Optimal Pricing and Seat Allocation in Airline Industry Under Market Competition

PhD Thesis

Oral Exam Presentation

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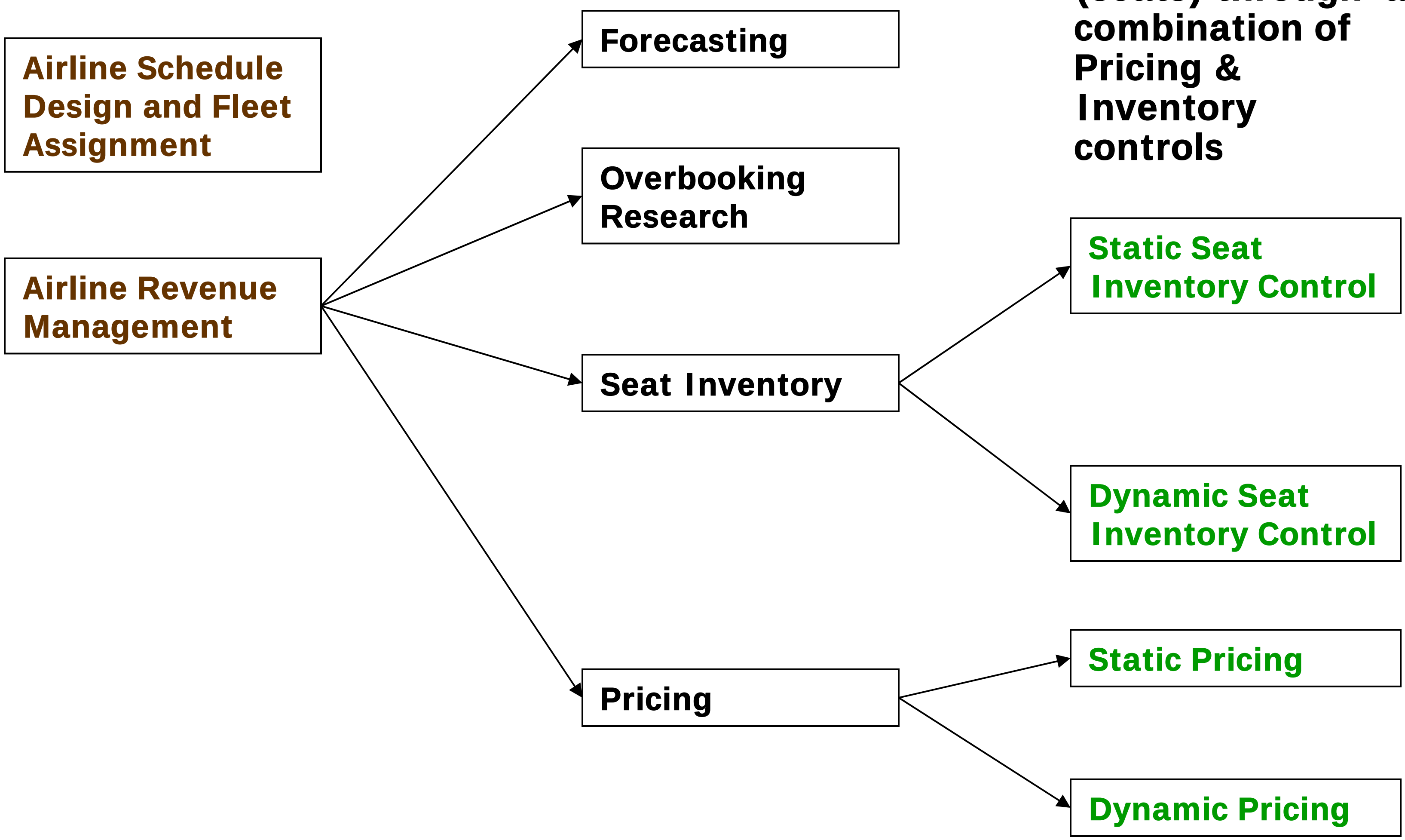
Outline

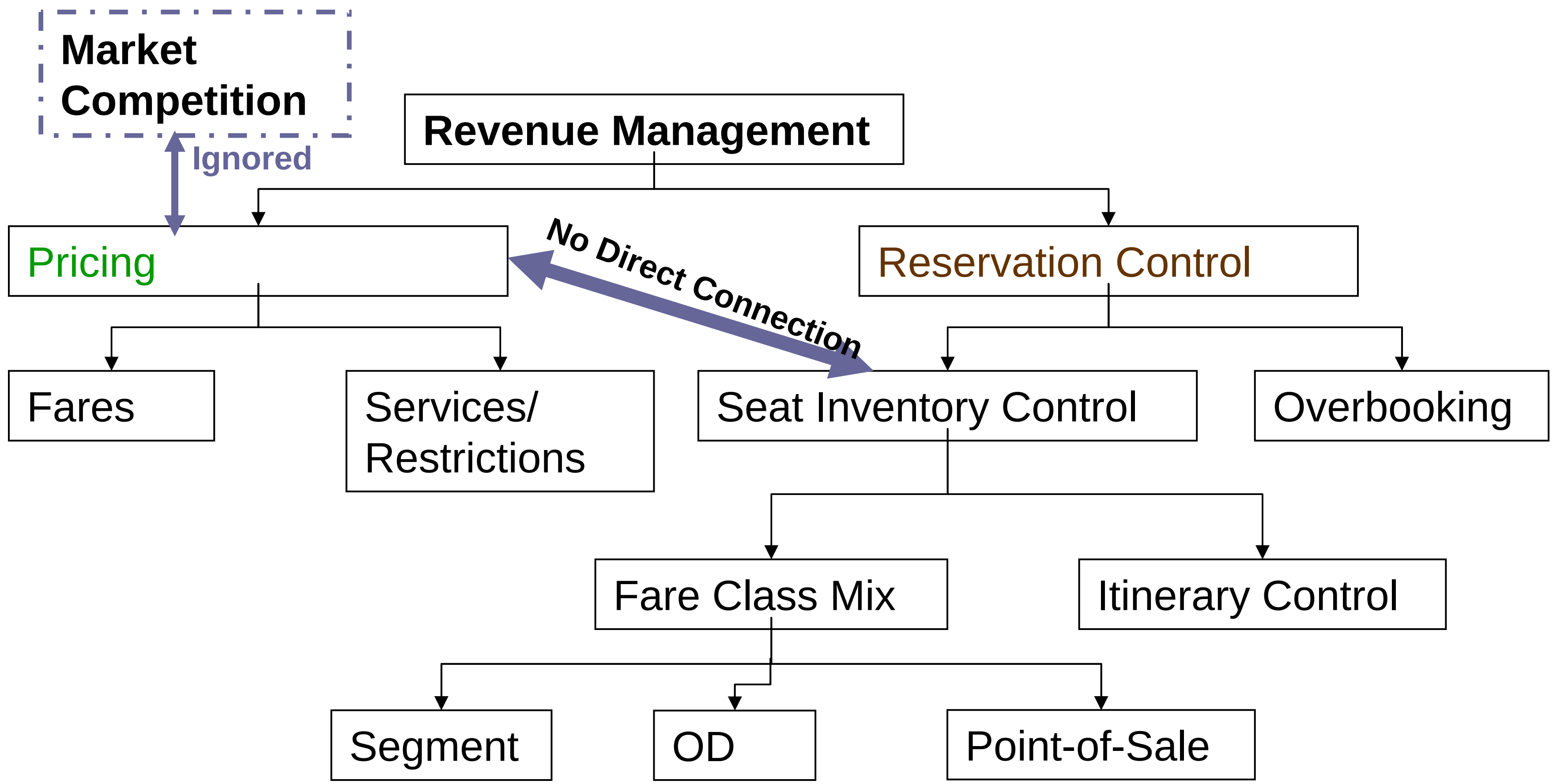
- Revenue Management (RM)
- Background and Literature
- Model and Results
- Conclusion and Future Works
- Questions

