Evaluation of sub-component alternatives in product design processes

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Received 1 July 2000; received in revised form 1 June 2001; accepted 1 August 2001

Abstract

In this paper, a sub-component selection methodology for product design is described. The described technique incorporates the analytic hierarchy process and linear goal programming into the process of evaluating alternatives for sub-components and parts, which enables the design of products by satisfying customer, technical, and financial requirements. Also, an additional comparison technique for comparing sub-component alternatives is developed, called “scoring matrix”. In this technique, pair-wise comparisons are performed within one matrix for all possible criteria to measure the strength of one-to-one relationship between sub-component alternatives. This technique is more appropriate than traditional analytical hierarchy process in addressing problems such as the comparison of sub-component alternatives. An illustrative example demonstrates the application of our methodology to the design of a computer system. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Analytic hierarchy process; Product design; Linear programming

1. Introduction

Most products having more than one part can be represented in terms of their components and sub-components in a hierarchy (an example of a PC hierarchical representation is given in Fig. 1). The one-to-one interaction between components and sub-components in this hierarchy influences the ultimate performance of the product. When designing a product, it is common that basic design features (general attributes that are related to the size, shape, functionality etc. of the product) are defined first, and then components and sub-components are selected. In this case, selected design features are the key inputs to the sub-component selection process.

The first step of the design process, definition of product features, must account for both technical design requirements and customer requirements. Success in today’s marketplace is dependent on the level of customer satisfaction. If the designed product enables more features, which are required by the customer, the level of satisfaction is increased. As a result, consumer preferences will be the major input to the selection of design features. Due to technical and financial constraints, it would not be possible to accommodate all the customer requirements in the final product. Hence, the product design team should be capable to cope with tradeoffs in the selection of design features, which results in the highest possible level of customer satisfaction subject to the given constraints [1]. Next step of the design process is selecting the components and sub-components. Since the product design technique requires initial definition of product features, the sub-component selection will be based on these predefined features.

There are cases when a selected best input1 (considered individually) might not lead to the best final product performance in combination with other selected inputs. Typically, inputs that are supplied from outside sources are not exclusively produced for the particular product being designed but rather for more generic purposes. Such inputs can also be supplied from more than one source. When the inputs of a product are supplied from a number of different sources, there is a high probability of quality loss in the final product. To get around this disadvantage, we have developed a bottom-up methodology for selecting the optimal combination of sub-components from multiple alternatives.

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1 Either a candidate part raw material or a sub-component alternative.
alternatives that are incrementally carried from the lowest to the highest level in the hierarchy. Taking customer requirements into consideration starting at the lowest levels of the product hierarchy results in a designed product that is an optimal combination of all the inputs that in the end maximizes the level of customer satisfaction.

The sub-component alternative selection methodology comprises the following steps:

- Reduction in the number of possible alternatives by means of score matrices.
- Weighting the alternatives according to certain performance criteria by means of analytical hierarchy process (AHP).
- Selecting the best number of alternatives (in our case three) using the information provided by the previous two steps, by means of Linear Goal Programming.

The remainder of this paper is organized as follows. In Section 2, related work is reviewed. Section 3 describes the sub-component selection methodology. In Section 4, an example of applying our technique is given and conclusions are drawn in Section 5.

2. Related work

To date, studies in product design focus on ranking customer requirements and setting design features for non-decomposed products. The most widely used technique for this purpose has been AHP [2–6]. AHP provides a mathematically structured means for comparing attributes and alternatives. As part of this decision-making technique, Saaty [2] includes a consistency ratio to quantify the randomness of specified comparisons in development applications. This approach can be used to weight the customer requirements and rank them according to their importance. Armacost et al. [7], Doukas et al. [8], Wasserman [1], and Lu et al. [9] provide alternate integrated quality function deployment (QFD) and AHP methods for ranking customer requirements, user quality factor requirement, design requirements, and marketing goals, respectively.

The purpose of including consumer preferences in design is maximizing customer satisfaction. The number of features considered in design is directly related to the level of customer satisfaction. Due to constraints such as financial, technical etc., a tradeoff between these constraints and the level of customer satisfaction arises. Since design team’s goal is maximizing customer satisfaction subject to given constrains, linear programming (LP) could be thought of as the proper technique for obtaining the optimal design feature mix. Wasserman [1] used LP to select the mix of design features, which results in the highest level of customer satisfaction. Also, Barbarosoglu et al. [10] combined LP and AHP to find the capital rationing in the public sector. Solutions to the LP formulations described in these studies are unique and, therefore, not flexible enough. When LP is used for customer satisfaction, input values for objective function will be the AHP weights, which are obtained by quantifying perceptual relationships such as “better” or “worse”. Since these relationships are not easily quantifiable, design feature mix of the product, which is obtained via LP, can fail to give optimal results. Our LP model for evaluation of AHP weights in this paper differs in two ways from Wasserman’s method. First, the integer-programming (IP) model does not select one optimal set, but the best $n$ sets. Second, the LP model used in this paper is not for selecting a set of design features, but is aimed at selecting the set of sub-components used to construct the components placed at the immediately superior level in the hierarchy. The optimization of the sub-component selection is carried out for all levels of the product hierarchy.
3. Sub-component alternatives selection

The proposed technique requires the definition of the product with its major components and sub-components in a hierarchical scheme. From this scheme, parts and components that have the least amount of effect to the overall characteristics of the product are first discarded. The reason for this elimination is to reduce the computational overhead in our mathematical model. AHP technique is used for the identification of less effective inputs to the overall characteristics of the product. For any given input, a sequence of techniques (mentioned in Section 1) to select the best alternatives is employed. First, using scoring matrices, the number of possible combinations of alternatives will be further limited to a manageable number. Second, using AHP, possible alternatives of a component are weighted and the weights are normalized. Finally, to be able to select the best $n$ ($n = 3$ in our case) alternatives to carry to the next (upper) level, an IP model is utilized.

