Optimal Pricing and Seat Allocation in Airline Industry Under Market Competition

PhD Thesis
Oral Exam Presentation
Syed Asif Raza
Outline

- Revenue Management (RM)
- Background and Literature
- Model and Results
- Conclusion and Future Works
- Questions
Revenue Management is simply the practice of maximising Revenue from Perishable assets (seats) through a combination of Pricing & Inventory controls.
Missing Connections in Airline RM Studies

- Market Competition
  - Ignored

Revenue Management

- Pricing
  - Fares
  - Services/Restrictions

- Reservation Control
  - Seat Inventory Control
  - Overbooking
    - Fare Class Mix
      - Segment
    - Itinerary Control
      - OD
      - Point-of-Sale
Review Articles:

- McGill and Van Ryzin (1999)- Revenue Management: Research overview and prospects.
- Pak and Piersma (2002)- Overview of OR techniques in airline RM.
- Silver (2004)- An overview of heuristic solution methods
- Barnhart et al. (2003) – Application of operations research in the air transport industry.
- Elmagraby and Keskinocak (2003) - Dynamic pricing in the presence of inventory considerations: Research overview, current practices, and future directions
Competition in RM

- Cote et al. (2003)- Mathematical optimization approach for integrated seat allocation and pricing in airline industry.
  - Bi-level mathematical optimization model is presented for joint determination of fare pricing and seat allocation
  - The customer demand is considered price insensitive and the model is non dynamic
- Dai et al. (2005)- Multiple firms competition in RM context.
  - Competitive prices are determined for both the deterministic and stochastic price sensitive demand for a known capacity.
  - Mostly the analysis consider duopoly competition
- Chen et al.(2005) – Newsvendor pricing game
  - Jointly determined the competitive pricing and capacity for the competing firms for a single commodity
  - Unique Nash equilibrium is determined analytically
Competition in Airline RM:

- Hotelling (1929)- Scheduling decision under competition
- Borenstein et al. (2003)- An empirical paper, focus on broad competition problem but ignore any seat allocation/pricing modeling.
- Belobaba and Wilson (1997)- Simulation model to study scheduling strategy under market competition.
- Teodorovic and Krcmar-Nozic (1989)- Multi-criteria model to determine flight frequencies on an airline network under competitive conditions.
Objectives of thesis

- Develop new models for joint determination of competitive fare pricing and seat allocation in airline Industry
- Develop new solution methodologies for joint determination of pricing and capacity in RM
Airline RM Competition Models

- Airline RM game is developed mainly in duopoly market
- Two game theoretic models are presented
- The models are studied for both the cooperative and non-cooperative game settings.
- Numerical analysis with statistical DOE
Distribution Free Approach is RM

- An alternative approach for joint determination of pricing and capacity under monopoly
- Determines lower bound (worst possible) revenue estimate
- The approach is addressed to:
  - Standard Newsvendor problem
  - Extension to Shortage cost Penalty
  - Extension to Shortage and Holding cost
  - Extension to recourse case
  - Extension random yield
  - Extension to multiple items
Competition Models in Airline RM:

\[ \Pi_i = P_{Li} \min\{S_{Li}, B_i\} + P_{Hi} \min\{S_{Hi}, C_i - \min\{S_{Li}, B_i\}\}, i = \{1, 2\} \]
### Modeling Approaches