Airline RM

Revenue Management is simply the practice of maximising Revenue from Perishable assets (seats) through a combination of Pricing & Inventory controls



Missing Connections in Airline RM Studies



- Review Articles:
  - McGill and Van Ryzin (1999)- Revenue Management: Research overview and prospects.
  - Pak and Piersma (2002)- Overview of OR techniques in airline RM.
  - Silver (2004)- An overview of heuristic solution methods
  - Barnhart et al. (2003) – Application of operations research in the air transport industry.
  - Elmagraby and Keskinocak (2003) -Dynamic pricing in the presence of inventory considerations: Research overview, current practices, and future directions
  - Bitran and Caldentey (2003)- An overview of pricing models for Revenue Management.

Competition in RM

- Cote et al. (2003)- Mathematical optimization approach for integrated seat allocation and pricing in airline industry.
  - Bi-level mathematical optimization model is presented for joint determination of fare pricing and seat allocation
  - The customer demand is considered price insensitive and the model is non dynamic
- Dai et al. (2005)- Multiple firms competition in RM context.
  - Competitive prices are determined for both the deterministic and stochastic price sensitive demand for a known capacity.
  - Mostly the analysis consider duopoly competition
- Chen et al.(2005) – Newsvendor pricing game
  - Jointly determined the competitive pricing and capacity for the competing firms for a single commodity
  - Unique Nash equilibrium is determined analytically

- Competition in Airline RM:
  - Hotelling (1929)-Scheduling decision under competition
  - Borenstein et al. (2003)- An empirical paper, focus on broad competition problem but ignore any seat allocation/ pricing modeling.
  - Belobaba and Wilson (1997)- Simulation model to study scheduling strategy under market competition.
  - Lippman and McCardle (1997)- two-firm pricing competition.
  - Teodorovic and Krcmar-Nozic (1989)- Multi-criteria model to determine flight frequencies on an airline network under competitive conditions.

Objectives of thesis

- Develop new models for joint determination of competitive fare pricing and seat allocation in airline Industry
- Develop new solution methodologies for joint determination of pricing and capacity in RM

