Decentralized Control Design for Interconnected Systems Based on
A Centralized Reference Controller

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Abstract—This paper deals with the decentralized control of interconnected systems, where each subsystem has models of all other subsystems (subject to uncertainty). A decentralized controller is constructed based on a reference centralized controller. It is shown that when a priori knowledge of each subsystem about the other subsystems’ models is exact, then the decentralized closed-loop system can perform exactly the same as its centralized counterpart. An easy-to-check necessary and sufficient condition for the internal stability of the decentralized closed-loop system is obtained. Moreover, the stability of the closed-loop system in presence of the perturbation in the parameters of the system is investigated, and it is shown that the decentralized control system is likely more robust than its centralized counterpart. A proper cost function is then defined to measure the closeness of the decentralized closed-loop system to its centralized counterpart. This enables the designer to obtain the maximum allowable standard deviation for the modeling errors of the subsystems to achieve a satisfactorily small performance deviation with a sufficiently high probability. The effectiveness of the proposed method is demonstrated in one numerical example.

I. INTRODUCTION

In control of large-scale interconnected systems, it is often desired to have some form of decentralization. Typical interconnected control systems have several local control stations, which observe only local outputs and control only local inputs, according to the prescribed restrictions in the information flow structure. All the controllers are involved, however, in the overall control operation.

In the past several years, the problem of decentralized control design has been investigated extensively in the literature [1-9]. These works have studied decentralized control problem from two different viewpoints as follows:

1. The local input and output information of any subsystem is private, and is not accessible by other subsystems [3], [5]. In this case, each local controller should attempt to attenuate the degrading effect of the incoming interconnections on its corresponding subsystem, in addition to contributing to the performance of the overall control system.

2. All output measurements cannot be transmitted to every local control station. Problems of this kind appear, for example, in electric power systems, communication networks, flight formation, robotic systems, to name only a few. In this case, each subsystem can have certain beliefs about other subsystems’ models. A local controller is then designed for each subsystem, based on this a priori knowledge.

More recently, the problem of optimal decentralized control design has been studied to obtain a high-performance control law with some prescribed constraints for the system. The main objective in this problem is to find a decentralized feedback law for an interconnected system in order to attain a sufficiently small performance index. This problem has been investigated in the literature from the two different viewpoints discussed above [3], [6], [7], [8], [9]. However, there is no efficient approach currently to address the problem from the second viewpoint. The relevant works often attempt to present a near-optimal decentralized controller instead of an optimal one. Furthermore, it is often assumed that the decentralized controller to be designed is static [7]. This assumption can significantly degrade the performance of the overall system. In other words, the overall performance of the system can be improved considerably, if a dynamic feedback law is used instead of a static one.

This paper deals with the decentralized control problem from the second viewpoint discussed above. Hence, it is assumed that each subsystem of the interconnected system has some beliefs about the parameters of the other subsystems as well as their initial states. A decentralized controller is then constructed based on a given reference centralized controller which satisfies the design specifications. This decentralized control law relies on the expected values of any subsystem’s initial state from any other subsystem’s view and the beliefs of each subsystem about the other subsystems’ parameters.

Some important issues regarding the proposed decentralized control law are also studied in this work. First, an easy-to-check necessary and sufficient condition for the internal stability of the interconnected system under the proposed decentralized control law is presented. Note that although the expected values of the initial states are part of the local control functions, it is shown that they never affect the internal stability of the overall system. Second, it is shown that if the knowledge of any subsystem about the other subsystems’ parameters is accurate, then the decentralized closed-loop system can perform exactly the same as the centralized closed-loop system. However, since the exact knowledge of the subsystem’s parameters is very unlikely.

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to be available in practice, a performance index is defined to evaluate the closeness of the proposed decentralized closed-loop system to its centralized counterpart. In addition, the stability of the decentralized closed-loop system in presence of perturbation in the parameters of the system is studied. Finally, a near-optimal decentralized control law is introduced, and its properties are investigated.

This paper is organized as follows. Given a centralized controller with desired properties, a decentralized control law with similar performance is derived from it in Section II. The robustness of the proposed controller is studied and compared to its centralized counterpart in Section III. Performance evaluation is then presented in Section IV. An illustrative example is given in Section V and finally some concluding remarks are drawn in Section VI.

II. MODEL-BASED DECENTRALIZED CONTROL

Consider a stabilizable LTI system \( S \) consisting of \( \nu \) subsystems \( S_1, S_2, ..., S_\nu \). Suppose that the state-space equation for the system \( S \) is as follows:

\[
\dot{x}(t) = Ax(t) + Bu(t)
\]

\[
y(t) = Cx(t)
\]

where \( x \in \mathbb{R}^n \), \( u \in \mathbb{R}^m \), \( y \in \mathbb{R}^r \) are the state, the input, and the output of the system \( S \), respectively.