<table>
<thead>
<tr>
<th>Additive Model</th>
<th>Multiplicative Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{Li} = D_{Li} + \xi_{Li}$, $\forall i = {1, 2}$</td>
<td>$S_{Li} = D_{Li} \cdot \xi_{Li}$, $\forall i = {1, 2}$</td>
</tr>
<tr>
<td>$S_{Hi} = D_{Hi} + \xi_{Hi}$, $\forall i = {1, 2}$</td>
<td>$S_{Hi} = D_{Hi} \cdot \xi_{Hi}$, $\forall i = {1, 2}$</td>
</tr>
<tr>
<td>Where</td>
<td>Where</td>
</tr>
<tr>
<td>$E[\xi_{Li}] = E[\xi_{Hi}] = 0$</td>
<td>$E[\xi_{Li}] = E[\xi_{Hi}] = 1$</td>
</tr>
<tr>
<td>$\xi_{Li} \in [\underline{\xi}<em>{Li}, \overline{\xi}</em>{Li}]$</td>
<td>$\xi_{Li} \in [\underline{\xi}<em>{Li}, \overline{\xi}</em>{Li}]$</td>
</tr>
<tr>
<td>$\xi_{Hi} \in [\underline{\xi}<em>{Hi}, \overline{\xi}</em>{Hi}]$</td>
<td>$\xi_{Hi} \in [\underline{\xi}<em>{Hi}, \overline{\xi}</em>{Hi}]$</td>
</tr>
</tbody>
</table>

Both $\xi_{Li}$ and $\xi_{Hi}$ follow Increasing Generalized Failure Rate (IGFR) A Uniform Distribution is assumed
Assumptions

The Demand is modeled with linear price sensitive determinisitic demand functions

\[ D_{Li} = \alpha_{Li} - \beta_{Li} P_{Li} + \theta_{Lij} P_{Lj}, \quad D_{Hi} = \alpha_{Hi} - \beta_{Hi} P_{Hi} + \theta_{Hij} P_{Hj} \]

\[ \beta_{Li} > \theta_{Lij}, \beta_{Li}, \theta_{Lij} > 0, i \neq j, i, j = \{1,2\}, \]

\[ \beta_{Hi} > \theta_{Hij}, \beta_{Hi}, \theta_{Hij} > 0, i \neq j, i, j = \{1,2\}, \]

It is also assumed that

\[ \frac{\partial D_{L1}}{\partial P_{L1}} \leq 0, \quad \frac{\partial D_{L1}}{\partial P_{L2}} \geq 0, \quad \frac{\partial D_{L1}}{\partial P_{L1}} \geq 0 \quad \text{and} \quad \frac{\partial D_{H1}}{\partial P_{H1}} \leq 0, \quad \frac{\partial D_{H1}}{\partial P_{H2}} \geq 0, \quad \frac{\partial D_{H1}}{\partial P_{H1}} \geq 0 \]
Additive Model

\[ \Pi_{Li} = P_{Li} E_{\xi_{Li}} \left[ \min \{ S_{Li}, B_i \} \right] \]
\[ = P_{Li} E_{\xi_{Li}} \left[ S_{Li} \right] - P_{Li} E_{\xi_{Li}} \left[ S_{Li} - B_i \right]^+ \]
\[ = P_{Li} B_i - P_{Li} \int_{\xi_{Li}}^{B_i-D_{Li}} \Phi_{Li}(\xi_{Li}) d\xi_{Li} \]

\[ \Pi_{Hi} = E_{\xi_{Hi}} E_{\xi_{Li}} \left[ P_{Hi} \min \{ S_{Hi}, C_i - \min \{ S_{Li}, B_i \} \} \right] \]
\[ = P_{Hi} \left( C_i - B_i + \int_{\xi_{Li}}^{B_i-D_{Li}} \Phi_{Li}(\xi_{Li}) d\xi_{Li} - \int_{\xi_{Hi}}^{\xi_{Li}} \Phi_{Hi}(\xi_{Hi}) d\xi_{Hi} \right) \]