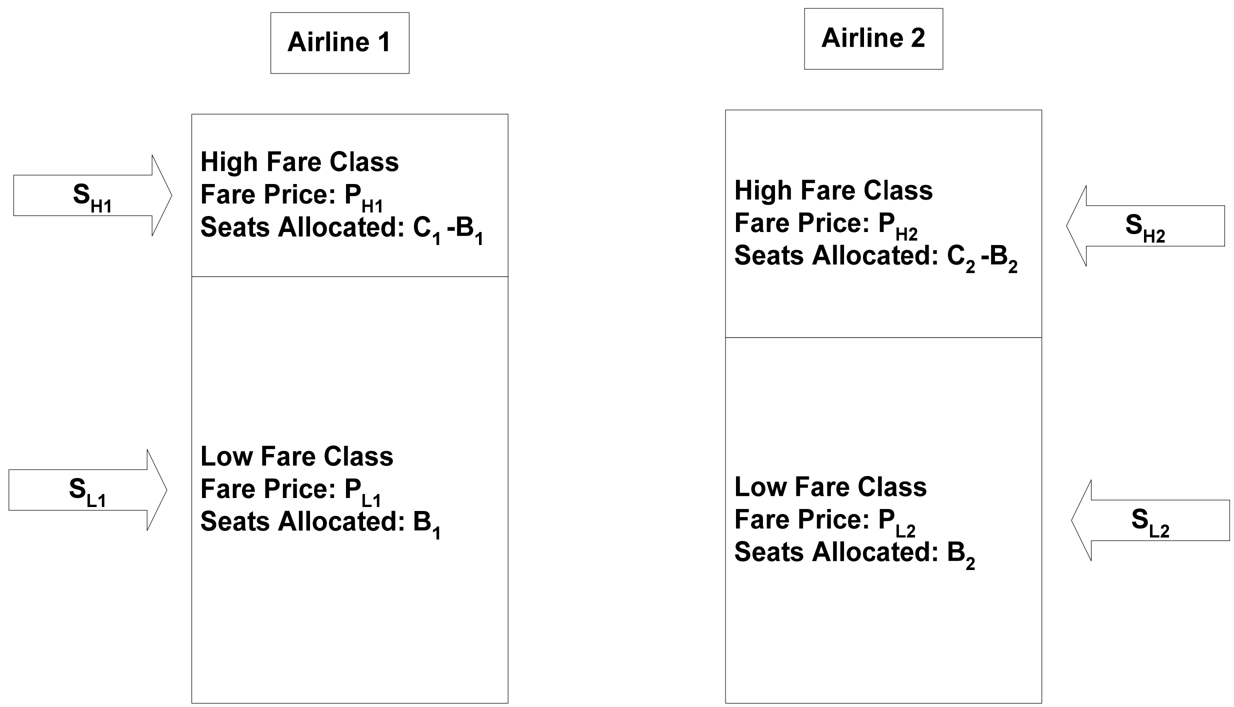
Airline RM Competition Models

- Airline RM game is developed mainly in duopoly market
- Two game theoretic models are presented
- The models are studied for both the cooperative and non-cooperative game settings.
- Numerical analysis with statistical DOE

Distribution Free Approach is RM

- An alternative approach for joint determination of pricing and capacity under monopoly
- Determines lower bound (worst possible) revenue estimate
- The approach is addressed to:
  - Standard Newsvendor problem
  - Extension to Shortage cost Penalty
  - Extension to Shortage and Holding cost
  - Extension to recourse case
  - Extension random yield
  - Extension to multiple items

Competition Models in Airline RM:



$$\Pi_i = P_{Li} \min\{S_{Li}, B_i\} + P_{Hi} \min\{S_{Hi}, C_i - \min\{S_{Li}, B_i\}\}, i = \{1,2\}$$

Modeling Approaches

Additive Model

$$S_{Li} = D_{Li} + \xi_{Li}, \forall i = \{1,2\}$$

$$S_{Hi} = D_{Hi} + \xi_{Hi}, \forall i = \{1,2\}$$

Where

$$E[\xi_{Li}] = E[\xi_{Hi}] = 0$$

$$\xi_{Li} \in [\underline{\xi}_{Li}, \overline{\xi}_{Li}]$$

$$\xi_{Hi} \in [\underline{\xi}_{Hi}, \overline{\xi}_{Hi}]$$

Multiplicative Model

$$S_{Li} = D_{Li}\,\xi_{Li}, \forall i = \{1,2\}$$

$$S_{Hi} = D_{Hi}\,\xi_{Hi}, \forall i = \{1,2\}$$

Where

$$E[\xi_{Li}] = E[\xi_{Hi}] = 1$$

$$\xi_{Li} \in [\underline{\xi}_{Li}, \overline{\xi}_{Li}]$$

$$\xi_{Hi} \in [\underline{\xi}_{Hi}, \overline{\xi}_{Hi}]$$

Both ξ_{Li} and ξ_{Hi} follow Increasing Generalized Failure Rate (IGFR)

A Uniform Distribution is assumed

Assumptions

The Demand is modeled with linear price sensitive determinsitic demand functions

$$D_{Li} = \alpha_{Li} - \beta_{Li} P_{Li} + \theta_{Lij} P_{Lj},\ D_{Hi} = \alpha_{Hi} - \beta_{Hi} P_{Hi} + \theta_{Hij} P_{Hj}$$

$$\beta_{Li} > \theta_{Lij}, \beta_{Li}, \theta_{Lij} > 0, i \neq j, i, j = \{1,2\},$$

$$\beta_{Hi} > \theta_{Hij}, \beta_{Hi}, \theta_{Hij} > 0, i \neq j, i, j = \{1,2\},$$

It is also assumed that

$$\frac{\partial \mathrm{D_{L1}}}{\partial P_{L1}} \leq 0, \frac{\partial \mathrm{D_{L1}}}{\partial P_{L2}} \geq 0, \frac{\partial \mathrm{D_{L1}}}{\partial P_{L1} \partial P_{L2}} \geq 0 \text{ and } \frac{\partial \mathrm{D_{H1}}}{\partial P_{H1}} \leq 0, \frac{\partial \mathrm{D_{H1}}}{\partial P_{H2}} \geq 0, \frac{\partial \mathrm{D_{H1}}}{\partial P_{H1} \partial P_{H2}} \geq 0$$

Additive Model

$$\Pi_{Li} = P_{Li} E_{\xi_{Li}} [\min\{S_{Li}, B_i\}]$$
$$= P_{Li} E_{\xi_{Li}} [S_{Li}] - P_{Li} E_{\xi_{Li}} [S_{Li} - B_i]^{+}$$
$$= P_{Li} B_i - P_{Li} \int_{\underline{\xi}_{Li}}^{B_i - D_{Li}} \Phi_{Li}(\xi_{Li})\, d\xi_{Li}$$

$$\Pi_{Hi} = E_{\xi_{Li}} E_{\xi_{Hi}} [P_{Hi} \min\{S_{Hi}, C_i - \min\{S_{Li}, B_i\}\}]$$
$$= P_{Hi} \left(C_i - B_i + \int_{\underline{\xi}_{Li}}^{B_i - D_{Li}} \Phi_{Li}(\xi_{Li})\, d\xi_{Li} - \int_{\underline{\xi}_{Hi}}^{y_i} \Phi_{Hi}(\xi_{Hi})\, d\xi_{Hi} \right)$$

Where

$$y_i = C_i + \int_{\underline{\xi}_{Li}}^{B_i - D_{Li}} \Phi_{Li}(\xi_{Li})\, d\xi_{Li} - D_{Hi} - B_i$$

$$\Pi_i = P_{Hi} C_i - (P_{Hi} - P_{Li}) B_i + (P_{Hi} - P_{Li}) \int_{\underline{\xi}_{Li}}^{B_i - D_{Li}} \Phi_{Li}(\xi_{Li})\, d\xi_{Li}$$
$$- P_{Hi} \int_{\underline{\xi}_{Hi}}^{y_i} \Phi_{Hi}(\xi_{Hi})\, d\xi_{Hi}$$

