Voronoi diagram for a group of 15 unidentical sensors in a network [16].](image)

### III. Deployment Protocols

In this section, three different protocols are developed for a distributed sensor network. The proposed algorithms are iterative, where in each iteration every sensor first broadcasts its sensing radius and location to other sensors, and then constructs its MW-Voronoi region based on the similar information it receives from other sensors. It checks the region subsequently to detect the possible coverage holes. If any coverage hole exists, the sensor calculates its target location (but does not move there) using a proper scheme, which guarantees that by moving there the coverage hole would be eliminated, or at least its size would be reduced by a certain threshold. Once the new target location is calculated, the coverage area w.r.t. this location (in the previously constructed MW-Voronoi region) is obtained. If this coverage area is greater than the current one, the sensor moves to the new location; otherwise it remains in its present position. In order to terminate the algorithm in finite time, a proper coverage improvement threshold \( \varepsilon \) is defined such that the algorithm will continue only if there is a sensor in the network whose coverage increases at least by \( \varepsilon \) in the next iteration.

Finally, when none of the sensors' weighted coverage or sensors' dynamic coverage area in its corresponding Voronoi polygon would be increased by a certain threshold level, there is no need to continue the iterations. As noted above, one of the important characteristics of the sensor deployment strategies proposed in this report is that each sensor moves to its new destination point only if its coverage area w.r.t. the new location in the old MW-Voronoi region increases. The following theorem shows that the total coverage is increased under this type of deployment scheme.

**Theorem 1.** Consider the set \( S \) of \( n \) sensors in the plane, and let their positions be denoted by \( P = \{P_1, P_2, \ldots, P_n\} \) with the corresponding MW-Voronoi regions \( \Pi_1, \Pi_2, \ldots, \Pi_n \). Assume the sensors move to new positions \( \tilde{P} = \{\tilde{P}_1, \tilde{P}_2, \ldots, \tilde{P}_n\} \) such that \( \tilde{P}_i \neq P_i \) if and only if \( i \in K \), where \( K \) is a non-empty subset of \( n \). If the \( i \)-th coverage area w.r.t. \( \tilde{P}_i \) in the previously constructed MW-Voronoi region \( \Pi_i \) is greater than the \( i \)-th local coverage area (i.e., the coverage area w.r.t. \( P_i \)) for all \( i \in K \), then the total coverage in the network increases.

**Proof:** Let the total uncovered area of the sensing field when the sensors are located in \( P \) and \( \tilde{P} \) be denoted by \( \theta \) and \( \theta' \), respectively. From the characterization of the MW-Voronoi diagram:

\[
\theta = \sum_{i=1}^{n} \theta_i^P
\]

It is straightforward to show that by increasing the coverage area in \( \Pi_i, i \in K \), the corresponding coverage hole will be decreased. Since it is assumed that the \( i \)-th coverage area w.r.t. \( \tilde{P}_i \) is greater than the \( i \)-th local coverage area for any \( i \in K \),
one can conclude that:
\[
\theta_{i}^{*} < \theta_{i}^{n}, \quad \forall i \in K
\]  
(5)

In addition, note that if \( \hat{P}_i = P_i \) then:
\[
\theta_{i}^{\hat{P}} = \theta_{i}^{P}, \quad \forall i \in n|K
\]  
(6)

On the other hand, it is possible that some of the points in \( \theta_{i}^{\hat{P}} \) are also covered by other mobile sensors located at \( \hat{P}_j \), for some \( j \in n\backslash\{i\} \). Hence:
\[
\hat{\theta} \leq \sum_{i=1}^{n} \theta_{i}^{\hat{P}}
\]  
(7)

Furthermore, from (5), (6) and (7) one arrives at the following inequality:
\[
\hat{\theta} < \sum_{i=1}^{n} \theta_{i}^{P}
\]  
(8)

Now, it is concluded from (4) and (8) that:
\[
\hat{\theta} < \theta
\]  
(9)

which means that the total coverage area increases using the underlying deployment scheme.

**Notation 1.** Given an MW-Voronoi diagram with \( n \) regions (each one corresponding to a node), the number of boundary curves and vertices (corners of the boundary, associated with the intersections of the boundary curves) of the \( i \)-th region \((i \in n)\) are denoted by \( e_i \) and \( m_i \), respectively. It is easy to verify that \( m_i = e_i \) for the case when the corresponding region has at least two vertices.