3.1. Representation of the product in the hierarchical scheme

The described methodology focuses on selecting inputs. Raw materials or parts are usually the initial inputs to a product. Their composition with other parts creates sub-components, which are used to construct components. Finally, the assembly of components results in a marketable product. If a hierarchy is built to represent a product, at level zero, the product itself is placed (the levels in the hierarchy are numbered in increasing order from top to bottom). Then, the major components of the product will be placed at level one. These will be followed by their sub-components, until the lowest level is reached. Some of these parts and components may not have high correlation with the functional characteristics of the product. If the product has too many components and sub-components, representing every single detail in the hierarchical scheme and applying all the mathematical techniques would be time consuming, complex and unnecessary. The marginal contribution of this extra effort would not be economic [11]. To be able to reduce the number of components and sub-components, two aspects are considered:

(a) If there is any major component supplied from an outside source, representing its sub-components in the hierarchical scheme and applying the given evaluation techniques on them may not be necessary (for example, for a computer system design, we might not need to decompose the screen into its sub-components).
(b) Since the inputs have to comply with specific requirements, by using the AHP technique, they are weighted for every component of the product. As a result, inputs that are less correlated with the specific requirements of the component will be eliminated from the scheme and will not be taken into account for further evaluations.

To be able to implement item (b) mentioned above, customer and engineering requirements for the product and its sub-components are collected. AHP technique is used to evaluate these requirements, together with the inputs. Consequently, certain components and sub-components (the most compliant to the requirements) placed in product hierarchy are selected. Fig. 2 is an example of the hierarchical representation of a product (computer system) with its major components.

3.2. Evaluation of alternatives

After constructing the product hierarchy, the next step is to obtain alternatives for the lowest level inputs (parts or raw material). These parts can be supplied from outside sources or manufactured in-house. Combinations of the initially selected alternatives result in components with different specifications. The goal of designers is to select the best possible combination of these alternatives to carry it to the next (upper) level in the hierarchy. Since it cannot be guaranteed that the best alternative at a lower level can also be the best at the upper level, it is necessary to transfer more than one combination for higher level evaluations. Although carrying more than one alternative to a higher level results in a better final design, when the product has a large number of components, evaluation of components will result in too many combinations at the higher levels. A solution to this problem is to restrict the number of alternatives to be carried to the upper levels. The AHP technique would help the designers to pick the alternatives most suitable to their requirements.

When the hierarchical scheme is prepared, functional criteria for every component and sub-component are defined. Using the same criteria, alternatives of sub-components in any level are compared and the one that is the most compliant to these criteria is selected by using the AHP technique. Since carrying more than one alternative to higher levels is required for this procedure, a technique was developed to compare multiple alternatives for every component, and to select the best $n$ alternatives of a sub-component ($n$ is the number of alternatives used for the AHP and LP evaluations). The steps involved in this technique are explained later in this section.

In Fig. 3, a level of the product hierarchy, along with the functional criteria and the sub-component alternatives, is depicted.

The notations employed in Fig. 3 are summarized in Table 1.

The assumption is that the criteria to be fulfilled by the alternatives of component $AC_i$ are chosen from among the criteria required of the sub-components $SC_k$. Although all the required criteria are applied to $AC_i$,
Table 1
Notations employed in Fig. 3

<table>
<thead>
<tr>
<th>Notation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_i, i = 1, 2, \ldots, c$</td>
<td>Input requirements (functional criteria)</td>
</tr>
<tr>
<td>$SC_k, k = 1, 2, \ldots, w$</td>
<td>Sub-components (inputs) of any particular component</td>
</tr>
<tr>
<td>$A_{ij}$</td>
<td>$j$th possible alternative for part (sub-component) $i$</td>
</tr>
<tr>
<td>$AC_l, l = 1, 2, \ldots, t$</td>
<td>Possible combinations of inputs that form alternatives for assembling the component in question</td>
</tr>
</tbody>
</table>

Fig. 2. Hierarchical representation of a computer system.

Fig. 3. A level of the product (computer system) hierarchy.
some may not be applicable to $SC_k$ due to different characteristics of the sub-components. For example, criterion $C_1$ may not be measured on sub-component $SC_3$. Therefore, when the alternatives of $SC_3$ are compared, criterion $C_1$ is not included in the AHP model. Once the number of alternatives is limited for all the components at any level, the same process is repeated for the next (upper) level.

The process of evaluating alternatives is mathematically described next.