Multiplicative Model

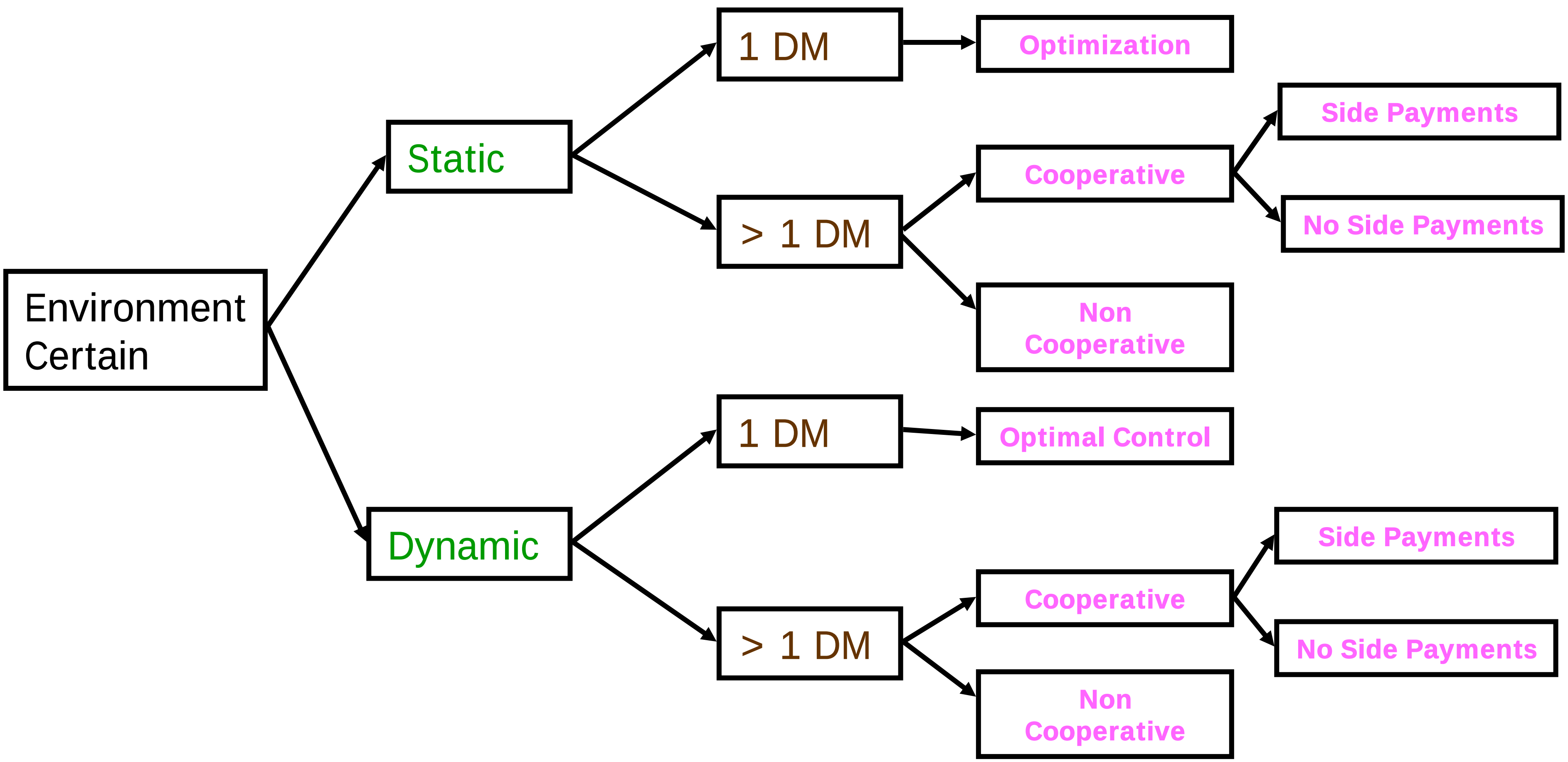
$$\Pi_i = P_{Hi}C_i - \left(P_{Hi} - P_{Li}\right)B_i + \left(P_{Hi} - P_{Li}\right)D_{Li} \int_0^{B_i / D_{Li}} \Phi_{Li}(\xi_{Li})\, d\xi_{Li}$$

$$- P_{Hi}\, D_{Hi} \int_0^{y_i / D_{Hi}} \Phi_{Hi}(\xi_{Hi})\, d\xi_{Hi}$$

Where

$$y_i = C_i - B_i + D_{Li} \int_0^{B_i / D_{Li}} \Phi_{Li}(\xi_{Li})\, d\xi_{Li}$$

Game Theory


Environment Certain
Static
Dynamic
1 DM
> 1 DM
Optimization
Cooperative
Non Cooperative
Side Payments
No Side Payments
1 DM
> 1 DM
Optimal Control
Cooperative
Non Cooperative
Side Payments
No Side Payments

Competition Analysis

- Non-cooperative analysis
  - Uniqueness of Nash Equilibrium (NE)
  - Numerical analysis with a Statistical Design Of Experiment (s) (DOE).
- Cooperative analysis
  - Bargaining game with No Side Payment options (NSP)
  - Bargaining game with Side Payment options (SP)

Nash Equilibrium

A pair of strategies (x_1^N, x_2^N) is said to constitute a Nash Equilibrium (NE) if the following pair of inequalities are satisfied for all $x_1 \in X_1$ and for all $x_2 \in X_2$:

$$f_1(x_1^N, x_2^N) \geq f_1(x_1, x_2^N) \text{ and } f_2(x_1^N, x_2^N) \geq f_2(x_1^N, x_2)$$

Where X_1 and X_2 represent the strategy sets of player 1 and player 2 respectively.