Let:

\[
u(t) := \begin{bmatrix} u_1(t)^T & u_2(t)^T & \ldots & u_\nu(t)^T \end{bmatrix}^T, \quad (2a)
\]

\[
x(t) := \begin{bmatrix} x_1(t)^T & x_2(t)^T & \ldots & x_\nu(t)^T \end{bmatrix}^T, \quad (2b)
\]

\[
y(t) := \begin{bmatrix} y_1(t)^T & y_2(t)^T & \ldots & y_\nu(t)^T \end{bmatrix}^T \quad (2c)
\]

where \( x_i \in \mathbb{R}^{n_i}, u_i \in \mathbb{R}^{m_i}, y_i \in \mathbb{R}^{r_i}, i \in \bar{\nu} := \{1, 2, ..., \nu\} \), are the state, the input, and the output of the \( i \)-th subsystem \( S_i \), respectively. Let for any \( i, j \in \bar{\nu} \), the \((i, j)\) block entry of the matrix \( A \) be denoted by \( A_{ij} \in \mathbb{R}^{n_i \times n_j} \). Assume that the matrices \( B \) and \( C \) are block diagonal with the \((i, i)\) block entries \( B_i \in \mathbb{R}^{n_i \times m_i} \) and \( C_i \in \mathbb{R}^{r_i \times n_i} \), respectively, for any \( i \in \bar{\nu} \). Assume also that a centralized LTI controller \( K_c \) is designed for the system \( S \), which satisfies the desired design specifications, and denote its state-space equation as follows:

\[
\dot{z}(t) = Az(t) + Bz(t) + C x(t)
\]

\[
u(t) = Cz(t) + Dz(t)
\]

where \( z \in \mathbb{R}^q \) is the state of the controller. It is now aimed to design a decentralized controller for the system \( S \) so that it performs approximately (and under certain conditions, exactly) the same as the centralized controller \( K_c \). The following definitions are used to obtain the desired decentralized control law.

**Definition 1:** Define \( x^{ij}_0 \), \( i, j \in \bar{\nu}, i \neq j \), as the expected value of the initial state of the subsystem \( S_i \) from the viewpoint of the subsystem \( S_j \).

**Definition 2:** For any \( i, j \in \bar{\nu}, i \neq j \), the modified subsystem \( S^j_i \) is defined to be a system obtained from the subsystem \( S_i \) by replacing the parameters \( A_{ij}, B_i, C_i \) of its state equations with \( A^j_{il}, B^j_i, C^j_i \), respectively, for any \( l \in \bar{\nu} \). In addition, the initial state of \( S^j_i \) is \( x^{ij}_0 \) (instead of \( x_i(0) \)).

Denote the state, the input, and the output of the modified subsystem \( S^j_i \) with \( x^{ij}_1, u^{ij}_1 \) and \( y^{ij}_1 \), respectively.

Note that \( S^j_i \) represents the belief of subsystem \( j \) about the model of subsystem \( i \). In other words, it is desired to have \( S^j_i = S_i \), for all \( j \in \bar{\nu}, j \neq i \).

**Definition 3:** For any \( i \in \bar{\nu}, S^i \) is defined to be a system obtained from \( S \), after replacing the subsystem \( S_j \) with the corresponding modified subsystem \( S^j_i \), for any \( j = 1, 2, ..., i - 1, i + 1, ..., \nu \).

It is be noted that \( S^i \) represents the belief of subsystem \( i \) about the model of the system \( S \).

**Definition 4:** Consider the system \( S^i \) under the centralized feedback law (3). Remove all interconnections going to the \( i \)-th subsystem \( S_i \) from the other subsystems to obtain a new closed-loop system. One can decompose this closed-loop system to two portions:

i) the system \( S_i \);

ii) all other interconnected components (consisting of \( S^j_i, j = 1, 2, ..., i - 1, i + 1, ..., \nu \), and \( K_c \), and the corresponding interconnections). Define this interconnected system as augmented local controller for the subsystem \( S_i \), and denote it with \( K_{di} \). Denote the state of the controller \( K_c \) (inside the controller \( K_{di} \)) with \( z^i \).

Define the controller consisting of all augmented local controllers \( K_{di}, K_{d2}, ..., K_{d\nu} \) as the augmented decentralized controller \( K_d \). Let this decentralized controller be applied to the interconnected system \( S \) (i.e., \( K_{di} \) is applied to \( S_i \), for all \( i \in \bar{\nu} \)). Note that to obtain the augmented local controller \( K_{di} \), in addition to the output \( y_i \), all of the interconnections coming out of the subsystem \( S_i \) are also assumed to be available for the local controller \( K_{di} \). Since these interconnections contain, in fact, the information of the subsystem \( S_i \), this is a feasible assumption in many practical applications such as flight formation and power systems, and does not violate the decentralized information flow constraint for the control law. The following theorem presents one of the main properties of the proposed decentralized controller.

**Theorem 1:** Consider the interconnected system \( S \) with the augmented decentralized controller \( K_d \). If \( A^j_{il} = A_{il}, B^j_i = B_i, C^j_i = C_i \), and \( x^{ij}_0 = x_i(0) \) for all \( i,j,l \in \bar{\nu}, i \neq j \), then the input, the output, and the state of the overall decentralized closed-loop system will be the same as those of the system \( S \) under the centralized controller \( K_c \).