### 3.2.1. Scoring matrix

Excluding the lowest level inputs, all the sub-components and components are combinations of parts or sub-components. The one-to-one interaction between sub-components or parts is directly related to the final characteristics of a particular component. The degree of interaction between sub-components is estimated by means of the scoring matrix method. An example of a scoring matrix is given in Fig. 4. Pair-wise comparison enables one to measure the strength of relationship between inputs, which are combined together to construct a part/sub-component. Also, this comparison would help to separate mutually exclusive parts. Using this idea, at the beginning of the sub-component selection process, the number of possible combinations can be reduced. Subsequently, alternative couples (an alternative couple consists of two alternatives from two distinct sub-components) are scored to measure the strength of their relationship (relationship can be in terms of compatibility, cost, time etc). In this paper, only compatibility is used for one-to-one scoring, but other criteria can also be used to find the overall score of alternatives in one group. Some possible additional criteria are as follows:

(a) Is there any cost reduction or increase when these two alternatives are used together?

(b) What is the time required for assembling these two parts by comparison to the other alternative pairs?

Since some of the criteria may have more effect on the design features, when there are two or more criteria, they have to be weighted according to their influence on the final performance of the product. The AHP technique would be the best way of weighting these criteria according to their importance in the design process.

The following procedures have to be performed in the scoring matrix method:

1. For every alternative of any sub-component, find the MUST and NOT relations with other components’ alternatives.
   (a) Select the mutually exclusive alternatives.
   (b) Select the alternatives that must be used together.
2. Score the alternative couples according to their level of compatibility. The level (weight) of compatibility is determined (on a scale of 1–9) a priori by the design team based on customer input, previous experience, technological specifications etc. An alternative couple has one weight for each functional criterion considered in the design.

Example:

Alternative couples | Weight of compatibility ($\Psi_j$)
--- | ---
$A_{11}-A_{21}$ | 5
$A_{11}-A_{22}$ | 7
$A_{21}-A_{31}$ | 3

... ... ...

3. Given MUST and NOT conditions, derive all possible combinations of alternatives for a higher level component (one possible combination contains one alternative for each sub-component of the component under analysis).

Example: \{\{A_{11}, A_{21}, A_{33}, \ldots, A_{M1}\} (possible alternative for the component)

4. Pair-wise comparison of criteria. This operation is performed in order to weight the functional criteria of the product, by using AHP technique.

5. Find the global weighted score for all alternative pairs.

$$\Psi_j(A_{ij}, A_{ik}) = \sum_{q=1}^{m} W_q \cdot \Psi_q(A_{ij}, A_{ik}),$$

### Table

<table>
<thead>
<tr>
<th>Sub-components</th>
<th>Monitor</th>
<th>PC</th>
<th>Graphic Card</th>
<th>Sound Card</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monitor</td>
<td>$A_{11}$</td>
<td>$A_{12}$</td>
<td>$A_{13}$</td>
<td>$A_{21}$</td>
</tr>
<tr>
<td>$A_{11}$</td>
<td>9</td>
<td>5</td>
<td>ME</td>
<td>5</td>
</tr>
<tr>
<td>$A_{12}$</td>
<td>7</td>
<td>ME</td>
<td>5</td>
<td>ME</td>
</tr>
<tr>
<td>$A_{13}$</td>
<td>M</td>
<td>9</td>
<td>5</td>
<td>ME</td>
</tr>
<tr>
<td>PC</td>
<td>$A_{21}$</td>
<td>9</td>
<td>7</td>
<td>ME</td>
</tr>
<tr>
<td>$A_{22}$</td>
<td>5</td>
<td>ME</td>
<td>9</td>
<td>ME</td>
</tr>
<tr>
<td>$A_{23}$</td>
<td>ME</td>
<td>5</td>
<td>5</td>
<td>ME</td>
</tr>
<tr>
<td>Graphic Card</td>
<td>$A_{31}$</td>
<td>5</td>
<td>ME</td>
<td>9</td>
</tr>
<tr>
<td>$A_{32}$</td>
<td>7</td>
<td>5</td>
<td>ME</td>
<td>5</td>
</tr>
<tr>
<td>$A_{33}$</td>
<td>5</td>
<td>5</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>Sound Card</td>
<td>$A_{41}$</td>
<td>ME</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>$A_{42}$</td>
<td>3</td>
<td>8</td>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

Fig. 4. Application of scoring matrix for compatibility. $A_{ij}$: Sub-component alternatives. ME: mutually exclusive alternatives.
where \( r \) is the index of the alternative pair, \( \Psi_r \), the total score for alternative pair \( A_j - A_k \), indexed with \( r \), \( W_q \) the weight of criterion \( q \) (found in step 4 above), and \( \Psi_q(A_j, A_k) \) the score of the alternative pair \( A_j - A_k \) w.r.t. criterion \( q \) (found in step 2).

6. Find the total score for every alternative, as the sum of scores of possible alternative pairs contained in a combination of alternatives. For example, the combination illustrated in step 3 would have the following total score (\( \Psi_1 \)):

\[
\Psi_1 = \Psi(A_{11}, A_{21}) + \Psi(A_{11}, A_{33}) + \ldots + \Psi(A_{33}, A_{41}).
\]

(2)

7. Up to this point, we have alternative sets and their total scores, for a component of the product hierarchy. Instead of dealing with all possible combinations, only a certain number of alternatives will be evaluated. The computed total scores can help in selecting the top \( n \) alternatives. The number of alternatives (\( n \)) could be 7, which is the ideal number for AHP [3].

3.2.2. AHP technique

The purpose of the scoring matrix technique is to select the best combination of alternatives out of a large number of such possible combinations. However, the scoring matrix does not capture any information about the importance (weight) of any sub-component to the overall performance of the component under consideration. In order to incorporate such information in the selection process, an AHP technique is designed to weight the sub-component alternatives individually w.r.t. the importance (weight) of each sub-component to the overall performance of the component under consideration and w.r.t. each functional criterion of the component.