**Notation 2.** Throughout this report, a circle of radius \( r \), centered at the point \( O \), will be represented by \( \Omega(O,r) \).

The above-mentioned procedure will be used in the reminder of this section to develop the weighted vector based, Minmax-curve, and Maxmin-curve algorithms. It is to be noted that the target point for each sensor is determined using the techniques presented in Subsections III-A, III-B and III-C.

### A. Weighted Vector Based (WVB) Strategy

This method tends to move the sensors out of the densely covered areas. Denote with \( d_{ij} \) the distance between the sensors \( S_i \) and \( S_j \). Define a new (virtual) sensor network with \( h := \lceil \sum_{j=1}^{\frac{n}{2}} w_j^2 \rceil \) sensors of unit sensing radius, evenly distributed in the target area. Let \( d \) be the distance between a sensor and its nearest neighboring sensor in this new network (this distance can be calculated off-line). In this strategy, if the distance between the two sensors \( S_i \) and \( S_j \) in the original network \((i, j \in n)\) is less than \( \frac{w_i + w_j}{2} d \) and none of them covers its MW-Voronoi region completely, then a virtual force between the two sensors will tend to push them away, as if it wants to move the sensor \( S_i \) by \( \frac{w_i}{w_i + w_j} D_{ij} \) and the sensor \( S_j \) by \( \frac{w_j}{w_i + w_j} D_{ij} \), where \( D_{ij} = \frac{w_i + w_j}{2} d - d_{ij} \). If, however, one of the two sensors, say \( S_i \), covers its region completely, then it will not move, but will push the other sensor \( S_j \) by a virtual force as if it wants to move \( S_j \) by \( D_{ij} \). In the case when both sensors cover their regions completely, then they will not apply any virtual force to one another. In other words, for every pair of sensors, if there is a coverage hole in any of the corresponding two regions, then one or two virtual forces tend to push the sensors away from each other by \( \frac{w_i + w_j}{2} d \). On the other hand, virtual forces are also applied in a similar manner from each boundary to any sensor which is closer than a certain distance to it. More precisely, if the distance \( d_{hi} \) between \( S_i \) and a certain boundary is less than \( \frac{w_i}{2} d \), then a virtual force tends to push the sensor away from the boundary by \( \frac{w_i}{2} d - d_{hi} \). Eventually, each sensor is moved by the vector sum of all virtual forces applied to it from the boundaries and from other sensors.

Fig. 3 shows an operational example of the WVB strategy. In this example, 27 sensors are randomly deployed on a \( 50m \times 50m \) flat surface: 15 with a sensing radius of 6m, 6 with a sensing radius of 5m, 3 with a sensing radius of 7m, and 3 with a sensing radius of 9m. Moreover, the communication range of each sensor is assumed to be \( 10/3 \) times greater than its sensing range. In this figure, three snapshots are provided, and in each one both local coverage of the sensors (filled circles) and the MW-Voronoi regions are depicted. The initial coverage is 66.7\%, but after the first round it is improved to 71.9\%, and the final coverage is 85.1\%.

### B. Minmax-Curve Strategy

The idea behind the Minmax-curve technique is that normally for optimal coverage, each sensor should not be too far from any of its Voronoi curves. The Minmax-curve strategy selects the target location for each sensor as a point inside the corresponding MW-Voronoi region which has the smallest distance from the farthest curve. This point will be referred to as the Minmax-curve centroid, and will be denoted by \( \hat{O}_i \) for the \( i \)-th region, \( i \in n \). Furthermore, the distance between this point and the farthest curve from it will be represented by \( \hat{r}_i \). The Minmax-curve circle is defined in the sequel.
Consider the circle

**Lemma 2.**

**Definition 5.** The Minmax-curve circle of an MW-Voronoi region is the smallest circle centered inside or on the boundary of that region, intersecting or touching the region’s all curves (or their extensions). This circle is, in fact, \( \Omega(O_i, r_i) \), for the \( i \)-th region, and is generically unique. However, in some special configurations there can be more than one Minmax-curve circles.