**Proof:** Consider the system \( S \) under the decentralized controller \( K_d \). One can easily write the following equations for the state of the subsystem \( S_i \) under its augmented local controller \( K_{di} \):

\[
\dot{x}^i(t) = \tilde{A}^i x^i(t) + B^i u^i(t) + \sum_{j \in \bar{\nu}, j \neq i} \tilde{A}^{ij} x^j(t)
\]

\[
y^i(t) = C^i x^i(t)
\]

\[
\dot{z}^i(t) = A_k z^i(t) + B_k y^i(t)
\]

\[
u^i(t) = C_k z^i(t) + D_k y^i(t)
\]
where:

\[
x^i = \begin{bmatrix} x_1^T & \ldots & x_{i-1}^T & x_i & x_{i+1}^T & \ldots & x_{\nu}^T \end{bmatrix}^T
\]

\[
y^i = \begin{bmatrix} y_1^T & \ldots & y_{i-1}^T & y_i & y_{i+1}^T & \ldots & y_{\nu}^T \end{bmatrix}^T
\]

\[
u^i = \begin{bmatrix} u_1^T & \ldots & u_{i-1}^T & u_i & u_{i+1}^T & \ldots & u_{\nu}^T \end{bmatrix}^T
\]

and where:

1. \( \tilde{A}_i \) is obtained from \( A \) by replacing all of the entries of its \( i \)th row block except \( A_{ii} \) (i.e. \( A_{ij}, j \in \nu, i \neq j \)) with zeros, and replacing its entry \( A_{ij} \) with \( A_{ij} \) for any \( l, j \in \nu, l \neq i \).
2. \( A^{ij} \) (\( i \neq j \)) is obtained from \( A \) by setting all of its block entries except \( A_{ij} \), to zero.
3. \( B^i \) and \( C^i \) are obtained from \( B \) and \( C \), after replacing their entries \( B_l \) and \( C_l \) with \( B^i_l \) and \( C^i_l \), respectively, for \( l = 1, 2, \ldots, i-1, i+1, \ldots, \nu \).

The equation (4) can be rewritten in a matrix form as follows:

\[
\begin{bmatrix}
\dot{x}_i^i(t) \\
\dot{z}_i^i(t)
\end{bmatrix} = \begin{bmatrix}
\tilde{A}_i + B^iD_kC^i \\
B^iC^i
\end{bmatrix} \begin{bmatrix}
x_i^i(t) \\
z_i^i(t)
\end{bmatrix}
+ \sum_{j \in \nu, j \neq i} \begin{bmatrix}
\tilde{A}_{ij} \\
0_{n \times n}
\end{bmatrix} \begin{bmatrix}
x_j^i(t) \\
z_j^i(t)
\end{bmatrix}
\]

(6)

So far, the states of the subsystem \( S_i \) and its corresponding augmented local controller \( K_d \) are obtained. In order to simplify (6), the following vector and matrices are defined:

\[
x_g^i(t) := \begin{bmatrix} x_i^i(t) \\
z_i^i(t)
\end{bmatrix}, \quad \tilde{A}_g^i := \begin{bmatrix}
\tilde{A}_i + B^iD_kC^i \\
B^iC^i
\end{bmatrix}, \quad \tilde{A}_{ij} := \begin{bmatrix}
\tilde{A}_{ij} \\
0_{n \times n}
\end{bmatrix}, \quad i, j \in \nu, i \neq j
\]

(7)

Note that the subscript "g" denotes the parameters and variables of the augmented equations. According to (6) and (7), the state of the system \( S \) under the proposed decentralized controller \( K_d \) satisfies the following equation:

\[
\dot{x}_g(t) = A_g^d x_g(t)
\]

(8)

where:

\[
x_g(t) := \begin{bmatrix}
x_g^1(t) \\
x_g^2(t) \\
\vdots \\
x_g^\nu(t)
\end{bmatrix}, \quad A_g^d := \begin{bmatrix}
A_1^d & \tilde{A}_1^{12} & \ldots & \tilde{A}_1^{1\nu} \\
A_2^d & A_2^d & \ldots & \tilde{A}_2^{2\nu} \\
\vdots & \vdots & \ddots & \vdots \\
A_\nu^d & A_\nu^d & \ldots & A_\nu^d
\end{bmatrix}
\]

(9)

and where the superscript "d" represents the decentralized nature of the control system. On the other hand, one can easily verify that the state of the system \( S \) under the centralized controller \( K_c \) satisfies the following:

\[
\dot{x}_g(t) = A_g^c x_g(t)
\]

(10)

where:

\[
A_g^c := \begin{bmatrix}
A + BD_kC & BC_k \\
B_kC & A_k
\end{bmatrix}
\]

(11)

Since it is assumed that \( A_{il}^l = A_{ii}, B_l^l = B_l, \) and \( C_l^l = C_l \) for all \( i, j, l \in \nu, i \neq j \), one can write:

\[
\tilde{A}^i_g = A_g^c - \sum_{j=1, j \neq i}^\nu \tilde{A}_{ij}^g
\]

(12)

According to (9) and (12), \( A_g^c \) can be rewritten as the sum of two components \( \Theta \) and \( \Gamma \), where

\[
\Theta := \begin{bmatrix}
A_1^g & 0 & \cdots & 0 \\
0 & A_2^g & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & A_\nu^g
\end{bmatrix}
\]

\[
\Gamma := \begin{bmatrix}
-\sum_{j=1, j \neq 1}^\nu \tilde{A}_{1j}^g & -\sum_{j=1, j \neq 2}^\nu \tilde{A}_{2j}^g & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
-\sum_{j=1, j \neq \nu}^\nu \tilde{A}_{\nu j}^g
\end{bmatrix}
\]