When the hierarchical scheme is prepared, the requirements of every sub-component are set. Using these requirements, AHP will be performed in seven successive steps (refer to Fig. 3), as follows:

1. Pair-wise comparison of sub-components to obtain weights \( W_{ij} \) w.r.t. each criterion \( W_{ij} \) is the weight of sub-component \( j \) w.r.t. criterion \( i \).

2. Pair-wise comparison of criteria to obtain weight of each criterion \( WC_i \) denoted as the weight of \( i \)th criterion. These weights quantify the degree of influence of every criterion on the overall performance of the product.

3. Normalization of sub-component weights, as follows:

\[
W_i = W_{ij} WC_i,
\]

(3)

where \( W_i \) is the normalized weight of sub-component \( i \). The meaning of the other two factors has been explained above.

4. Pair-wise comparison of the sub-component alternatives to obtain the weights \( W_{ik} \) (the weight of \( k \)th alternative of \( j \)th sub-component w.r.t. \( i \)th criterion). Since any sub-component can be considered as a different product from the standpoint of the component being analyzed, the criteria used to compare sub-component alternatives may differ from the criteria used to compare sub-components.

5. Pair-wise comparison of criteria imposed on sub-component alternatives; obtain the weight \( WC_{ij} \) (the weight of \( j \)th criterion imposed on \( i \)th sub-component’s alternative).

6. Normalization of sub-component alternative weights, as follows:

\[
W_{jk} = WC_{ij} W_{ik},
\]

(4)

where \( W_{jk} \) is the normalized weight of the \( k \)th alternative of the \( j \)th sub-component.

7. Normalize the weight of sub-component alternatives w.r.t. the weight of the sub-components found in step 3.

\[
WA_{ij} = W_{jk} W_i,
\]

(5)

where \( WA_{ij} \) is the normalized weight of the \( j \)th alternative of the \( i \)th sub-component.

3.2.3. Integer programming

After the scoring matrix method is applied and alternatives are weighted, next step is selecting the best \( p \) alternatives. To carry \( p \) (less than \( n \), could be 3) alternatives to the next higher level in the product hierarchy, integer programming (IP) is used. For this purpose, an IP model, which compares alternatives in case of different criteria, has been developed. In this model, the major objective function will be to maximize sub-components’ compliance to the requirements. Other objective functions can be defined, such as minimization of alternatives’ cost etc. In LP models, when we have more than one objective function, the effect of each function to the overall goal might differ from each other. The LP model we have developed can consist of more than one objective function. As such, according to the characteristics of the design, the importance of each objective function should be determined by using an AHP-like pair-wise comparison technique. The values of the objective functions coefficients should be normalized. Linear goal programming models in which multiple goals are prioritized have already been implemented in conjunction with AHP [12].

The mathematical formulation looks as follows:

\[
\text{Max } WO_1 Z_1 - WO_2 Z_2
\]

(6)

Subject to

\[
Z_1 = \sum (WA_k AC_k), \ k = 1, 2, \ldots, K (K = 7 \text{ in this paper}),
\]

(8)
\[
Z_2 = \sum (PA_k AC_k), \quad k = 1, 2, \ldots, K, \quad (9)
\]

\[
\sum AC_k = p, \quad k = 1, 2, \ldots, K, \quad (10)
\]

\[
\text{Total} = \sum \sum P_{ij}, \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m, \quad (11)
\]

\[
P_{ij} = P_{ij}/\text{Total}; \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m \quad (12)
\]

(Normalization of coefficients),

\[
WA_k = \sum \sum W_{ij}, \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m; \quad k = 1, 2, \ldots, K, \quad (13)
\]

\[
PA_k = \sum \sum P_{ij}, \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m; \quad k = 1, 2, \ldots, K, \quad (14)
\]

\[
AC_k = 0 \text{ or } 1; \quad k = 1, 2, \ldots, K, \quad (15)
\]

\[
Z_1 >= 0, \quad (16)
\]

\[
Z_2 >= 0, \quad (17)
\]

where \(Z_1\) is the objective function that maximizes the customer satisfaction, \(Z_2\) is the any other objective function. \((Z_2\) is cost minimization in our case), \(WO_1\) is the degree of importance of the first objective (compliance to requirements), \(WO_2\) is the degree of importance of the second objective (cost), \(p\) is number of alternatives carried to the upper level, \(K\) is number of alternatives found in scoring matrix analysis \((K = 7\) in our case), \(AC_k\) is a combination of sub-component alternatives, found by scoring matrix. \(WA_k\) is the total weight of \(AC_k\). \(PA_k\) is the penalty (cost in our case) incurred by selecting \(AC_k\) (combination of alternatives), \(W_{ij}\) is the weight of sub-component alternative \(A_{ij}\) (found by the AHP technique as described in Section 3.2.1), \(P_{ij}\) is the penalty incurred by selecting \(A_{ij}\) as a sub-component alternative, and \(\text{Total}\) is the total penalty (cost in our case).

The IP model illustrated above is able to select a number of best alternatives (in our case three) of the final product. The design team, based on its experience, makes the final decision over the selected alternatives for the final product.

The overall methodology of sub-component selection can be summarized as in Fig. 5.