For a booking limit pre-commitment

$$\left|\frac{\partial^2 \Pi_{L1}}{\partial P_{L1}{}^2}\right| \geq \left|\frac{\partial^2 \Pi_{L1}}{\partial P_{L1} \partial P_{L2}}\right| \Leftrightarrow \text{Unique Nash equilibrium}$$

$$\left|\frac{\partial^2 \Pi_{H1}}{\partial P_{H1}{}^2}\right| \geq \left|\frac{\partial^2 \Pi_{H1}}{\partial P_{H1} \partial P_{H2}}\right| \Leftrightarrow \text{Unique Nash equilibrium}$$

Nash Bargain Solution

No Side Payment Option

$$\text{Max}\left(\Pi_1 - \Pi_1^{NE}\right)\left(\Pi_2 - \Pi_2^{NE}\right)$$

Subject to

$$\Pi_1 \geq \Pi_1^{NE}$$

$$\Pi_2 \geq \Pi_2^{NE}$$

With Side Payments Option

$$\text{Max } \Pi_1 + \Pi_2$$

Subject to

$$\Pi_1 \geq \Pi_1^{NE}$$

$$\Pi_2 \geq \Pi_2^{NE}$$

Side Payments

$$\text{SP} = \frac{1}{2}\left(\Pi_1^{FC} - \Pi_1^{NE} + \Pi_2^{NE} - \Pi_2^{FC}\right)$$

Numerical Analysis

$$C_i = 100, \underline{P}_{Li} = 0, \overline{P}_{Li} = 100, \underline{P}_{Hi} = 100, \overline{P}_{Hi} = 200, \forall i = \{1,2\}$$

$$\alpha_{Li} = 60, \alpha_{Hi} = 40, \beta_{Li} = 0.25, \beta_{Hi} = 0.15, \forall i = \{1,2\}$$

$$\theta_{Lij} = 0.15, \theta_{Hij} = 0.10, \forall i, j = \{1,2\}, i \neq j$$

For Additive Model

$$[\underline{\xi}_{Li}, \overline{\xi}_{Li}] = [\underline{\xi}_{Hi}, \overline{\xi}_{Hi}] = [-30, 30], \forall i = \{1,2\}$$

For Multiplicative Model

$$[\underline{\xi}_{Li}, \overline{\xi}_{Li}] = [\underline{\xi}_{Hi}, \overline{\xi}_{Hi}] = [0, 2], \forall i = \{1,2\}$$

Non-Cooperative Analysis (Symmetric market)

Model	Distribution	Airline	Booking Limit	Low Fare Price	High Fare Price	Payoff
Additive	Uniform	Airline 1	72.35	176.53	205.18	13570.21
		Airline 2	72.35	176.53	205.18	13570.21
	Normal	Airline 1	72.89	171.47	200.04	13349.00
		Airline 2	72.90	171.47	200.04	13349.00
Multiplicative	Uniform	Airline 1	84.90	175.50	208.32	13608.25
		Airline 2	84.90	175.50	208.32	13608.25
	Normal	Airline 1	85.91	171.56	200.26	13355.13
		Airline 2	85.88	171.56	200.26	13355.11

Non-Cooperative Analysis (Asymmetric market)

Parameters	Levels
C_2	[80 (-), 120 (+)]
β_{L2}	[0.2 (-), 0.3 (+)]
β_{H2}	[0.1(-), 0.2 (+)]
❑$_{L21}$	[0.1(-), 0.2 (+)]
❑$_{H21}$	[0.05 (-), 0.15 (+)]
Model Type (I)	[Additive (-), Multiplicative (+)]

DOE-Non Cooperative Game

$$\hat{\Pi}_1 = 14861 - 598C_2 - 537\beta_{L2} - 1337\beta_{H2} + 867\theta_{H21} - 922\beta_{H2}\theta_{H21}$$

$$\hat{\Pi}_2 = 15234 - 1182\beta_{L2} - 3080\beta_{H2} + 1182\theta_{L21} + 2821\theta_{H21} - 1724\beta_{H2}\theta_{H21}$$

$$\hat{B}_1 = 76.061 - 1.329\beta_{L2} + 1.019\beta_{H2} + 5.172\mathrm{I} + 0.858\beta_{H2}\theta_{H21}$$

$$\hat{B}_2 = 69.998 + 10.033C_2 + 9.677\beta_{H2}\theta_{H21} - 7.946\beta_{H2}\mathrm{I} + 7.605\theta_{H21}\mathrm{I}$$

$$\hat{P}_{L1} = 178.343 - 6.050C_2 - 7.156\beta_{L2} - 6.256\beta_{H2} + 4.449\theta_{L21} + 4.882\theta_{H21}$$

$$\hat{P}_{L2} = 169 - 13.63C_2 - 18.33\beta_{L2} + 10.83\theta_{L21}$$

$$\hat{P}_{H1} = 235.63 - 6.83C_2 - 23.59\beta_{H2} + 13.79\theta_{H21} + 7.99\mathrm{I} - 18.05\beta_{H2}\theta_{H21}$$

$$\hat{P}_{H2} = 273.53 - 12.85C_2 - 54.36\beta_{H2} + 30.74\theta_{H21} - 40.48\beta_{H2}\theta_{H21}$$

Cooperative Analysis (Bargaining Game)

Additive

	Airline 1				Airline 2			
	B_1	P_{L1}	P_{H1}	Payoff	B_2	P_{L2}	P_{H2}	Payoff
Non-cooperative	72.35	176.53	205.18	13570.21	72.35	176.53	205.18	13570.21
NBS no side payments	54.05	169.57	255.81	13570.21	55.11	156.04	259.43	13570.21
NBS with side payments	70	200	368.28	15732.42	70.00	200	368.28	15732.42
Gain Of Cooperation	2162.2				2162.2			

Cooperative Analysis (Bargaining Game)

Multiplicative

	Airline 1				Airline 2			
	B_1	P_{L1}	P_{H1}	Payoff	B_2	P_{L2}	P_{H2}	Payoff
Non-cooperative	84.90	175.50	208.32	13608.25	84.90	175.50	208.32	13608.25
NBS no side payments	80	200	310.102	15212.24	80.00	200.00	310.10	15212.24
NBS with side payments	80	200	310.102	15212.24	80.00	200.00	310.10	15212.24
Gain Of Cooperation	1604.00				1604.00			