**Notation 3.** The set of all boundary curves \( e_{i1}, e_{i2}, \ldots, e_{ij} \) of the \( i \)-th MW-Voronoi region will hereafter be denoted by the boldfaced symbol \( e_i \). In the present subsection, intersecting/tangent to/touching a boundary curve \( e_{ij} \) means intersecting/tangent to/touching \( e_j \) or its extension (\( i \in \{1, \ldots, e_i\} \)). It is to be noted that the extension of the boundary curve \( e_{ij} \) belongs to the same Apollonian circle as \( e_j \).

**Definition 6.** The bisector of two curves \( e_{i1} \) and \( e_{i2} \) is defined as the locus of any point \( E \) whose distance from \( e_{i1} \) is equal to that from \( e_{i2} \). The bisector of the curves \( e_{i1} \) and \( e_{i2} \) is denoted by \( \Gamma_{e_{i1}, e_{i2}} \).

**Lemma 1.** Consider two circles \( \Omega_1(O_1, r_1) \) and \( \Omega_2(O_2, r_2) \). The bisector of \( \Omega_1 \) and \( \Omega_2 \) is:

i) A branch of a hyperbola or the perpendicular bisector of \( O_1O_2 \), if \( \Omega_2 \) is outside \( \Omega_1 \).

ii) An ellipse, if \( \Omega_1 \) is inside \( \Omega_2 \).

iii) The union of a branch of a hyperbola or the perpendicular bisector of \( O_1O_2 \) and an ellipse, if \( \Omega_1 \) intersects \( \Omega_2 \).

**Lemma 2.** Consider the circle \( \Omega(O, r) \) and line \( \Delta \). The bisector of \( \Omega \) and \( \Delta \) is:

i) A parabola, if \( \Delta \) does not intersect \( \Omega \).

ii) The union of two parabolas, if \( \Delta \) intersects \( \Omega \).

**Lemma 3.** Consider two points \( A, B \), and a circle \( \Omega(O, r) \) (which in the particular case can be a straight line). Let the distance between \( A \) and \( \Omega(O, r) \) be denoted by \( \sigma \), and that between \( B \) and this circle by \( \rho \). Let also the distance between \( A \) and \( B \) be denoted by \( \xi \). Then:

\[
\sigma - \xi \leq \rho \leq \sigma + \xi
\]  

(10)

**Lemma 4.** If an MW-Voronoi region has more than one boundary curve, then the corresponding Minmax-curve circle is tangent to at least two of the boundary curves.

**Definition 7.** For any two curves \( e_{i1} \) and \( e_{i2} \), the sets \( \Psi_{e_{i1}, e_{i2}}^{\text{max}} \) and \( \Psi_{e_{i1}, e_{i2}}^{\text{min}} \) are defined as follows:

\[
\Psi_{e_{i1}, e_{i2}}^{\text{min}} = \{X \in \Gamma_{e_{i1}, e_{i2}} \mid \exists \delta > 0 \colon \forall Y \in \Gamma_{e_{i1}, e_{i2}}, |Y - X| \leq \delta \Rightarrow d(X, e_{i1}) \leq d(Y, e_{i1})\}
\]  

(11)

\[
\Psi_{e_{i1}, e_{i2}}^{\text{max}} = \{X \in \Gamma_{e_{i1}, e_{i2}} \mid \exists \delta > 0 \colon \forall Y \in \Gamma_{e_{i1}, e_{i2}}, |Y - X| \leq \delta \Rightarrow d(X, e_{i1}) \geq d(Y, e_{i1})\}
\]  

(12)

**Definition 8.** Let \( e_{i1} \) and \( e_{i2} \) be two arbitrary circular arcs of circles \( \Omega_1 \) and \( \Omega_2 \) respectively. The curves \( e_{i1} \) and \( e_{i2} \) are called parallel if the circles \( \Omega_1 \) and \( \Omega_2 \) are concentric.
Lemma 5. Consider an MW-Voronoi diagram, and assume that the i-th region has at least three boundary curves. Then the Minmax-curve circle of this region is tangent to at least two boundary curves. Furthermore, if the Minmax-curve circle is tangent to exactly two boundary curves, say $\ell_1$ and $\ell_2$, then at least one of the following conditions hold:

i) $\ell_1$ and $\ell_2$ are parallel;

ii) $\ell_i \in \Psi_{\ell_1,\ell_2}$, or

iii) $\ell_i$ is the intersection of the bisector of $\ell_1$, $\ell_2$ and one boundary curve of the region.