(13)

Note that the matrix \( \Theta \) consists of \( \nu \) block matrices \( A_g^c \) on its main diagonal and that each of the diagonal block entries of \( \Gamma \) is equal to the negated sum of the other block entries of its own block row. Consider now the equation (8) in the Laplace domain:

\[
X_g(s) = (sI - A_g^d)^{-1}x_g(0) = ((sI - \Theta) - \Gamma)^{-1}x_g(0)
\]

(14)

It is known that for any arbitrary square matrices \( \Omega_1 \) and \( \Omega_2 \):

\[
(\Omega_1 + \Omega_2)^{-1} = \Omega_1^{-1} - \Omega_1^{-1}(I + \Omega_2\Omega_1^{-1})^{-1}\Omega_2\Omega_1^{-1}
\]

(15)

provided \( \Omega_1 \) and \( (I + \Omega_2\Omega_1^{-1}) \) are nonsingular. It can be concluded from (14) and (15) that:

\[
X_g(s) = (sI - \Theta)^{-1}x_g(0) + (sI - \Theta)^{-1}x_g(0)
\]

\[
\times (I - \Gamma(sI - \Theta)^{-1})^{-1}((sI - \Theta)^{-1}x_g(0)
\]

(16)

Moreover, the assumption \( x_{il}^j = x_i(0), \ i, j \in \nu, i \neq j \) yields that \( x^i = x(0), \ i \in \nu \). Therefore:

\[
x_g^i(0) = \begin{bmatrix} x(0)^T \\
z(0)^T
\end{bmatrix}^T, \ i \in \nu
\]

(17)

Hence, \( x_{il}^j(0) = x_{il}^j(0), \ i, j \in \nu \). From the equation (17) and the definitions of \( \Theta \) and \( \Gamma \) given by (13), the \( i \)th entry of the vector \((sI - \Theta)^{-1}x_g(0)\) can be obtained as follows:

\[
\tilde{A}_{i1}^g (sI - A_g^c)^{-1}x_g^i(0) + \ldots + \tilde{A}_{i,\nu}^g (sI - A_g^c)^{-1}x_g^\nu(0) + \tilde{A}_{i,i}^g = 0
\]

(18)
This means that $\Gamma(sI - \Theta)^{-1}x_g(0) = 0$. Hence, it can be concluded from (16) that $X_g(s) = (sI - \Theta)^{-1}x_g(0)$. Consequently, from (17):

$$X_g^i(s) = (sI - A_g^i)^{-1}x_g^j(0) = (sI - A_g^0)^{-1}\begin{bmatrix} x(0) \\ z(0) \end{bmatrix}$$  \tag{19}

On the other hand, it follows from (10) that the state of the system $S$ under the centralized controller $K_c$ satisfies the following equation in the Laplace domain:

$$X(s) = (sI - A_g)^{-1}\begin{bmatrix} x(0) \\ z(0) \end{bmatrix}$$  \tag{20}

From (5) and (7), one can observe that the state of the subsystem $S_i$ of the system $S$ under the decentralized controller $K_d$ is the $i$th block entry of $x_g^j(t)$ which is, according to (19), the $i$th block entry of $(sI - A_g)^{-1}\begin{bmatrix} x(0) \\ z(0) \end{bmatrix}$ in the Laplace domain. On the other hand, equation (20) expresses that the state of the subsystem $S_i$ of the system $S$ under the centralized controller $K_c$ is also the $i$th block entry of the same matrix. This means that the state of the system $S$ under the decentralized and the centralized control schemes given above are equivalent. Using this result, it is straightforward now to show that the output and the input of these two closed-loop systems are the same.

Theorem 1 states that under certain conditions, the state, the input, and the output of the system $S$ under the centralized controller $K_c$ are equal to those of the system $S$ under the decentralized controller $K_d$. These conditions are met when the belief of each subsystem about the model and the initial state of any other subsystem is precise. Since this assumption never holds in practice, it is important to obtain stability conditions for the case when the corresponding beliefs are inaccurate. This issue is addressed in the following Corollary.

**Corollary 1**: The interconnected system $S$ under the proposed decentralized controller $K_d$ is internally stable, if and only if all the eigenvalues of the matrix $A_d^i$ given in (9) are located in the open left-half $s$-plane.

**Proof**: The proof follows immediately from the fact that the modes of the system $S$ under the decentralized controller $K_d$ are the eigenvalues of the matrix $A_d^i$, according to the equation (8).

**Remark 1**: Note that stability of the decentralized closed-loop system is verified by simply checking the location of the eigenvalues of $A_d^i$. This means that the stability of the resultant system is independent of the values $x_0^i$, $i,j \in \nu$, $i \neq j$.