Where

\[ y_i = C_i + \int_{\xi_{Li}}^{B_i-D_{Li}} \Phi_{Li}(\xi_{Li}) d\xi_{Li} - D_{Hi} - B_i \]

\[ \Pi_i = P_{Hi} C_i - (P_{Hi} - P_{Li}) B_i + (P_{Hi} - P_{Li}) \int_{\xi_{Li}}^{B_i-D_{Li}} \Phi_{Li}(\xi_{Li}) d\xi_{Li} \]
\[ - P_{Hi} \int_{\xi_{Hi}}^{y_i} \Phi_{Hi}(\xi_{Hi}) d\xi_{Hi} \]
Multiplicative Model

\[ \Pi_i = \left( P_{Hi} - P_{Li} \right) B_i + \left( P_{Hi} - P_{Li} \right) D_{Li} \int_0^{B_i/D_{Li}} \Phi_{Li}(\xi_{Li}) \, d\xi_{Li} \]

\[ -P_{Hi} D_{Hi} \int_0^{y_i/D_{Hi}} \Phi_{Hi}(\xi_{Hi}) \, d\xi_{Hi} \]

Where

\[ y_i = C_i - B_i + D_{Li} \int_0^{B_i/D_{Li}} \Phi_{Li}(\xi_{Li}) \, d\xi_{Li} \]
Competition Analysis

- **Non-cooperative analysis**
  - Uniqueness of Nash Equilibrium (NE)
  - Numerical analysis with a Statistical Design Of Experiment (s) (DOE).

- **Cooperative analysis**
  - Bargaining game with No Side Payment options (NSP)
  - Bargaining game with Side Payment options (SP)
Nash Equilibrium

A pair of strategies \((x_1^N, x_2^N)\) is said to constitute a Nash Equilibrium (NE) if the following pair of inequalities are satisfied for all \(x_1 \in X_1\) and for all \(x_2 \in X_2\):

\[
f_1(x_1^N, x_2^N) \geq f_1(x_1, x_2^N) \quad \text{and} \quad f_2(x_1^N, x_2^N) \geq f_2(x_1^N, x_2)
\]

Where \(X_1\) and \(X_2\) represent the strategy sets of player 1 and player 2 respectively.

For a booking limit pre-commitment

\[
\left| \frac{\partial^2 \Pi_{L1}}{\partial P_{L1}^2} \right| \geq \left| \frac{\partial^2 \Pi_{L1}}{\partial P_{L1} \partial P_{L2}} \right| \iff \text{Unique Nash equilibrium}
\]

\[
\left| \frac{\partial^2 \Pi_{H1}}{\partial P_{H1}^2} \right| \geq \left| \frac{\partial^2 \Pi_{H1}}{\partial P_{H1} \partial P_{H2}} \right| \iff \text{Unique Nash equilibrium}
\]
Nash Bargain Solution

No Side Payment Option
Max \( \Pi_1 - \Pi_1^{NE} \)(\( \Pi_2 - \Pi_2^{NE} \))
Subject to
\[ \Pi_1 \geq \Pi_1^{NE} \]
\[ \Pi_2 \geq \Pi_2^{NE} \]

With Side Payments Option
Max \( \Pi_1 + \Pi_2 \)
Subject to
\[ \Pi_1 \geq \Pi_1^{NE} \]
\[ \Pi_2 \geq \Pi_2^{NE} \]
Side Payments
\[ SP = \frac{1}{2} \left( \Pi_1^{FC} - \Pi_1^{NE} + \Pi_2^{NE} - \Pi_2^{FC} \right) \]
Numerical Analysis

\[ C_i = 100, P_{Li} = 0, P_{Li} = 100, P_{Hi} = 100, P_{Hi} = 200, \forall i = \{1, 2\} \]

\[ \alpha_{Li} = 60, \alpha_{Hi} = 40, \beta_{Li} = 0.25, \beta_{Hi} = 0.15, \forall i = \{1, 2\} \]

\[ \theta_{Lij} = 0.15, \theta_{Hij} = 0.10, \forall i, j = \{1, 2\}, i \neq j \]

For Additive Model

\[ [\underline{\xi}_{Li}, \bar{\xi}_{Li}] = [\underline{\xi}_{Hi}, \bar{\xi}_{Hi}] = [-30, 30], \forall i = \{1, 2\} \]