As noted earlier, the Minmax-curve circle is generically unique, and only in some special configurations there can be more than one such circle. The next lemma addresses the case where there are more than one Minmax-curve circle.

Lemma 6. If a Minmax-curve circle is tangent to two parallel curves, then generically there are more than one such circle, all of which are also tangent to these parallel curves.

Remark 1. Consider an MW-Voronoi region with at least three boundary curves, two of which are assumed to be parallel. If one of the Minmax-curve circles is tangent to these parallel curves, then all Minmax-curve circles are also tangent to these two curves. At least one of these circles is tangent to some other boundary curves too, and one of such circles is arbitrarily chosen as the Minmax-curve circle in this case.

Definition 9. For convenience of notation, the circle touching two curves $\ell_g$ and $\ell_h$ of the i-th MW-Voronoi region, centered at the intersection of the bisector of $\ell_g$ and $\ell_h$ and the curve $\ell_{ik}$, is denoted by $\Omega_{g,h}^k$, for any $k, g, h \in e_i := \{1, \ldots, l_i\}$. Also, the circle touching the two curves $\ell_{g,s}$ and $\ell_{s,h}$ of the i-th MW-Voronoi region, centered at the point $a \in \Psi_{\ell_{g,s},\ell_{s,h}}$, is denoted by $\Omega_{g,s}^h$, for any $r, s \in e_i$.

Theorem 2. Consider an MW-Voronoi diagram, and suppose the i-th region has at least three boundary curves. Let $D_i$ and $\hat{D}_i$ be the sets of all circles $\Omega_{g,h}^k, \forall k, g, h \in e_i$, and $\Omega_{g,s}^h, \forall r, s \in e_i, a \in \Psi_{\ell_{g,s},\ell_{s,h}}$ such that: (i) their centers lie inside the region or on its boundaries, and (ii) they intersect or are tangent to all of the boundary curves of the region (or their extensions, as noted before). Let also $\tilde{D}_i$ be the set of all circles such that: (i) they are tangent to at least three boundary curves of the i-th region; (ii) their centers lie inside the region or on its boundaries, and (iii) they intersect or are tangent to all of the boundary curves of the MW-Voronoi region. Define $D_i := \hat{D}_i \cup \tilde{D}_i$, then the Minmax-curve circle belongs to $D_i$, and is the smallest circle in this set.

Remark 2. If an MW-Voronoi region has exactly one boundary curve, then this curve is a circle and it is, in fact, the Minmax-curve circle. If, on the other hand, it has exactly two boundary curves, then according to Lemma 4 the Minmax-curve circle is tangent to both curves.

Using the result of Theorem 2 and discussions in Remarks 1 and 2, one can develop a procedure with a complexity of order $O(e_i^2)$ to calculate the Minmax-curve centroid in the i-th MW-Voronoi region. Since typically an MW-Voronoi region does not have "too many" boundary curves, the computational complexity for calculating the Minmax-curve centroid is normally not very high.

As an example, consider the initial deployment of Fig. 3, and let the Minmax-curve strategy be used. After the first round, the coverage is improved to 78.5%, and finally it reaches 90.8%, as depicted in Fig. 4.
C. Maxmin-Curve Strategy

The main idea behind this strategy is that normally for optimal coverage, each sensor should not be too close to any of its Voronoï curves. The Maxmin-curve strategy selects the target location for each sensor as a point inside the corresponding MW-Voronoi region which has the largest distance from the nearest curve. This point will be referred to as the Maxmin-curve centroid, and will be denoted by \( \tilde{O}_i \) for the \( i \)-th region, \( i \in \mathbf{n} \). Furthermore, the distance between this point and the nearest curve to it will be represented by \( \tilde{r}_i \). The Maxmin-curve circle is defined next.

**Definition 10.** The Maxmin-curve circle of an MW-Voronoi region is the largest circle inside that region. This circle is, in fact, \( \Omega(\tilde{O}_i, \tilde{r}_i) \), for the \( i \)-th region. Similar to the Minmax-curve circle, the Maxmin-curve circle is also generically unique, but in some special cases there can be multiple such circles.

**Lemma 7.** If an MW-Voronoi region has more than one boundary curve, then the corresponding Maxmin-curve circle is tangent to at least two of the curves.