**Remark 2**: The degree of the local controller $K_d$, for the $i$th subsystem of $S$ is $n+\mu$ minus the degree of the subsystem $S_i$, for any $i \in \nu$. Moreover, it can be concluded from Theorem 1, that the local controller $K_d$ implicitly includes an observer to estimate the states of the other subsystems. In contrast, if an explicit decentralized observer for each subsystem of $S$ is to be designed by the existing methods, then the degrees of the observers $1, 2, \ldots, \nu$ (for subsystems $1, 2, \ldots, \nu$) will be $n, 2n, \ldots, \nu n$, respectively. Note that in a practical setup with explicit observers, each local controller consists of a compensator and an observer. This means that the local controllers proposed here include implicit observers, whose degrees are much less than those of the conventional decentralized observers.

### III. Robustness Analysis

In the previous section, a method was proposed for designing a decentralized controller based on a reference centralized controller and its stability condition was discussed in detail. Suppose that there are some uncertainties in the parameters of the system $S$ given by (1). Since the decentralized controller $K_d$ is designed in terms of the nominal parameters of the system, the resultant decentralized closed-loop system may become unstable. One of the main objectives of this section is to find a necessary and sufficient condition for internal stability of the perturbed decentralized closed-loop system.

**Definition 5**: For any arbitrary matrix $M$, denote its perturbed version with $\tilde{M}$, and define its perturbation matrix as $\Delta M := \tilde{M} - M$.

Consider now the system $\tilde{S}$ as the perturbed version of the system $S$ whose state-space representation is as follows:

$$\begin{align*}
\dot{x}(t) &= \tilde{A}x(t) + \tilde{B}u(t) \\
y(t) &= Cx(t)
\end{align*}$$  \tag{21}

Let for any $i, j \in \nu$, the $(i, j)$ block entry of the matrix $\tilde{A}$ be denoted by $\tilde{A}_{ij}$. Assume that the matrices $\tilde{B}$ and $C$ are block diagonal with the block entries $B_1, B_2, \ldots, B_\nu$ and $C_1, C_2, \ldots, C_\nu$, respectively. As discussed earlier, to construct the decentralized controller $K_d$, it is assumed that in addition to the output of any subsystem $S_i$ all of the interconnections going out of subsystem $i$ are also available for the local controller $K_{di}$. Hence, it is required to make some assumptions on the interconnection signals. Consider again the unperturbed system $S$. Denote the interconnection signal coming out of subsystem $i$ and going into subsystem $j$ with $\zeta_{ji}(t)$. Since $\zeta_{ji}(t)$ can be considered as an output for subsystem $i$, there is a matrix $\Pi_{ji}$ such that $\zeta_{ji}(t) = \Pi_{ji}x_i(t)$. Similarly, since $\zeta_{ij}(t)$ can be considered as an input for subsystem $j$, there is a matrix $\Gamma_{ji}$ such that $A_{ji}x_j(t) = \Gamma_{ji}\zeta_{ij}(t)$. As a result, $A_{ji} = \Gamma_{ji}\Pi_{ji}$. Denote now the perturbed matrices corresponding to $\Pi_{ji}$ and $\Gamma_{ji}$ with $\tilde{\Pi}_{ji}$ and $\Gamma_{ji}$, respectively. Hence, $\tilde{A}_{ji} = \Gamma_{ji}\tilde{\Pi}_{ji}$.

**Definition 6**:

1. Define $\tilde{B}^i$ and $\tilde{C}^i, i \in \nu$, as the matrices obtained from $B^i$ and $C^i$ by replacing their block entries $B_i$ and $C_i$ with $\tilde{B}_i$ and $\tilde{C}_i$, respectively.
2. Define $\tilde{A}_{ij}^i, i,j \in \nu, i \neq j$, as the perturbed matrix of $A_{ij}^i$, derived by replacing the block entry $A_{ij}$ of $A_{ij}^i$ with $\tilde{A}_{ij}$.
3. Define $\tilde{A}^i, i \in \nu$, as the matrix derived from $\tilde{A}$ as follows:
   - Replace the entry $A_{ii}$ with $\tilde{A}_{ii}$.
   - For all $j \in \nu, j \neq i$, replace the block entry $A_{ji}^i$ with $\Gamma_{ji}\tilde{\Pi}_{ji}$.
Theorem 2: Suppose that the decentralized controller $K_d$ which is designed based on the nominal parameters of the system $S$ as well as the centralized controller $K_c$, is applied to the perturbed system $\bar{S}$. The resultant decentralized closed-loop control system is internally stable if and only if all of the eigenvalues of the matrix $A_{d}^{\nu}$ are located in the open left-half of the complex plane, where:

$$A_{d}^{\nu} := \begin{bmatrix}
A_{d1}^{\nu} & A_{d2}^{\nu} & \ldots & A_{d\nu}^{\nu} \\
A_{g1} & A_{g2} & \ldots & A_{g\nu} \\
\vdots & \vdots & \ddots & \vdots \\
A_{g1} & A_{g2} & \ldots & A_{g\nu}
\end{bmatrix}$$

and:

$$\bar{A}_{i}^{j} := \begin{bmatrix}
\hat{A}_{i}^{j} + \hat{B}_{i}^{j}D_{k}\hat{C}_{i} & \hat{B}_{i}^{j}C_{k} \\
B_{k}\hat{C}_{i} & A_{k}
\end{bmatrix}, \quad i \in \bar{\nu}$$

Proof: The proof of this theorem is omitted due to its similarity to the proof of Corollary 1.