For Multiplicative Model

\[ [\underline{\xi}_{Li}, \bar{\xi}_{Li}] = [\underline{\xi}_{Hi}, \bar{\xi}_{Hi}] = [0, 2], \forall i = \{1, 2\} \]
### Non-Cooperative Analysis (Symmetric market)

<table>
<thead>
<tr>
<th>Model</th>
<th>Distribution</th>
<th>Airline</th>
<th>Booking Limit</th>
<th>Low Fare Price</th>
<th>High Fare Price</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Additive</td>
<td>Uniform</td>
<td>Airline 1</td>
<td>72.35</td>
<td>176.53</td>
<td>205.18</td>
<td>13570.21</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Airline 2</td>
<td>72.35</td>
<td>176.53</td>
<td>205.18</td>
<td>13570.21</td>
</tr>
<tr>
<td></td>
<td>Normal</td>
<td>Airline 1</td>
<td>72.89</td>
<td>171.47</td>
<td>200.04</td>
<td>13349.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Airline 2</td>
<td>72.90</td>
<td>171.47</td>
<td>200.04</td>
<td>13349.00</td>
</tr>
<tr>
<td>Multiplicative</td>
<td>Uniform</td>
<td>Airline 1</td>
<td>84.90</td>
<td>175.50</td>
<td>208.32</td>
<td>13608.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Airline 2</td>
<td>84.90</td>
<td>175.50</td>
<td>208.32</td>
<td>13608.25</td>
</tr>
<tr>
<td></td>
<td>Normal</td>
<td>Airline 1</td>
<td>85.91</td>
<td>171.56</td>
<td>200.26</td>
<td>13355.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Airline 2</td>
<td>85.88</td>
<td>171.56</td>
<td>200.26</td>
<td>13355.11</td>
</tr>
</tbody>
</table>
Non-Cooperative Analysis (Asymmetric market)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_2 )</td>
<td>[80 (-), 120 (+)]</td>
</tr>
<tr>
<td>( \beta_{L2} )</td>
<td>[0.2 (-), 0.3 (+)]</td>
</tr>
<tr>
<td>( \beta_{H2} )</td>
<td>[0.1(-), 0.2 (+)]</td>
</tr>
<tr>
<td>( \Omega_{L21} )</td>
<td>[0.1(-), 0.2 (+)]</td>
</tr>
<tr>
<td>( \Omega_{H21} )</td>
<td>[0.05 (-), 0.15 (+)]</td>
</tr>
<tr>
<td>Model Type (I)</td>
<td>[Additive (-), Multiplicative (+)]</td>
</tr>
</tbody>
</table>
DOE-Non Cooperative Game

\[ \hat{\Pi}_1 = 14861 - 598 C_2 - 537 \beta_{L2} - 1337 \beta_{H2} + 867 \theta_{H21} - 922 \beta_{H2} \theta_{H21} \]

\[ \hat{\Pi}_2 = 15234 - 1182 \beta_{L2} - 3080 \beta_{H2} + 1182 \theta_{L21} + 2821 \theta_{H21} - 1724 \beta_{H2} \theta_{H21} \]

\[ \hat{B}_1 = 76.061 - 1.329 \beta_{L2} + 1.019 \beta_{H2} + 5.172 I + 0.858 \beta_{H2} \theta_{H21} \]

\[ \hat{B}_2 = 69.998 + 10.033 C_2 + 9.677 \beta_{H2} \theta_{H21} - 7.946 \beta_{H2} I + 7.605 \theta_{H21} I \]

\[ \hat{P}_{L1} = 178.343 - 6.050 C_2 - 7.156 \beta_{L2} - 6.256 \beta_{H2} + 4.449 \theta_{L21} + 4.882 \theta_{H21} \]

\[ \hat{P}_{L2} = 169 - 13.63 C_2 - 18.33 \beta_{L2} + 10.83 \theta_{L21} \]

\[ \hat{P}_{H1} = 235.63 - 6.83 C_2 - 23.59 \beta_{H2} + 13.79 \theta_{H21} + 7.99 I - 18.05 \beta_{H2} \theta_{H21} \]

\[ \hat{P}_{H2} = 273.53 - 12.85 C_2 - 54.36 \beta_{H2} + 30.74 \theta_{H21} - 40.48 \beta_{H2} \theta_{H21} \]
## Cooperative Analysis (Bargaining Game)