Comparing NSP with NE

Airline 1

- Revenue improves about 5.4%
- 1.9% more seats for High fare customers
- Low and high fare prices increase by 4.8% and 35% respectively

Airline 2

- Revenue improves 6.5%
- 5.3% more seats for high fare class
- Low and high fare prices increase is 9.6% and 28.4%

Comparing SP with NSP

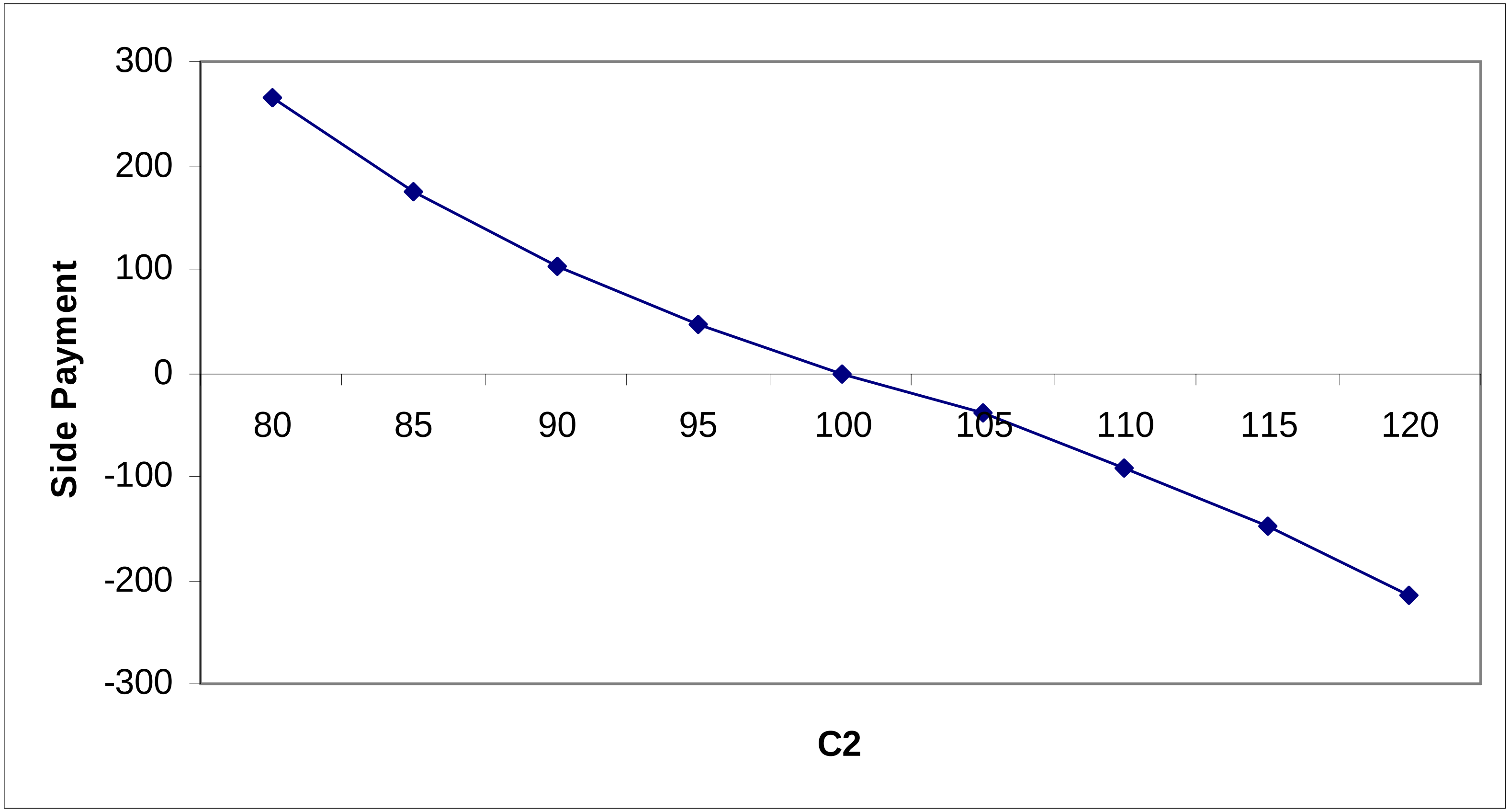
Airline 1

- Revenue improves about 6%
- 1.9% more seats for High fare customers
- Low and high fare prices increase by 6.7% and 8.6% respectively

Airline 2

- Revenue improves 5.2%
- 1.6% more seat to high fare class but not significant.
- Low and high fare prices increase is 6.3% and 5.8% respectively

Side Payments



Newsvendor Problem and RM

- Yao (2002)' s thesis investigates newsvendor problem (NVP) with pricing in the context of SCM and reported that the elegant structure of the problem also finds applications in RM.
- In standard newsvendor problem optimal quantity is determined for a single perishable asset with stochastic demand having an objective to maximize the revenue

Proposed Extension to NVP

- Investigate alternative solution approach to NVP with pricing using Distribution Free Approach.
- Using Distribution Free approach study several extensions to NVP with application in RM

Distribution Based Vs. Distribution Free Approach

- Distribution Based
  - Modeling requires precise information about demand distribution
  - It maximizes the revenue under a known distribution

- Distribution Free
  - Modeling requires only the mean and standard error estimate of the demand.
  - It maximizes the revenue under the worst possible demand distribution.