Moreover, it can be easily verified that the modes of the perturbed system $\bar{S}$ under the centralized controller $K_c$ are the eigenvalues of the matrix $A_{c}^{\nu}$, where:

$$A_{c}^{\nu} := \begin{bmatrix}
\hat{A} + \hat{B}_{i}D_{k}\hat{C}_{i} & \hat{B}_{i}C_{k} \\
B_{k}\hat{C}_{i} & A_{k}
\end{bmatrix}$$

Therefore, robustness analysis with respect to the perturbation in the parameters of the system can be summarized as follows:

- For decentralized case, the location of the eigenvalues of the matrix $A_{d}^{\nu}$ should be checked.
- For centralized case, the location of the eigenvalues of the matrix $A_{c}^{\nu}$ should be checked.

The equalities $A_{ij}^{l} = A_{ij}$, $B_{ij}^{l} = B_{ij}$, $C_{ij}^{l} = C_{ij}$, $i,j,l \in \bar{\nu}$, $i \neq l$ will hereafter be assumed to simplify the presentation of the properties of the decentralized controller proposed in this paper. It is to be noted that most of the results obtained under the above assumption can simply be extended to the general case.

Theorem 3: The Frobenius norms of the perturbation matrices for the decentralized and the centralized cases satisfy the following inequalities:

$$\|\Delta(A_{d}^{\nu})\| \leq N_{1}$$

$$\|\Delta(A_{c}^{\nu})\| \leq N_{2}$$

where:

$$(N_{1})^{2} = 8 \sum_{i=1}^{\nu} \|\Delta B_{i}\|^{2} \|D_{k}\|^{2} \|\Delta C_{i}\|^{2} + 8 \sum_{i=1}^{\nu} \|\Delta A_{ii}\|^{2} + 8 \sum_{i,j \in \bar{\nu}, i \neq j} \|\Delta B_{ij}\| \|\Delta C_{i}\|^{2} + 8 \sum_{i,j \in \bar{\nu}, i \neq j} \|\Delta B_{ij}\| \|\Delta C_{j}\|^{2} + 8 \sum_{i,j \in \bar{\nu}, i \neq j} \|\Delta B_{ij}\| \|\Delta C_{i}\|^{2} + 8 \sum_{i,j \in \bar{\nu}, i \neq j} \|\Delta B_{ij}\| \|\Delta C_{j}\|^{2} + 8 \sum_{i,j \in \bar{\nu}, i \neq j} \|\Delta B_{ij}\| \|\Delta C_{i}\|^{2} + 8 \sum_{i,j \in \bar{\nu}, i \neq j} \|\Delta B_{ij}\| \|\Delta C_{j}\|^{2}$$

and:

$$(N_{2})^{2} = 32 \sum_{i,j \in \bar{\nu}, i \neq j} \|\Delta B_{ij}\| \|\Delta C_{i}\|^{2} + \|\Delta B\|^{2} \|C_{k}\|^{2} + 8 \sum_{i,j \in \bar{\nu}, i \neq j} \|\Delta B_{ij}\| \|\Delta C_{j}\|^{2} + 8 \sum_{i,j \in \bar{\nu}, i \neq j} \|\Delta B_{ij}\| \|\Delta C_{i}\|^{2} + 8 \sum_{i,j \in \bar{\nu}, i \neq j} \|\Delta B_{ij}\| \|\Delta C_{j}\|^{2} + 8 \sum_{i,j \in \bar{\nu}, i \neq j} \|\Delta B_{ij}\| \|\Delta C_{i}\|^{2} + 8 \sum_{i,j \in \bar{\nu}, i \neq j} \|\Delta B_{ij}\| \|\Delta C_{j}\|^{2}$$

and where $\| \cdot \|$ represents the Frobenius norm.

Proof: The proof is omitted due to space restrictions.

The robustness analysis for the decentralized case can be described as follows. Consider a Hurwitz matrix $A_{d}^{\nu}$ and assume that it is desired to find admissible variations for the independent perturbation matrices $\Delta B_{ij}$, $\Delta C_{i}$, $\Delta\Pi_{ij}$, $\Delta\Gamma_{ij}$, $\Delta A_{ii}$, $i,j \in \bar{\nu}$, $i \neq j$ such that the perturbed matrix $A_{d}^{\nu}$ remains Hurwitz. Analogously, for the centralized case one should check if $A_{c}^{\nu}$, the perturbed version of the Hurwitz matrix $A_{c}^{\nu}$, is also Hurwitz. This is an algebraic problem which has been addressed in the literature using different approaches [11, 12].

To obtain some admissible variations for the independent perturbation matrices $\Delta B_{ij}$, $\Delta C_{i}$, $\Delta\Pi_{ij}$, $\Delta\Gamma_{ij}$, $\Delta A_{ii}$, $i,j \in \bar{\nu}$, $i \neq j$, one can choose any existing result for the matrix perturbation problem, e.g., Bauer-Fike Theorem, and substitute the bound $N_{1}$ for the norm of the perturbation matrix to compute the admissible perturbations.