### Additive

<table>
<thead>
<tr>
<th></th>
<th>Airline 1</th>
<th></th>
<th></th>
<th>Airline 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B_1$</td>
<td>$P_{L_1}$</td>
<td>$P_{H_1}$</td>
<td>Payoff</td>
<td>$B_2$</td>
</tr>
<tr>
<td>Non-cooperative</td>
<td>72.35</td>
<td>176.53</td>
<td>205.18</td>
<td>13570.21</td>
<td>72.35</td>
</tr>
<tr>
<td>NBS no side payments</td>
<td>54.05</td>
<td>169.57</td>
<td>255.81</td>
<td>13570.21</td>
<td>55.11</td>
</tr>
<tr>
<td>NBS with side payments</td>
<td>70</td>
<td>200</td>
<td>368.28</td>
<td>15732.42</td>
<td>70.00</td>
</tr>
<tr>
<td>Gain Of Cooperation</td>
<td>2162.2</td>
<td></td>
<td></td>
<td></td>
<td>2162.2</td>
</tr>
</tbody>
</table>
Cooperative Analysis (Bargaining Game)

**Multiplicative**

<table>
<thead>
<tr>
<th></th>
<th>Airline 1</th>
<th>Airline 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( B_1 ) ( P_{L_1} ) ( P_{H_1} ) Payoff</td>
<td>( B_2 ) ( P_{L_2} ) ( P_{H_2} ) Payoff</td>
</tr>
<tr>
<td>Non-cooperative</td>
<td>84.90 175.50 208.32 13608.25</td>
<td>84.90 175.50 208.32 13608.25</td>
</tr>
<tr>
<td>NBS no side payments</td>
<td>80 200 310.102 15212.24</td>
<td>80.00 200.00 310.10 15212.24</td>
</tr>
<tr>
<td>NBS with side payments</td>
<td>80 200 310.102 15212.24</td>
<td>80.00 200.00 310.10 15212.24</td>
</tr>
<tr>
<td>Gain Of Cooperation</td>
<td>1604.00</td>
<td>1604.00</td>
</tr>
</tbody>
</table>
## Comparing NSP with NE

<table>
<thead>
<tr>
<th>Airline 1</th>
<th>Airline 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revenue improves about 5.4%</td>
<td>Revenue improves 6.5%</td>
</tr>
<tr>
<td>1.9% more seats for High fare customers</td>
<td>5.3% more seats for high fare class</td>
</tr>
<tr>
<td>Low and high fare prices increase by 4.8% and 35% respectively</td>
<td>Low and high fare prices increase is 9.6% and 28.4%</td>
</tr>
</tbody>
</table>
Comparing SP with NSP

Airline 1
- Revenue improves about 6%
- 1.9% more seats for High fare customers
- Low and high fare prices increase by 6.7% and 8.6% respectively

Airline 2
- Revenue improves 5.2%
- 1.6% more seat to high fare class but not significant.
- Low and high fare prices increase is 6.3% and 5.8% respectively
Side Payments

**Graph:**
- X-axis: C2
- Y-axis: Side Payment
- The graph shows a decreasing trend as C2 increases.

**Data Points:**
- (80, 250)
- (85, 220)
- (90, 200)
- (95, 180)
- (100, 160)
- (105, 140)
- (110, 120)
- (115, 100)
- (120, 80)
Newsvendor Problem and RM

- Yao (2002)’s thesis investigates newsvendor problem (NVP) with pricing in the context of SCM and reported that the elegant structure of the problem also finds applications in RM.