4. Case study

The Industrial Virtual Reality Institute (IVRI) of The University of Illinois at Chicago considers purchasing a graphic card, a sound card and a new monitor to update one of its three existing PCs (Dell-Pentium III 400 Mhz, HP-Pentium III 400 Mhz, and IBM-Pentium III 450 Mhz). IVRI is a virtual reality (VR) lab currently implementing most of its projects on Silicon Graphics workstations. A new project requires a PC to create telecollaborative VR environments between the PC and the workstations. As a result, IVRI lab members decided to update one of their existing PCs to design a VR-compatible computer system.

After reviewing the current needs of the lab, researchers found that the following criteria are essential for completing the project: 3D Graphic Support, Network Capabilities, Fast Rendering and Quality Sound System. After investigating the computer market, lab
members determined a number of alternatives for each sub-component (selected alternatives for the computer system design are given in Fig. 6). The goal of IVRI is to design a computer system with minimum cost, that maximizes the overall technical expectation (market prices of the sub-component alternatives are given in Table 2). Finally, the lab members evaluated the alternatives of sub-components to design a computer system for the IVRI’s current needs, using the proposed technique in this paper.

The methodology of selecting the optimal computer system consists of the three steps of our sub-component selection technique and is described next.

4.1. Scoring matrix

1. MUST and NOT relationships between sub-component alternatives are shown in Fig. 4.
2. According to the level of compatibility, sub-component alternatives are pair-wise compared as in Fig. 4.
3. Possible alternatives for the computer system under consideration and their total scores from the scoring matrix are shown in Table 3.
4. Since the compatibility is the only criterion used in our computer system design process, comparison of the criteria is not applicable in this example.
5. Because of the same reason given in step 4, global weighted scores for the alternatives are the same as the weights found in step 2.
6. Total weights of the computer system alternatives are given in Table 3.
7. From the list of alternatives given in Table 3, the top 7 alternatives are selected for the future evaluation, which is performed by using the LP model described in Section 3.2.3.

4.2. AHP application

The ultimate goal of IVRI is to maximize the system performance according to its needs. Some of the sub-components have more effect on the lab’s requirement. Using the scoring matrix technique, IVRI identifies the one-to-one relationship between sub-components alternatives. Yet, design team did not consider the level of effect of the sub-components into computer system design.
Using the AHP technique, priority weights of the criteria and the sub-components are obtained (Table 4).

In the second part of the AHP technique, the design team weighted each alternative to ascertain the importance of alternatives on the overall performance of the computer system. Priority weights are obtained for each sub-component alternative, as in Table 5.

The methodology of obtaining the weights given in Table 5 is described in detail in Appendix A.

### 4.3. Linear programming model

The goal of IVRI is to design a computer system satisfying the functional requirements with a minimum cost. To combine these two objectives, linear goal programming is employed (see Appendix B) and the result obtained from running the model is given below.

In the LP model, all the coefficients of objective functions should be normalized (sum of the coefficients belonging to an objective function should be one). If there is a different importance of each objective functions, objective functions should be compared to each other and weighted. In this paper, objective function \( Z_1 \) that maximizes the customer satisfaction has priority over the objective function \( Z_2 \) that minimizes the cost, so a 7 to 3 ratio is enforced between \( Z_1 \) and \( Z_2 \) (Objective function is written as Max 0:7 \( Z_1 \) − 0:3 \( Z_2 \)).

Results of the LP model are summarized in Tables 5 and 6.

From the solution of the LP model, the design team has three measures (total AHP weights, total price and LP output for each component alternative) to take into account when selecting one of the given three alternatives. Even though the price differences are quite high, the total weight difference between alternatives 3 and 5 is not significant. Yet, the alternative 7 has a total weight that is significantly higher than the other two. Since the priority here is designing a computer system most compliant to the functional requirements, the lab management decided to select the computer system described by alternative 7. This decision is supported by the LP result (\( Z_1 − Z_2 \) value for \( A_7 \) is the highest (0.1946) among the three alternatives given above).
5. Conclusions

A methodology (based on the hierarchical representation of a product) for optimally selecting product sub-components during the product design process has been described. The purpose of this technique is to maximize the level of customer satisfaction with the final product. When sub-components are selected, their one-to-one interactions are taken into account. The strength of one-to-one interactions is mathematically represented using scoring matrices. The ability of sub-components to comply with functional requirements is measured using AHP technique. The outputs of these two processes are embedded in an Integer Programming model in order to select the most compliant sub-components from several alternatives.

The major advantage of the described methodology is its ability to capture the optimal combination of sub-components that form higher-level components in the product hierarchy. By selecting more than one alternative for any given component, our technique avoids situations when a selected alternative is the best for a particular component (considered separately from the overall product), but fails to enable the observance of requirements at higher levels of the product hierarchy. This, in turn, provides the design team with more flexibility in making decisions.

The described technique can be expandable to inventory selection process. When any input in a warehouse is used for more than one product family of a company, design requirements of the product families would be different from each other. To select the input maximizing the overall fitness to all the products’ requirements, the technique described in this paper can be used. This is a further research direction.

Appendix A. AHP application

The following tables show pair-wise comparisons between either criteria or inputs to measure their importance to the final goal. A 1–9 scale is used to compare pairs. The value 1 means that the pairs are equally important, and the value 9 shows the absolute importance. Priority weights \( W_i \) are calculated using the AHP technique. \( \lambda \) is the largest eigenvalue of the pair-wise comparison matrix, which specifies the consistency of the scoring (if \( \lambda \) is too high, the relationship between scores in the same matrix is inconsistent). The consistency index (CI) and the consistency ratio (CR) are calculated to observe the consistency of the judges (lab member in our case) comparison decisions [3].