Remark 3: Sensitivity of the eigenvalues of a matrix to the variation of its entries depends, in general, on several factors such as the structure of the matrix (represented by condition number or eigenvalue condition number [12]), repetition or distinction of the eigenvalues, and the most important of all, the norm of the perturbation matrix. On the other hand, it can be easily concluded from (26) and (27) that the bound $N_{1}$ for the decentralized case is less than or equal to the bound $N_{2}$ for the centralized case. Hence, it is expected that the decentralized closed-loop system be more robust to the parameter variation compared to the centralized closed-loop system.

IV. PERFORMANCE EVALUATION

Since the perfect matching condition given in Theorem 1 does not hold in practice, it is desired now to evaluate the degradation in the performance of the decentralized closed-loop system with respect to its centralized counterpart. Define $x_{i}(t)$ as the state of the $i^{th}$ subsystem $S_{i}$ of the closed-loop system $S$ under $K_{d}$ minus the state of the $i^{th}$ subsystem $S_{i}$ of the closed-loop system $S$ under $K_{c}$, for any $i \in \bar{\nu}$. Note that $x_{i}(t)$ is, in fact, the deviation in the state of the $i^{th}$ subsystem due to the mismatch between the real initial states and their expected values in the proposed decentralized control law.

To evaluate the closeness of the decentralized closed-loop system to its centralized counterpart, the following
performance index is defined:

$$J_{dev} = \int_0^\infty (\Delta x(t)^T Q \Delta x(t)) dt$$  \hspace{1cm} (28)$$
where:

$$\Delta x(t) = [ \Delta x_1(t)^T \Delta x_2(t)^T \cdots \Delta x_v(t)^T ]^T$$  \hspace{1cm} (29)$$
and where $Q \in \mathbb{R}^{n \times n}$ is a positive definite matrix. Consider now the system $S$ under the decentralized controller $K_d$.

The state vector of this closed-loop system is $x_g(t)$, which consists of the states of the subsystems as well as those of the local controllers. However, only the states of the subsystems contribute to the performance index (28). Thus, it is desirable to derive $x(t)$ from $x_g(t)$. Define the following matrix:

$$\Phi_i = \begin{bmatrix} 0_{n_i \times n_1} & 0_{n_i \times n_2} & \cdots & 0_{n_i \times n_{i-1}} & I_{n_i \times n_1} \\
0_{n_i \times n_{i+1}} & \cdots & 0_{n_i \times n_{n_v}} & 0_{n_i \times n_{i}} \end{bmatrix}, \quad i \in \vec{\nu}$$  \hspace{1cm} (30)$$

From the definition of $x_g^i(t)$ given in (7), one can write:

$$x_i(t) = \Phi_i x_g^i(t), \quad i \in \vec{\nu}$$  \hspace{1cm} (31)$$
Now, define the block diagonal matrix $\Phi$ as follows:

$$\Phi = \text{diag} (\Phi_1, \Phi_2, \ldots, \Phi_{\nu})$$  \hspace{1cm} (32)$$
It can be concluded from (31) and the above matrix that $\Phi x_g(t) = x(t)$.

Definition 7: Define $\Delta x_0$ as follows:

$$\Delta x_0 = \begin{bmatrix} (\Delta x_0^1)^T & (\Delta x_0^2)^T & \cdots & (\Delta x_0^v)^T \end{bmatrix}^T$$  \hspace{1cm} (33)$$
where:

$$\Delta x_0^i = \begin{bmatrix} (\Delta x_{0,1}^i)^T & \cdots & (\Delta x_{0,1}^{i-1})^T & (0_{n_i \times 1})^T \\
(\Delta x_{0,1}^{i+1})^T & \cdots & (\Delta x_{0,1}^{v,i})^T & (0_{n_\nu \times n_{i+1}})^T \end{bmatrix}, \quad i \in \vec{\nu}$$  \hspace{1cm} (34)$$
and:

$$\Delta x_{0,ij} = x_{0,ij} - x_i(0), \quad i, j \in \vec{\nu}, \quad i \neq j$$  \hspace{1cm} (35)$$
$\Delta x_0$ can be considered as the prediction error of the initial states from different subsystems’ view.

Theorem 4: Suppose that the decentralized closed-loop system is internally stable. Then, the performance index $J_{dev}$ given by (28) is equal to $\Delta x_0^T P_d \Delta x_0$, where the matrix $P_d$ is the solution of the following Lyapunov equation:

$$(A_c^g)^T P_d + P_d A_c^g + \Phi^T Q \Phi = 0$$  \hspace{1cm} (36)$$
Proof: The proof is omitted due to space restrictions. □

Consider now the interconnected system $S$ under the centralized control law $K_c$, and define the following performance index for it:

$$J_c = \int_0^\infty x(t)^T Q x(t) dt$$  \hspace{1cm} (37)$$
It is straightforward to use a similar approach and apply it to (10) to show that $J_c = x(0)^T P_c x(0)$, where the matrix $P_c$ is the solution of the following Lyapunov equation:

$$(A_c)^T P_c + P_c A_c + Q = 0$$  \hspace{1cm} (38)$$
Remark 4: One can use Theorem 4 and the equation (38) to obtain statistical results for the relative performance deviation $\frac{J_c}{J_{dev}}$ in terms of the expected values of the initial states of the subsystems. This can be achieved by using Chebyshev’s inequality. This enables the designer to determine the maximum allowable standard deviation for $\Delta x_0$ to achieve a relative performance deviation within prespecified margins with a sufficiently high probability (e.g. 95%).