- In standard newsvendor problem optimal quantity is determined for a single perishable asset with stochastic demand having an objective to maximize the revenue.
Proposed Extension to NVP

- Investigate alternative solution approach to NVP with pricing using Distribution Free Approach.
- Using Distribution Free approach study several extensions to NVP with application in RM
Distribution Based Vs. Distribution Free Approach

**Distribution Based**
- Modeling requires precise information about demand distribution.
- It maximizes the revenue under a known demand distribution.

**Distribution Free**
- Modeling requires only the mean and standard error estimate of the demand.
- It maximizes the revenue under the worst possible demand distribution.
Newsvendor problem with Pricing

Additive Model:

\[ RD = D + \xi \]

\[ \Pi(P, Q) = E_\xi[P \min\{Q, RD\} - CQ] \]

\[ = (P - C)Q - P E_\xi[Q - RD]^+ \]

Where

\[ E_\xi[Q - RD]^+ \leq \frac{(\sigma^2 + (D - Q)^2)^{1/2} - (D - Q)}{2} \]

\[ \Pi_{LB}(P_{LB}^*, Q_{LB}^*) = (P_{LB} - C)Q_{LB} - P_{LB} \left(\frac{\sigma^2 + (D - Q_{LB})^2}{2}\right)^{1/2} - (D - Q_{LB}) \]
Newsvendor problem with Pricing

Multiplicative Model:

\[ RD = D \xi \]

\[ \Pi(P, Q) = E_\xi[P \min\{Q, RD\} - C \cdot Q] \]

\[ = (P - C) \cdot Q - P \cdot D \cdot E_\xi[Q / D - \xi]^+ \]

Where

\[ E_\xi[Q / D - \xi]^+ \leq \frac{\left(\sigma^2 + \left(1 - \frac{Q}{D}\right)^2\right)^{1/2} - \left(1 - \frac{Q}{D}\right)}{2} \]

\[ \Pi_{LB}(P_{LB}^*, Q_{LB}^*) = (P_{LB} - C) \cdot Q_{LB} - P_{LB} \cdot D \cdot \frac{\left(\sigma^2 + \left(1 - \frac{Q}{D}\right)^2\right)^{1/2} - \left(1 - \frac{Q}{D}\right)}{2} \]
Proving concavity

$\Pi_{LB}$ is quasi – concave if,

\[ i) \frac{\partial^2 \Pi_{LB}}{\partial P_{LB}^2} \leq 0, \frac{\partial^2 \Pi_{LB}}{\partial Q_{LB}^2} \leq 0 \]

\[ ii) \left| H \right| = \frac{\partial^2 \Pi_{LB}}{\partial P_{LB}^2} \frac{\partial^2 \Pi_{LB}}{\partial Q_{LB}^2} - \frac{\partial^2 \Pi_{LB}}{\partial P_{LB} \partial Q_{LB}} \geq 0 \]

For Additive model

$\Pi_{LB}$ is quasi – concave; if \( i) Q_{LB} \geq D, ii) P_{LB} \geq -\frac{\sigma}{4D'} \)

For Multiplicativc model

$\Pi_{LB}$ is quasi – concave; if \( i) D \leq Q_{LB} \leq 2D, ii) P_{LB} \geq \frac{D\sigma}{2D'(\sigma - 2)} \)
Extension to Holding and Shortage cost case:

$$\Pi(P, Q) = E_\xi[P \min\{Q, RD\} - CQ - G[Q - RD]^+ - S[RD - D]^+]$$

$$= (P + S - C)Q - (P + S + G)E_\xi[Q - RD]^+ - SD$$

$$\Pi_{LB}(P_{LB}^*, Q_{LB}^*) = (P_{LB} + S - C)Q_{LB} - SD$$

$$\quad - (P_{LB} + G + S)\left(\frac{\sigma^2 + (D - Q_{LB}^2)^2}{2}\right) - (D - Q_{LB}^*)$$
Proving concavity

