FAST AUTOMATED STOPPING-TIME AND EDGE-STRENGTH ESTIMATION FOR ANISOTROPIC DIFFUSION

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ABSTRACT

Anisotropic diffusion (ATD) is an edge-oriented, scale-space based, and iterative image-smoothing process. Two main challenges of ATD are how to automatically stop the iterative process, so to avoid blurring, and how to determine the scale (or edge-strength) parameter, so to best differentiate between edge and noise. In this paper, we propose 1) an automatic noise-adaptive stopping-time estimator and 2) a robust scale parameter (or edge strength) estimator. With these two novel estimators, our adaptive ATD method effectively reduces noise (high PSNR gain) and preserves structures (significantly less blurring) than conventional ATD.

***Index Terms*—** Denoising, anisotropic diffusion, stopping-time estimation, edge-strength, edge preservation.

1. INTRODUCTION

Anisotropic diffusion (ATD) [1, 2, 3] has become a widely used tool for multi-scale non-linear image processing such as denoising or segmentation. It is an iterative scale-space approach that is edge dependent by defining a diffusion coefficient as a function of the local gradient to encourage intra-region over inter-region smoothing. Two main challenges of ATD are 1) how to automatically stop the iterative process, i.e. determine an optimal stopping time T so to avoid too-much smoothing (blurring) and 2) how to determine the scale (or edge-strength) parameter σ_e so that noise and edge are not confused, even in very high or very low noisy images.

In scale-space methods such as ATD, researchers often set the stopping time T to a large value (ideally infinity) and observe how the diffused function evolves with time and converges to a constant. For real-time or fast denoising, we require an effective fast stopping-time function: effective to stop the diffusion process and avoid blurring and fast to not overload the computationally expensive ATD.

Stopping-time criteria are proposed in the literature but they are either image-dependent, developed for 1-dimensional signals, or require extensive computations (e.g., [4, 5]).

In the remainder of this paper, we discuss challenges and advantages of ATD in Section 2; in Section 3, we propose a novel stopping-time estimator to automatically stop the iterative ATD and a novel edge-strength estimator to adapt to edges and noise; in Section 4, we objectively and subjectively show the superior performance of the proposed estimators under Gaussian noise and high structure; we conclude the paper in Section 5.

This work is supported, in part, by the Natural Science and Engineering Research Council (NSERC) of Canada.

2. ATD BASED ON ROBUST STATISTICS

The general form of ATD [1] is

$$I_s^{t+1} = I_s^t + \lambda \sum_{p=1}^{\mathcal{N}_s} C_p(\| \nabla I_p \|) \cdot \nabla I_p, \tag{1}$$

where $|\mathcal{N}_s|$ is the number of directions (e.g., $|\mathcal{N}_s| = 4$ for north, south, west, and east) along which ATD is computed, s is the center pixel, $1 < t \leq T$ is the current scale, T is stopping time or number of iterations, $I_s^t = I_s(x, y, t) = I(s, t)$ is the image intensity at s and the current scale t, I_s^{t+1} is the intensity at the next scale $t + 1$ and is a scaled version of I^t, $\nabla I_p = (I_p - I_s)$ is the gradient of the image in the direction p, λ is a time step responsible for the stability of the function, and $C_p(\| \nabla I_p \|)$ is the conduction-coefficient (or edge-stopping) function in direction p. $0 \leq C(\cdot) \leq 1$ should approach zero when the diffusion is at edges and approach one when the diffusion at homogeneous areas.

Black et al. [2] develop a statistical interpretation of (1) by estimating a smoothed image from noisy data using tools of robust statistics. This estimation should satisfy,

$$min_I \sum_{s \in I} \sum_{p \in \mathcal{N}_s} \rho(I_p - I_s, \sigma), \tag{2}$$

where ρ is a robust error norm, $\mathcal{N}_s$ represent the spatial neighborhood of pixel s, σ is a scale (or edge strength) parameter that controls the shape of the edge-stopping function and hence determines residual errors (outliers or edges). Solving (2) using gradient descent gives

$$I_s^{t+1} = I_s^t + \frac{\lambda}{|\mathcal{N}_s|} \sum_{p \in \mathcal{N}_s} \psi(I_p - I_s^t, \sigma), \tag{3}$$

$|\mathcal{N}_s|$ is the number of neighbors, $\psi(I_p - I_s^t) = \rho'(I_p - I_s^t)$, the derivative of the error norm, is the influence function. To increase the robustness and preserve edges, $\psi(\cdot)$ should have a high value when $(I_p - I_s^t)$ is small, and should reach its smallest value when $(I_p - I_s)$ increases beyond an edge strength. In [2], $\psi(\nabla I) = C(\nabla I)\nabla I$, i.e., the edge-stopping function from a statistical point of view is the influence function divided by the image gradient $C(\nabla) = \frac{\psi(\nabla I)}{\nabla I}$.

Black *et al.* [2] suggest robust edge-stopping functions, Lorentzian and Tukey, as in

$$C_L(\nabla I) = \frac{1}{1+\frac{(\nabla I)^2}{2(\sigma)^2}}$$

$$C_T(\nabla I) = \begin{cases} \frac{1}{2}[1-(\frac{\nabla I}{\sigma})^2]^2 & \text{if} \quad |\nabla I| \le \sigma \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Note that σ, the scale parameter, controls the edge strength. As shown in Fig. 1, with $\sigma = \sigma_e$, the Tukey function is more sensitive (responsive) to edges than Lorentzian: in Tukey, when the gradient magnitude is over a specific value, the slope descends rapidly and reach the value zero while it reaches a value close to zero in Lorentzian.