**Lemma 8.** Consider an MW-Voronoi diagram, and suppose that the \( i \)-th region has at least three boundary curves. If a Maxmin-curve circle is tangent to not more than two boundary curves, say \( \varepsilon_1, \varepsilon_2 \), then these two curves are either parallel or \( \tilde{O}_i \in \Psi^{\max}_{\varepsilon_1, \varepsilon_2} \).

**Definition 11.** For convenience of notation, the circle tangent to two curves \( \varepsilon_r \) and \( \varepsilon_s \) of the \( i \)-th MW-Voronoi region, centered at the point \( A \in \Psi^{\max}_{\varepsilon_r, \varepsilon_s} \), is denoted by \( \Omega^{\max}_{\varepsilon_r, \varepsilon_s} \) for any \( r, s \in \varepsilon_i \).

**Lemma 9.** If a Maxmin-curve circle is tangent to two parallel curves, then generically there are more than one such circle, all of which are tangent to these parallel curves.

**Remark 3.** Consider an MW-Voronoi region with at least three boundary curves, two of which are parallel. If one of the Maxmin-curve circles is tangent to these parallel curves, then all Maxmin-curve circles are also tangent to these two curves. At least one of these circles is tangent to some other boundary curves too, and one of such circles is arbitrarily chosen as the Maxmin-curve circle in this case.

**Theorem 3.** Consider an MW-Voronoi diagram, and suppose that the \( i \)-th MW-Voronoi region has at least three boundary curves. Let \( \tilde{Z}_i \) be the set of all circles \( \Omega^{\max}_{\varepsilon_r, \varepsilon_s} \), \( \forall r, s \in \varepsilon_i \), \( A \in \Psi^{\max}_{\varepsilon_r, \varepsilon_s} \), that are inside the region. Let also \( \tilde{Z}_n \) be the set of all circles which: (i) are tangent to at least three curves of the \( i \)-th region, and (ii) are inside the region. Define \( \tilde{Z}_i := \tilde{Z}_i \cup \tilde{Z}_n \); then the Maxmin-curve circle belongs to \( \tilde{Z}_i \), and also it is the largest circle in this set.

**Remark 4.** If an MW-Voronoi region has exactly one boundary curve, then this curve is a circle as pointed out before, and it is, in fact, the Maxmin-curve circle.

Using the result of Theorem 3 and discussion in Remarks 3 and 4, one can develop a procedure with a complexity of order \( O(e^2) \) (which is typically not very high) to calculate the Maxmin-curve centroid for the \( i \)-th MW-Voronoi region. Given a group of sensors with the initial deployment of Fig. 3, let the Maxmin-curve strategy be used. It can be verified that after the first round the coverage increases to 84.9%, and eventually reaches 95.1%. This is depicted in Fig. 5, where it can be observed that, after the final round the sensors are distributed more evenly than the initial deployment, and that the coverage increases considerably.

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**Fig. 5.** Snapshots of the execution of the Maxmin-curve strategy. (a) Initial coverage; (b) coverage after the first round, and (c) final coverage.
Remark 5. In some special cases, even if the sensor moves toward its target location (according to any of the algorithms introduced above), the local coverage might not be improved. This happens when the target location of a sensor is too far from its present location. To overcome this problem, one can adopt a technique similar to the one provided in [11], and choose, for instance, the midpoint between the current location and the calculated target location for the sensor to move to. If the local coverage of the sensor is better from this point (compared to the calculated target point and the present location of the sensor), it stops there.

IV. Simulation Results

In this section, the three strategies proposed in Section III are applied to a flat space of size 50m × 50m. In these simulations, the algorithms are terminated when none of the sensors’ coverage area in its corresponding MW-Voronoi region would be increased by more than 1% in the next move. It is to be noted that the results presented in this section for network coverage are all the average values obtained by using 20 random initial deployments for the sensors. Furthermore, while the horizontal axis in all figures in this section represents a discrete parameter, the graphs are represented as continuous curves for clarity.

Assume first there are 36 sensors: 20 with a sensing radius of 6m, 8 with a sensing radius of 5m, 4 with a sensing radius of 7m, and 4 with a sensing radius of 9m. Moreover, the communication range of each sensor is assumed to be 10/3 times greater than its sensing range. The coverage factor of the sensor network (defined as the ratio of the covered area to the overall area) in each round is depicted in Fig. 6 for the methods proposed in this report. It can be seen from this figure that all three strategies result in satisfactory coverage in the first few rounds. It can also be observed that for this example, the Minmax-curve strategy performs better than the other two strategies as far as coverage is concerned.