For Additive model

\[ \Pi_{LB} \text{ is quasi – concave; if i) } Q_{LB} \geq D, \text{ ii) } P_{LB} \geq (G + S) - \frac{\sigma}{4D'} \]

For Multiplicative model

\[ \Pi_{LB} \text{ is quasi – concave; if i) } D \leq Q_{LB} \leq 2D, \text{ ii) } P_{LB} \geq \frac{D\sigma}{2D'(\sigma - 2)} - (G + S) \]
Extension to Shortage cost case:

Additive Model:

\[ RD = D + \xi \]

\[ \Pi(P, Q) = E_{\xi} \left[ P \min\{Q, RD\} - C \, Q - S [RD - D]^+ \right] \]

\[ = (P + S - C) \, Q - \left( P + S \right) E_{\xi} [Q - RD]^+ - S \, D \]

\[ \Pi_{LB}(P_{LB}^*, Q_{LB}^*) = \left( P_{LB} + S - C \right) Q_{LB} - S \, D \]

\[ - \left( P_{LB} + S \right) \left( \frac{\sigma^2 + (D - Q_{LB})^2}{2} \right)^{1/2} - (D - Q_{LB}) \]

\[ \Pi_{LB} \text{ is quasi-concave; if } i) \, Q_{LB} \geq D, \, ii) \, P_{LB} \geq S - \frac{\sigma}{4 \, D} \]
Recourse case:

Additive Model:

\[ RD = D + \xi \]

\[ \Pi(P, Q) = (P - C + C_r)Q - (P + C_r)E_\xi[Q - RD]^+ - C_r D \]

\[ \Pi_{LB}(P^*_LB, Q^*_LB) = (P_{LB} + C_r - C)Q_{LB} - C_r D \]

\[-(P_{LB} + C_r)\left( \sigma^2 + (D - Q_{LB})^2 \right)^{1/2} - (D - Q_{LB}) \]

\[ \Pi_{LB} \text{ is quasi-concave; if } i) Q_{LB} \geq D, \ ii) P_{LB} \geq -\frac{\sigma}{4D'}, \ iii) C_r < P_{LB} \]
Random yield case:

Additive Model:

\[ RD = D + \xi \]

\[ \Pi(P, Q) = E_{\xi}[P \min\{G(Q), RD\} - C Q] \]

\[ = (\rho P - C) Q - P E_{\xi}[G(Q) - RD]^+ \]

\[ \Pi_{LB}(P_{LB}^*, Q_{LB}^*) = (\rho P_{LB} - C) Q_{LB} \]

\[ - P_{LB} \left( \sigma^2 + \rho Q_{LB} (1 - \rho) + (\rho Q_{LB} - D)^2 \right)^{1/2} + (\rho Q_{LB} - D) \]

i) \[ \frac{\partial^2 \Pi_{LB}}{\partial P_{LB}^2} \leq 0 \text{ if } \sigma \geq 1/2 \]

ii) \[ \frac{\partial^2 \Pi_{LB}}{\partial Q_{LB}^2} \leq 0 \text{ if } Q_{LB} \geq \frac{D}{\rho}, D \text{ is IPE} \]
Multiple Product case:

\[ \Pi(P, Q) = \sum_{i=1}^{n} E_{\xi_i} [P_i \min\{Q_i, RD_i\} - C_i Q_i] \]

Subject to:

\[ \sum_{i=1}^{n} C_i Q_i \leq B \]

\[ \Pi_{LB}(P_{LB}^*, Q_{LB}^*) = \sum_{i=1}^{n} (P_{LB_i} - C_i) Q_{LB_i} - P_{LB_i} \left( \sigma_i^2 + (D_i - Q_{LB_i})^2 \right)^{1/2} - (D_i - Q_{LB_i}) \]