Newsvendor problem with Pricing

Additive Model:

$RD = D + \xi$

$$\Pi(P,Q) = E_{\xi}[P\min\{Q, RD\} - CQ]$$

$$= (P - C)Q - P\,E_{\xi}[Q - RD]^{+}$$

Where

$$E_{\xi}[Q - RD]^{+} \le \frac{\left(\sigma^2 + (D-Q)^2\right)^{1/2} - (D-Q)}{2}$$

$$\Pi_{LB}(P_{LB}^{*}, Q_{LB}^{*}) = (P_{LB} - C)Q_{LB} - P_{LB}\frac{\left(\sigma^2 + (D-Q_{LB})^2\right)^{1/2} - (D-Q_{LB})}{2}$$

Newsvendor problem with Pricing

Multiplicative Model:

$$RD = D\xi$$

$$\Pi(P,Q) = E_{\xi}[P\min\{Q, RD\} - CQ]$$

$$= (P-C)Q - PDE_{\xi}[Q/D-\xi]^{+}$$

Where

$$E_{\xi}[Q/D-\xi]^{+} \le \frac{\left(\sigma^2 + (1-Q/D)^2\right)^{1/2} - (1-Q/D)}{2}$$

$$\Pi_{LB}(P_{LB}^{*}, Q_{LB}^{*}) = (P_{LB}-C)Q_{LB} - P_{LB}D\frac{\left(\sigma^2 + (1-Q/D)^2\right)^{1/2} - (1-Q/D)}{2}$$

Proving concavity

$$\Pi_{\text{LB}} \text{ is quasi} - \text{concave if,}$$

$$i)\,\frac{\partial^2 \Pi_{\text{LB}}}{\partial P_{LB}^2} \le 0, \frac{\partial^2 \Pi_{\text{LB}}}{\partial Q_{LB}^2} \le 0$$

$$ii)\,\left|H\right| = \frac{\partial^2 \Pi_{\text{LB}}}{\partial P_{LB}^2}\frac{\partial^2 \Pi_{\text{LB}}}{\partial Q_{LB}^2} - \frac{\partial^2 \Pi_{\text{LB}}}{\partial P_{LB} \partial Q_{LB}} \ge 0$$

For Additive model

$$\Pi_{\text{LB}}\ is\ quasi - concave;\ if\ i)\,Q_{LB} \ge D,\ ii)\,P_{LB} \ge -\frac{\sigma}{4D'}$$

For Multiplicative model

$$\Pi_{\text{LB}}\ is\ quasi - concave;\ if\ i)\,D \le Q_{LB} \le 2D,\ ii)\,P_{LB} \ge \frac{D\sigma}{2D'(\sigma - 2)}$$

Extension to Holding and Shortage cost case:

$$\Pi(P,Q) = E_{\xi}[P \min\{Q, RD\} - CQ - G[Q - RD]^{+} - S[RD - D]^{+}]$$

$$= (P + S - C)Q - \left(P + S + G\right)E_{\xi}[Q - RD]^{+} - SD$$

$$\Pi_{LB}(P_{LB}^{*}, Q_{LB}^{*}) = \left(P_{LB} + S - C\right)Q_{LB} - SD$$

$$-\left(P_{LB} + G + S\right)\frac{\left(\sigma^{2} + \left(D - Q_{LB}\right)^{2}\right)^{1/2} - (D - Q_{LB})}{2}$$

Proving concavity

For Additive model

$$\Pi_{\mathrm{LB}}\ is\ quasi-concave;\ if\ i)\ Q_{LB} \geq D,\ ii)\ P_{LB} \geq (G+S) - \frac{\sigma}{4D'}$$

For Multiplicative model

$$\Pi_{\mathrm{LB}}\ is\ quasi-concave;\ if\ i)\ D \leq Q_{LB} \leq 2D,\ ii)\ P_{LB} \geq \frac{D\sigma}{2D'(\sigma-2)} - (G+S)$$

Extension to Shortage cost case:

Additive Model:

$$RD = D + \xi$$

$$\Pi(P,Q) = E_{\xi}[P\min\{Q, RD\} - C\,Q - S\,[RD - D]^{+}]$$

$$= (P + S - C)\,Q - (P + S)\,E_{\xi}[Q - RD]^{+} - S\,D$$

$$\Pi_{LB}(P_{LB}^{*}, Q_{LB}^{*}) = (P_{LB} + S - C)\,Q_{LB} - S\,D$$

$$- (P_{LB} + S)\frac{\left(\sigma^{2} + (D - Q_{LB})^{2}\right)^{1/2} - (D - Q_{LB})}{2}$$

$$\Pi_{\mathrm{LB}}\ is\ quasi-concave;\ if\ i)\ Q_{LB} \geq D,\ ii)\ P_{LB} \geq S - \frac{\sigma}{4\,D'}$$

Recourse case:

$$\text{Additive Model:}$$

$$RD = D + \xi$$

$$\Pi(P,Q) = (P - C + C_r)Q - \left(P + C_r\right)E_\xi[Q - RD]^+ - C_r D$$

$$\Pi_{LB}(P_{LB}^*, Q_{LB}^*) = \left(P_{LB} + C_r - C\right)Q_{LB} - C_r D$$

$$- \left(P_{LB} + C_r\right)\frac{\left(\sigma^2 + \left(D - Q_{LB}\right)^2\right)^{1/2} - (D - Q_{LB})}{2}$$

$$\Pi_{\text{LB}}\ is\ quasi-concave;\ if\ i)\,Q_{LB} \geq D,\ ii)\,P_{LB} \geq -\frac{\sigma}{4D'}\ iii)\,C_r < P_{LB}$$

Random yield case:

$$\text{Additive Model}:$$

$$RD = D + \xi$$

$$\Pi(P,Q) = E_{\xi}[P\min\{G(Q), RD\} - C\,Q]$$

$$= (\rho P - C)Q - P\,E_{\xi}[G(Q) - RD]^{+}$$

$$\Pi_{LB}(P_{LB}^{*}, Q_{LB}^{*}) = \left(\rho P_{LB} - C\right)Q_{LB}$$

$$- P_{LB}\frac{\left(\sigma^{2} + \rho Q_{LB}(1-\rho) + \left(\rho Q_{LB} - D\right)^{2}\right)^{1/2} + (\rho Q_{LB} - D)}{2}$$

$$i)\ \frac{\partial^{2}\Pi_{\mathrm{LB}}}{\partial P_{LB}^{2}} \le 0\ if; \sigma \ge 1/2$$

$$ii)\ \frac{\partial^{2}\Pi_{\mathrm{LB}}}{\partial Q_{LB}^{2}} \le 0\ if; Q_{LB} \ge \frac{D}{\rho}, D\ \text{is IPE}$$