1. Pair-wise comparison of criteria

\[
\begin{align*}
& C_1 \quad C_2 \quad C_3 \quad C_4 \quad \sum W_i \\
3D \text{ Graphic support } (C_1) & 1 \quad 3 \quad 1 \quad 7 \quad 0.4778 \\
\text{Network capabilities } (C_2) & 1/3 \quad 1 \quad 1 \quad 5 \quad 0.2495 \\
\text{High speed rendering } (C_3) & 1 \quad 1 \quad 1 \quad 5 \quad 0.2108 \\
\text{Sound system } (C_4) & 1/7 \quad 1/5 \quad 1 \quad 1 \quad 0.0610 \\
\lambda &= 4.1211; \text{ CI} = 0.040; \text{ CR} = 0.044
\end{align*}
\]

2. Pair-wise comparison of sub-components w.r.t. each criterion

(i) 3D graphic support

\[
\begin{align*}
& SC_1 \quad SC_2 \quad SC_3 \quad SC_4 \quad \sum W_i \\
\text{Monitor } (SC_1) & 1 \quad 1/3 \quad 1/5 \quad 7 \quad 0.1354 \\
\text{PC } (SC_2) & 3 \quad 1 \quad 1 \quad 9 \quad 0.2754 \\
\text{Graphic card } (SC_3) & 5 \quad 3 \quad 1 \quad 9 \quad 0.5549 \\
\text{Sound card } (SC_4) & 1/7 \quad 1/9 \quad 1/9 \quad 1 \quad 0.0342 \\
\lambda &= 4.2641; \text{ CI} = 0.087; \text{ CR} = 0.097
\end{align*}
\]

(ii) Network capabilities

\[
\begin{align*}
& SC_1 \quad SC_2 \quad SC_3 \quad SC_4 \quad \sum W_i \\
\text{Monitor } (SC_1) & 1 \quad 1/9 \quad 3 \quad 3 \quad 0.1398 \\
\text{PC } (SC_2) & 9 \quad 1 \quad 9 \quad 9 \quad 0.5549 \\
\text{Graphic card } (SC_3) & 1/3 \quad 1/9 \quad 1 \quad 1 \quad 0.0598 \\
\text{Sound card } (SC_4) & 1/3 \quad 1/9 \quad 1 \quad 1 \quad 0.0598 \\
\lambda &= 4.1545; \text{ CI} = 0.051; \text{ CR} = 0.056
\end{align*}
\]

(iii) High speed rendering

\[
\begin{align*}
& SC_1 \quad SC_2 \quad SC_3 \quad SC_4 \quad \sum W_i \\
\text{Monitor } (SC_1) & 1 \quad 3 \quad 1/3 \quad 9 \quad 0.2723 \\
\text{PC } (SC_2) & 1/3 \quad 1 \quad 1/5 \quad 9 \quad 0.1463 \\
\text{Graphic card } (SC_3) & 3 \quad 5 \quad 1 \quad 9 \quad 0.5491 \\
\text{Sound card } (SC_4) & 1/9 \quad 1/9 \quad 1/9 \quad 1 \quad 0.0321 \\
\lambda &= 4.3466; \text{ CI} = 0.11; \text{ CR} = 0.12
\end{align*}
\]
(iv) **Sound system**

<table>
<thead>
<tr>
<th>SC₁</th>
<th>SC₂</th>
<th>SC₃</th>
<th>SC₄</th>
<th>Wᵣ₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monitor (SC₁)</td>
<td>1</td>
<td>1/7</td>
<td>3</td>
<td>1/9</td>
</tr>
<tr>
<td>PC (SC₂)</td>
<td>7</td>
<td>1</td>
<td>9</td>
<td>1/5</td>
</tr>
<tr>
<td>Graphic card (SC₃)</td>
<td>1/3</td>
<td>1/9</td>
<td>1</td>
<td>1/9</td>
</tr>
<tr>
<td>Sound card (SC₄)</td>
<td>9</td>
<td>5</td>
<td>9</td>
<td>1</td>
</tr>
</tbody>
</table>

\( \lambda = 4.4129; \ CI = 0.140; \ CR = 0.142 \)

3. Normalization of sub-components weight

\[
\begin{array}{cccccc}
W_j & W_{j1} & W_{j2} & W_{j3} & W_{j4} & WC_j & W_j \\
\text{Monitor} & 0.1354 & 0.1398 & 0.2723 & 0.0629 & 0.4778 & 0.1953 \\
\text{PC (SC₂)} & 0.2754 & 0.7400 & 0.1463 & 0.2604 & * 0.2495 = 0.3629 \\
\text{Graphic card (SC₃)} & 0.555 & 0.0590 & 0.5491 & 0.0354 & 0.2108 & 0.3978 \\
\text{Sound card (SC₄)} & 0.0342 & 0.0598 & 0.0321 & 0.6411 & 0.0610 & 0.0770 \\
\end{array}
\]

4 and 5. Pair-wise comparison of sub-component alternatives w.r.t. corresponding criteria

To design a computer system, four major criteria were defined in advance. Yet, some of these criteria might not be highly correlated with all the sub-components. For example, we found that speed of computer and network capabilities are not correlated with the monitor and the graphic card. Also, the sound card is only correlated with the high sound quality criterion. During the evaluation of sub-component alternatives, the selected criteria are used.