Subject to:

\[ \sum_{i=1}^{n} C_i Q_{LB_i} \leq B \]
Numerical Analysis

Experimentation with each extension follows:
100 randomly generated problems

\[ C \sim U[20, 100], \alpha \sim U[100, 200], \beta \sim U[0.05, 0.3] \]
\[ S \sim U[0.1, 0.2] \times C, G \sim U[0.2, 0.3] \times C, C_r \sim U[1.2, 1.4] \times C \]
\[ \rho \sim U[0.5, 0.9], B \sim U[10,000, 15,000], D = \alpha - \beta P \]

For additive model, \( [\xi, \bar{\xi}], |\xi| = |\bar{\xi}| \sim U(0, 30) \)

For multiplicative model, \( [\xi, \bar{\xi}], |\xi| = |\bar{\xi}| \sim U(0, 2) \)
Standard Newsvendor Problem

Additive Model

- **Uniform**
  - EVAI Improves revenue 0.25%
  - 13% over-stocking
  - 0.85% under-pricing

- **Normal**
  - EVAI Improves revenue 0.13%
  - 3% over-stocking
  - 0.81% under-pricing

Multiplicative Model

- **Uniform**
  - EVAI Improves revenue 0.75%
  - 0.59% under-stocking
  - 0.01% over-pricing

- **Normal**
  - EVAI Improves revenue 0.60%
  - 0.13% over-stocking
  - 0.30% over-pricing
Extension to Shortage and Holding Cost

Additive Model
- **Uniform**
  - EVAI Improves revenue 0.20%
  - 0.48% under-stocking
  - 0.89% under-pricing

- **Normal**
  - EVAI Improves revenue 0.09%
  - 2.3% over-stocking
  - 0.85% under-pricing

Multiplicative Model
- **Uniform**
  - EVAI Improves revenue 0.48%
  - 2.45% under-stocking
  - 0.19% over-pricing

- **Normal**
  - EVAI Improves revenue 0.38%
  - 1.68% over-stocking
  - 0.30% over-pricing
**Extension ...**

**Recourse Case**

- **Uniform**
  - EVAI Improves revenue 0.29%
  - 0.86% over-stocking
  - 0.81% under-pricing

- **Normal**
  - EVAI Improves revenue 0.17%
  - 3.5% over-stocking
  - 0.77% under-pricing

**Random Yield Case**

- **Uniform**
  - EVAI Improves revenue 0.42%
  - 0.53% over-stocking
  - 0.80% under-pricing

- **Normal**
  - EVAI Improves revenue 0.22%
  - 3.27% over-stocking
  - 0.76% under-pricing
Conclusion

- Competition models in Airline Revenue Management (RM) are proposed using game theoretic approach.
- The models enable us to jointly determine the booking limits and fare pricing for airlines in the game.
- Both non-cooperative and cooperative bargaining games are considered.
Conclusion (Contd.)

- Numerical study shows that cooperation results superior revenue gain to airlines
- Cooperation with side payments is also found superior to cooperation with no side payments option in the bargaining game.
- The use of distribution free approach for pricing in RM is explored
Conclusion (Contd.)

- The approach establishes lower bound estimate on revenue and enables maximizing the revenue under worst possible demand distribution.
- The approach is applied to standard newsvendor problem and many other extension
Conclusion (Contd.)

- It is also reported that the use of the approach on extensions of standard newsvendor problem has direct implications on RM practice in aviation and other service industries
Future Works

- Dynamic pricing with seat allocation
- Pricing with:
  - Re-saleable return
  - Customer choice model
A typo on page 82

Replace $\lambda_2^2 = \left(1 - \frac{Q_{LB}}{D}\right)^2 + \sigma^2$ by $\lambda_2 = \left(1 - \frac{Q_{LB}}{D}\right)^2 + \sigma^2$


A. S. Raza, A. Akgunduz, “An Airline Revenue Management Fare Pricing Game with Seat Allocation” under second revision with *International Journal of Revenue Management*


A. S. Raza, A. Akgunduz, “The Impact of Fare Pricing Cooperation in Airline Revenue Management”, submitted to *INFOR*
Thank You
Questions?