Fig. 6. Network coverage per round for 36 sensors.

It is desired now to compare the performance of the proposed algorithms in terms of the number of deployed sensors $n$. To this end, consider three more set-ups: $n=18, 27$ and 45, in addition to the one discussed above. Let the changes in the number of identical sensors in the new setups be proportional to the changes in the total number of sensors (e.g., for the case of $n=27$ there will be 15 sensors with a sensing radius of 6m, 6 with a sensing radius of 5m, 3 with a sensing radius of 7m, and 3 with a sensing radius of 9m). Fig. 7 provides the coverage results for different number of sensors. It shows that although the WVB algorithm provides satisfactory coverage when there are a relatively small number of sensors, it is outperformed by other algorithms as the number of sensors increases.

Another important factor in the performance evaluation of different algorithms is how fast the desired coverage level is achieved. Notice that the sensor deployment time in each round is almost equal for all algorithms. Hence, to compare the deployment speed, it suffices to check the number of rounds it takes for the sensors to provide a prescribed coverage level. It is shown in Fig. 8 that in all three algorithms the number of rounds (required to meet a certain termination condition) increases by increasing the number of sensors up to a certain value, and then starts to decrease by adding more sensors. This is due mainly to the fact that when there are a small number of sensors in the network, the MW-Voronoi regions are large in comparison with the corresponding sensing circles. Hence, there is a good chance that each sensor’s local coverage area is inside its MW-Voronoi region, which in turn means that the sensor does not need to move in order to increase its coverage area. On the other hand, when there are a large number of sensors in the target field, there is a good chance that each sensor covers its MW-Voronoi region, which implies that the termination condition will be satisfied in a short period of time. It is also to be noted that in the WVB strategy the number of rounds required for the termination of the algorithm is larger than the other strategies. The number of rounds in the Minmax-curve algorithm is relatively low, making it a good candidate as far as the deployment time is concerned.

Another important means of assessing the performance of sensor deployment algorithms is the energy consumption of the sensors. The energy consumption of a mobile sensor highly depends on its traveling distance, as well as the number of times it stops before arriving at the destination (the latter is due to static friction). Thus, to compare the proposed methods
in terms of energy consumption, the traveling distance and number of movements of every sensor in the network should be taken into consideration. Fig. 9 depicts the average moving distance for different number of sensors. This figure shows that by increasing the number of sensors the average moving distance of the sensors is decreased in all scenarios. This can be justified for each algorithm as follows. In the WVB strategy, when the number of sensors increases, the distance between each sensor and its final position decreases, resulting in a decrease of average moving distance. In other algorithms, on the other hand, when the number of sensors increases, the MW-Voronoi regions become smaller. As a result, the distance between each sensor and its destination point in the corresponding MW-Voronoi region decreases, which in turn leads to a decrease in the average moving distance. It can be concluded from Fig. 9 that the average moving distance of all three algorithms are more or less the same when there are a large number of sensors in the network. The number of movements versus the number of sensors is illustrated in Fig. 10. It can be observed from this figure that when the number of sensors is more than a certain level (whose value is different for each algorithm), the number of movements decreases. This is due to the fact that for large number of sensors the MW-Voronoi regions become smaller, which helps the sensors cover their MW-Voronoi regions (as noted earlier). As a result, the coverage holes will be covered in a shorter period of time, decreasing the number of movements.
V. CONCLUSION

This report presents efficient sensor deployment algorithms to increase coverage in mobile sensor networks. It is assumed that the sensing radii of different sensors are not the same. A multiplicatively weighted Voronoi (MW-Voronoi) diagram is employed to develop three distributed deployment strategies, namely weighted vector boundary (WVB), Minmax-curve and Maxmin-curve. These strategies are based on some known facts about the general characteristics of an ideal configuration of sensors (e.g., no sensor should be too far or too close to any of the boundary curves of its corresponding MW-Voronoi region). Under these strategies, the sensors move iteratively to minimize coverage holes in the sensing field. Simulation results are presented to compare the proposed approaches for different number of sensors in the sensing field.

REFERENCES