(i) **Pair-wise comparison of monitor alternatives**

(a) **3D graphic support**

\[
\begin{array}{cccc}
A_{11} & A_{12} & A_{13} & W_{11k} \\
\text{DELL 17'' (A₁₁)} & 1 & 1 & 3 & 0.4285 \\
\text{DELL 19'' (A₁₂)} & 1 & 1 & 3 & 0.4285 \\
\text{HP 19'' (A₁₃)} & 1/3 & 1/3 & 1 & 0.1428 \\
\lambda = 3.00; \ CI = 0.0; \ CR = 0.0 \\
\end{array}
\]

(b) **Rendering**

\[
\begin{array}{cccc}
A_{11} & A_{12} & A_{13} & W_{11k} \\
\text{DELL 17'' (A₁₁)} & 1 & 3 & 1/3 & 0.2582 \\
\text{DELL 19'' (A₁₂)} & 1/3 & 1 & 1/5 & 0.1047 \\
\text{HP 19'' (A₁₃)} & 3 & 5 & 1 & 0.6369 \\
\lambda = 3.0385; \ CI = 0.019; \ CR = 0.033 \\
\end{array}
\]

(c) **Pair-wise comparison of criteria (3D graphic support vs. rendering)**

\[
\begin{array}{ccc}
C_{11} & C_{12} & WC_j \\
3D graphic (C₁₁) & 1 & 3 & 0.75 \\
Fast rendering (C₁₂) & 1/3 & 1 & 0.25 \\
\lambda = 2.00; \ CI = 0.0; \ CR = 0.0 \\
\end{array}
\]

(ii) **Pair-wise comparison of PC Alternatives**

(a) **3D graphic support**

\[
\begin{array}{cccc}
A_{21} & A_{22} & A_{23} & W_{12k} \\
\text{Dell-PentIII 400 Mhz (A₂₁)} & 1 & 3 & 1/3 & 0.2500 \\
\text{HP-PentIII 400 Mhz (A₂₂)} & 1/3 & 1 & 1/5 & 0.1047 \\
\text{IBM-PentIII 450 Mhz (A₂₃)} & 3 & 5 & 1 & 0.6369 \\
\lambda = 3.0385; \ CI = 0.019; \ CR = 0.033 \\
\end{array}
\]

(b) **Network capabilities**

\[
\begin{array}{cccc}
A_{21} & A_{22} & A_{23} & W_{22k} \\
\text{Dell-PentIII 400 Mhz (A₂₁)} & 1 & 1/3 & 1/3 & 0.1350 \\
\text{HP-PentIII 400 Mhz (A₂₂)} & 3 & 1 & 1/5 & 0.2800 \\
\text{IBM-PentIII 450 Mhz (A₂₃)} & 3 & 3 & 1 & 0.5841 \\
\lambda = 3.1356; \ CI = 0.067; \ CR = 0.166 \\
\end{array}
\]

(c) **High speed rendering**

\[
\begin{array}{cccc}
A_{21} & A_{22} & A_{23} & W_{32k} \\
\text{Dell-PentIII 400 Mhz (A₂₁)} & 1 & 1/3 & 1/3 & 0.4599 \\
\text{HP-PentIII 400 Mhz (A₂₂)} & 3 & 1 & 1/5 & 0.2211 \\
\text{IBM-PentIII 450 Mhz (A₂₃)} & 3 & 3 & 1 & 0.3189 \\
\lambda = 3.1356; \ CI = 0.067; \ CR = 0.117 \\
\end{array}
\]

(d) **Sound system**

\[
\begin{array}{cccc}
A_{21} & A_{22} & A_{23} & W_{42k} \\
\text{Dell-PentIII 400 Mhz (A₂₁)} & 1 & 1/5 & 1/3 & 0.1100 \\
\text{HP-PentIII 400 Mhz (A₂₂)} & 5 & 1 & 1/3 & 0.3229 \\
\text{IBM-PentIII 450 Mhz (A₂₃)} & 3 & 3 & 1 & 0.5665 \\
\lambda = 3.1356; \ CI = 0.140; \ CR = 0.254 \\
\end{array}
\]

(e) **Since all the criteria used for designing the computer system are applicable for the PC as well, we use the criteria weights found in step 1.**

(iii) **Pair-wise comparison of graphic cards**

(a) **3D graphic support**

\[
\begin{array}{cccc}
A_{31} & A_{32} & A_{33} & W_{13k} \\
\text{GE force256 (A₃₁)} & 1 & 5 & 3 & 0.6586 \\
\text{Diamond ViperII (A₃₂)} & 1/5 & 1 & 1 & 0.1561 \\
\text{ATI (A₃₃)} & 1/3 & 1 & 1 & 0.1851 \\
\lambda = 3.0290; \ CI = 0.014; \ CR = 0.025 \\
\end{array}
\]

(b) **High speed rendering**

\[
\begin{array}{cccc}
A_{31} & A_{32} & A_{33} & W_{23k} \\
\text{GE Force256 (A₃₁)} & 1 & 1/3 & 1/5 & 0.1047 \\
\text{Diamond ViperII (A₃₂)} & 3 & 1 & 1/3 & 0.2582 \\
\text{ATI (A₃₃)} & 5 & 3 & 1 & 0.6369 \\
\lambda = 3.0385; \ CI = 0.019; \ CR = 0.033 \\
\end{array}
\]