A. Near-optimal decentralized control law

Consider the interconnected system $S$ given by (1), and suppose that it is desired to find the controller $K_c$ given by (3) in order to minimize the following performance index:

$$J = \int_0^\infty (x(t)^T Q x(t) + u(t)^T R u(t)) dt$$  \hspace{1cm} (39)$$
where $R \in \mathbb{R}^{n \times n}$ and $Q \in \mathbb{R}^{n \times n}$ are positive definite and positive semi-definite matrices, respectively. Without loss of generality, assume that $Q$ and $R$ are symmetric. If $C$ is an identity matrix (i.e., if all states are available in the output), then the solution of this optimization problem is as follows:

$$u(t) = -K_{opt} y(t) = -K_{opt} x(t)$$  \hspace{1cm} (40)$$
where the matrix $K_{opt}$ is obtained from the Riccati equation. Assume that the controller $K_c$ given by (3) is the optimal centralized controller given by (40), i.e.:

$$A_k = 0, \quad B_k = 0, \quad C_k = 0, \quad D_k = -K_{opt}$$  \hspace{1cm} (41)$$
Construct the proposed decentralized controller $K_d$ based on the controller $K_c$ as described in Section II. It is straightforward to show that the controller $K_d$ is near-optimal (by virtue of having erroneous information). Moreover, the resultant control system has some important properties which can be inferred from the aforementioned theorems.

V. NUMERICAL EXAMPLE

Consider an interconnected system consisting of two SISO subsystems, with the following parameters:

$$A_1 = \begin{bmatrix} 1 & -2 \\
1 & 2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -1 & 0 \\
0 & 1 \end{bmatrix}, \quad C_1 = I_{2 \times 2}$$  \hspace{1cm} (42)$$
Suppose that a centralized controller $K_c$ is given for this system with the following state-space matrices:

$$A_k = \begin{bmatrix} -1 & -2 \\
1 & -1 \end{bmatrix}, \quad B_k = \begin{bmatrix} 1 & 1 \\
2 & 1 \end{bmatrix}, \quad C_k = I_{2 \times 2}, \quad D_k = \begin{bmatrix} 1 & 2 \\
1 & -3 \end{bmatrix}$$  \hspace{1cm} (43)$$
It is desired now to design a decentralized controller $K_d$, such that the system under $K_d$ behaves as closely as possible to the system under $K_c$. Using the method proposed in this paper, $K_d$ can be designed in terms of two constant values $x_{0,1}^{1,2}$ and $x_{0,1}^{2,1}$, which are the expected values of each subsystem’s initial state from the other subsystem’s view. For simplicity, suppose that $x_1(0) = x_2(0) = 1, z(0) = [1\ 1]^T$. Consider now the following two mismatching cases:
i) Suppose that $x_{0}^{2,1} = 0.5$ (−50% prediction error) and $x_{0}^{1,2} = 1.5$ (50% prediction error). Note that the numbers within the above parentheses show the percentage of errors in predicting the initial states. The output of the first subsystem is sketched with both centralized and decentralized controllers, in Figure 1 (note that the output of the second subsystem is not depicted here due to space restrictions). It is evident that the output trajectory of the decentralized closed-loop system is very close to that of the centralized case.

ii) Assume that $x_{0}^{2,1} = 0$ (−100% prediction error) and $x_{0}^{1,2} = 2$ (100% prediction error). Figure 2 illustrates the output of the first subsystem, with both centralized and decentralized controllers. Despite the large prediction errors, the outputs are close to each other.

Now, to evaluate the performance of the proposed decentralized controller with respect to its centralized counterpart, consider the performance index defined in (28), and assume that $Q = I$. It can be concluded from Theorem 4 that:

$$J_{dev} = 1.128 \left( x_{0}^{2,1} \right)^{2} + 1.107 \left( x_{0}^{1,2} \right)^{2} - 0.259 x_{0}^{2,1} x_{0}^{1,2} \quad (44)$$

On the other hand, it can be easily verified by using the equation (38) that $J_c = 18.147$. Now, one can find the expected value and the standard deviation of $\frac{J_{dev}}{J_c}$, provided some information about the distribution of $x_{0}^{2,1}$ and $x_{0}^{1,2}$ is available.

VI. CONCLUSIONS

A decentralized controller is proposed for interconnected systems based on the parameters of a given centralized controller and a priori knowledge about each subsystem’s model from any other subsystem’s view. It is shown that the proposed controller behaves exactly the same as its centralized counterpart, provided the knowledge of each subsystem about other subsystems has no error. Furthermore, a set of conditions for the stability of the decentralized closed-loop system in presence of inexact knowledge of the subsystems’ initial states as well as perturbation in the system’s parameters is presented. Moreover, it is shown that the decentralized control system is likely more robust than its centralized counterpart. In addition, a quantitative measure is given to statistically assess the closeness of the resultant decentralized closed-loop performance to its centralized counterpart. The proposed method is also used to design a near-optimal decentralized control law. Simulation results demonstrate the effectiveness of the proposed decentralized control law.

REFERENCES