Multiple Product case:

$$\Pi(P,Q)=\sum_{i=1}^{n}E_{\xi_i}[P_i\min\{Q_i,RD_i\}-C_i Q_i]$$

$$\text{Subject to :}$$

$$\sum_{i=1}^{n}C_i Q_i \le B$$

$$\Pi_{LB}(P_{LB}^{*},Q_{LB}^{*})=\sum_{i=1}^{n}\left(P_{LB_i}-C_i\right)Q_{LB_i}-P_{LB_i}\frac{\left(\sigma_i^2+\left(D_i-Q_{LB_i}\right)^2\right)^{1/2}-\left(D_i-Q_{LB_i}\right)}{2}$$

$$\text{Subject to :}$$

$$\sum_{\mathrm{i}=1}^{\mathrm{n}}\mathrm{C}_{\mathrm{i}}\, Q_{LB_i}\le B$$

Numerical Analysis

Experimentation with each extension follows:

100 randomly generated problems

$C \sim U[20, 100], \alpha \sim U[100, 200], \beta \sim U[0.05, 0.3]$

$S \sim U[0.1, 0.2] \times C, G \sim U[0.2, 0.3] \times C, C_r \sim U[1.2, 1.4] \times C$

$\rho \sim U[0.5, 0.9], B \sim U[10{,}000, 15{,}000], D = \alpha - \beta P$

For additive model, $[\underline{\xi}, \overline{\xi}], |\underline{\xi}| = |\overline{\xi}| \sim U(0, 30]$

For multiplicative model, $[\underline{\xi}, \overline{\xi}], |\underline{\xi}| = |\overline{\xi}| \sim U(0, 2]$

Standard Newsvendor Problem

Additive Model

Uniform
- EVAI Improves revenue 0.25%
- 13% over-stocking
- 0.85% under-pricing

Normal
- EVAI Improves revenue 0.13%
- 3% over-stocking
- 0.81% under-pricing

Multiplicative Model

Uniform
- EVAI Improves revenue 0.75%
- 0.59% under-stocking
- 0.01% over-pricing

Normal
- EVAI Improves revenue 0.60%
- 0.13% over-stocking
- 0.30% over-pricing

Extension to Shortage and Holding Cost

Additive Model

Uniform
- EVAI Improves revenue 0.20%
- 0.48% under-stocking
- 0.89% under-pricing

Normal
- EVAI Improves revenue 0.09%
- 2.3% over-stocking
- 0.85% under-pricing

Multiplicative Model

Uniform
- EVAI Improves revenue 0.48%
- 2.45% under-stocking
- 0.19% over-pricing

Normal
- EVAI Improves revenue 0.38%
- 1.68% over-stocking
- 0.30% over-pricing

Extension …

Recourse Case

Uniform

- ➢EVAI Improves revenue 0.29%
- ➢0.86% over-stocking
- ➢0.81% under-pricing

Normal

- ➢EVAI Improves revenue 0.17%
- ➢3.5% over-stocking
- ➢0.77% under-pricing

Random Yield Case

Uniform

- ➢EVAI Improves revenue 0.42%
- ➢0.53% over-stocking
- ➢0.80% under-pricing

Normal

- ➢EVAI Improves revenue 0.22%
- ➢3.27% over-stocking
- ➢0.76% under-pricing

Conclusion

- Competition models in Airline Revenue Management (RM) are proposed using game theoretic approach.
- The models enable us to jointly determine the booking limits and fare pricing for airlines in the game.
- Both non-cooperative and cooperative bargaining games are considered.

Conclusion (Contd.)

- Numerical study shows that cooperation results superior revenue gain to airlines
- Cooperation with side payments is also found superior to cooperation with no side payments option in the bargaining game.
- The use of distribution free approach for pricing in RM is explored

Conclusion (Contd.)

- The approach establishes lower bound estimate on revenue and enables maximizing the revenue under worst possible demand distribution.
- The approach is applied to standard newsvendor problem and many other extension

Conclusion (Contd.)

- It is also reported that the use of the approach on extensions of standard newsvendor problem has direct implications on RM practice in aviation and other service industries

Future Works

- Dynamic pricing with seat allocation
- Pricing with:
  - Re-saleable return
  - Customer choice model

A typo on page 82

Replace $\lambda_2^2 = \left(1 - \frac{Q_{LB}}{D}\right)^2 + \sigma^2$ by $\lambda_2 = \left(1 - \frac{Q_{LB}}{D}\right)^2 + \sigma^2$

Research Articles

- **A. S. Raza**, M. U. Al-Turki, S. Z. Selim: 2007, "Early Tardy Minimization of Joint Scheduling of Jobs and Maintenance Operations on a Single Machine" **special issue on scheduling in manufacturing, information and service industries in International Journal of Operations Research**, 4, 1-10
- **A. S. Raza**, A. Akgunduz, M.Y. Chen: 2006, "A Tabu Search Algorithm for Solving Economic Lot Scheduling Problem" **Journal of Heuristics**, 12, 413-426
- **A. S. Raza**, A. Akgunduz: 2005, "The Use of Meta-Heuristics to Solve Economic Lot Scheduling Problem", **Lecture Notes in Computer Science**, 3448, 190-201

Research Articles (Under review)

- **A. S. Raza**, A. Akgunduz, “An Airline Revenue Management Fare Pricing Game with Seat Allocation” under second revision with **International Journal of Revenue Management**
- **A. S. Raza**, A. Akgunduz, “A Comparative Study of Heuristic Algorithms on Economic Lot Scheduling Problem” submitted to **Computers and Industrial Engineering**
- **A. S. Raza**, A. Akgunduz, “A Distribution Free Approach for Jointly Controlling Price and Capacity in Revenue Management”, submitted to **International Journal of Production Economics**
- **A. S. Raza**, A. Akgunduz, “The Impact of Fare Pricing Cooperation in Airline Revenue Management”, submitted to **INFOR**

Thank You

Questions?