(iv) **Pair-wise comparison of sound cards**

(a) **Sound system**

\[
\begin{array}{cccc}
A_{41} & A_{42} & W_{14k} \\
\text{Sound blaster PC1512 (A₄₁)} & 1 & 7 & 0.8750 \\
\text{Turtle back montego (A₄₂)} & 1/7 & 1 & 0.1250 \\
\lambda = 2.00; \ CI = 0.0; \ CR = 0.0 \\
\end{array}
\]
6. Normalization of sub-component alternative weights w.r.t criteria correspond to the sub-components

(i) Monitors

\[
\begin{align*}
A_{1k}/C_{ij} & \quad W_{C_{ij}} & \quad W_{1k} \\
Dell 17'' & 0.4285 & 0.2582 & 0.75 & 0.3859 \\
Dell 19'' & 0.4285 & 0.1047 & * & 0.25 = 0.3475 \\
HP 19'' & 0.1428 & 0.6369 & & 0.2663 \\
\end{align*}
\]

(ii) PCs

\[
\begin{align*}
A_{3k}/C_{2j} & \quad W_{C_{2j}} & \quad W_{2k} \\
Dell-PentII & 0.2500 & 0.1350 & 0.4599 & 0.1100 & 0.4778 & 0.2567 \\
HP & 0.1047 & 0.2800 & 0.2211 & 0.3292 & * & 0.2495 = 0.1862 \\
IBM-PentIII & 0.6369 & 0.5841 & 0.3189 & 0.5665 & 0.2108 & 0.5518 \\
\end{align*}
\]

(iii) Graphic cards

\[
\begin{align*}
A_{3k}/C_{3j} & \quad W_{C_{3j}} & \quad W_{3k} \\
GE Force & 0.0.6586 & 0.1047 & 0.75 & 0.5201 \\
Diamond & 0.1561 & 0.2582 & * & 0.25 = 0.1816 \\
ATI & 0.1851 & 0.6369 & & 0.2980 \\
\end{align*}
\]

(iv) Sound cards

Since only one criterion was considered for the sound card evaluation, the normalization process is not required. The weights found above will be used in step 7.

7. Normalizing the sub-component alternative w.r.t weights of sub-components.

---

Priority weights of the sub-component alternatives

\[
\begin{align*}
W_{11} & = 0.07536 \\
W_{12} & = 0.06786 \\
W_{13} & = 0.05200 \\
W_{21} & = 0.09315 \\
W_{22} & = 0.06757 \\
W_{23} & = 0.20000 \\
W_{31} & = 0.20680 \\
W_{32} & = 0.07220 \\
W_{33} & = 0.11850 \\
W_{41} & = 0.06700 \\
W_{42} & = 0.00960 \\
\end{align*}
\]

\[
\begin{align*}
W_{1} & = W_{12} + W_{23} + W_{33} + W_{42}; \\
W_{2} & = W_{13} + W_{22} + W_{33} + W_{41}; \\
W_{3} & = W_{13} + W_{22} + W_{31} + W_{41}; \\
W_{4} & = W_{13} + W_{22} + W_{33} + W_{42}; \\
W_{5} & = W_{13} + W_{23} + W_{33} + W_{42}; \\
W_{6} & = W_{12} + W_{21} + W_{33} + W_{42}; \\
W_{7} & = W_{12} + W_{23} + W_{33} + W_{41}; \\
\end{align*}
\]

Priority weights found in step 2 (AHP), Top 7 alternatives for the computer system are selected in step 1 (scoring matrix), out of 15 alternatives.

Unit cost of sub-component alternatives

\[
\begin{align*}
P_{11} & = 650.00 \\
P_{12} & = 1030.00 \\
P_{13} & = 750.00 \\
P_{21} & = 0.00 \\
P_{22} & = 0.00 \\
P_{23} & = 0.00 \\
P_{31} & = 250.95 \\
P_{32} & = 150.00 \\
P_{33} & = 125.00 \\
P_{41} & = 230.00 \\
P_{42} & = 200.00 \\
\end{align*}
\]
Total = $\sum P_{ij}$; Sum of the prices ($3385.95$) of all the sub-component alternatives (used for normalization)

\[ P_{ij} = \frac{P_{ij}}{\text{Total}}; \quad i = 1, 2, \ldots, 4; \quad j = 1, \ldots, n \]

\((n = 3 \text{ for } i = 1, 2, \text{ and } 3, \text{ and } n = 2 \text{ for } i = 4)\)

\[ PA_1 = P_{12} + P_{23} + P_{33} + P_{42}; \]
\[ PA_2 = P_{13} + P_{22} + P_{33} + P_{41}; \]
\[ PA_3 = P_{13} + P_{22} + P_{31} + P_{41}; \]
\[ PA_4 = P_{13} + P_{22} + P_{33} + P_{42}; \]
\[ PA_5 = P_{13} + P_{23} + P_{33} + P_{42}; \]
\[ PA_6 = P_{12} + P_{21} + P_{33} + P_{42}; \]
\[ PA_7 = P_{12} + P_{23} + P_{33} + P_{41}; \]

\[ Z_1 >= 0; \]
\[ Z_2 >= 0; \]

\[ AC_i = \begin{cases} 1 & \text{if } AC_i \text{ is selected } \quad i = 1, 2, \ldots, 7 \\ 0 & \text{otherwise} \end{cases} \]

END

References