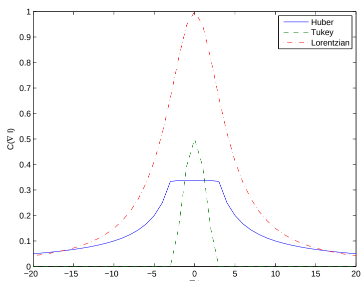


Fig. 1. Edge-stopping functions.

Note that (3) requires three parameters: λ for stability, σ for scaling (or edge-strength), and T for stopping time. [2] estimates σ separately for Tukey and Lorentzian functions based on the gradient of the image as in

$$\sigma_T = \sigma_e\sqrt{5} \quad \text{and} \quad \sigma_L = \frac{\sigma_e}{\sqrt{2}}, \\ \sigma_e = 1.4826 \cdot \text{MAAD}(\| \nabla I \|) \tag{5}$$

where MAAD $=$ median$_I(\| \nabla I \| - \text{median}(\| \nabla I \|))$ is the median absolute deviation, σ_e is the robust scale (or the edge strength), σ_T and σ_L are the robust scales for the Tukey and Lorentzian edge-stopping functions, respectively. [2] calculates λ using the influence function $\psi(\cdot)$ as in

$$\lambda = \frac{1}{\psi(\nabla I)} \quad \Rightarrow \lambda_T = \frac{25}{16\sigma_e} \quad \text{and} \quad \lambda_L = \frac{2}{\sigma_e}, \tag{6}$$

where λ_T and λ_L are the values of the stability parameter for Tukey and Lorentzian functions.

3. PROPOSED PARAMETER ESTIMATION

3.1. Stopping-time estimation

We analyzed the behavior of [2] ATD applied to noisy images and examined the image quality (in PSNR) in relation to the stopping time T and to the edge strength σ_e. Fig. 2 shows results of our noise/T analysis using Tukey edge-stopping function (similar behavior was found for Lorentzian function). As can be seen, for heavy noisy images, e.g., $PSNR = 14dB$, the higher T is, the higher the PSNR gain is; the lower the noise level is, e.g., $PSNR = 43dB$, the less number of iterations we need. We also found that Lorentzian function gives better PSNR in high noisy images than Tukey function (which gives better PSNR gain in not or little noisy images).

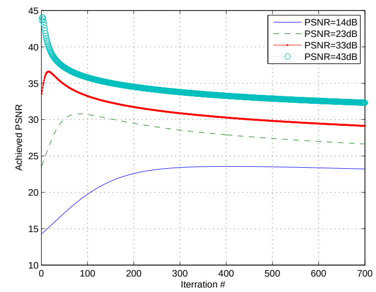


Fig. 2. PSNR gain versus T using $C_T(\nabla I)$.

Consequently, applying a higher or lower T than required will introduce significant blurring or annoying residual (spurious) noise. We therefore propose to adapt T exponentially to the estimated noise in PSNR as

$$T = (t_x - t_n) \cdot e^{-\frac{\widehat{P}}{a}} + t_n, \tag{7}$$

where t_x and t_n are the max. (e.g., 2500) and min. (e.g., 3) of T, $\widehat{P}$ is the estimated noise in PSNR (using [6]), and a is a constant. Fig. 3 plots the proposed stopping-time function for

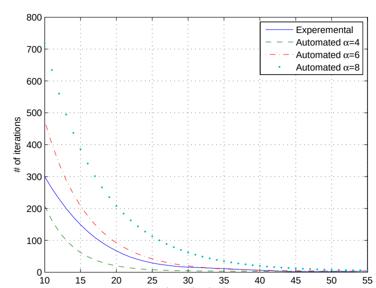


Fig. 3. Proposed stopping-time estimation.

different a. We set $a = 6$ because it outperforms (in PSNR) the manually tuned T (see Fig.3) for optimal ATD [2]. We applied (7) to all noise levels, including no-noise.

3.2. Robust edge-strength estimation

Black *et al.* estimate the edge strength σ_e using the MAAD of the image gradient. For Tukey edge-stopping function, this estimate gives good results in images with no or little noise and fails in noisy images. This is because Tukey edge stopping is very responsive to edges (see Fig. 1) and in the presence of noise, Tukey confuses noise with edges. Thus we require "less edge-responsive" (or "less robust") Tukey σ_e. On the other hand, the Lorentzian edge stopping is not well responsive to edges as Tukey is and may thus introduce blurring, especially in image with no or little noise. Thus we require a "more edge-adaptive" Lorentzian σ_e.

We propose to estimate σ_e using the mean absolute deviation (MEAD) instead of the [2] MAAD as follows.

$$\begin{gathered}\sigma_e = \beta \cdot \operatorname{mean}_I(\| \nabla I \| - \operatorname{mean}(\| \nabla I \|)),\\ \sigma_T = \sigma_T\sqrt{5} \quad \text{and} \quad \sigma_L = \sigma_e\sqrt{2}.\end{gathered} \tag{8}$$

We derive (8) from the relation between MEAD and the standard deviation StD as

$$\text{MEAD} \leq \text{StD} \Rightarrow \text{StD} = \beta \cdot \text{MEAD}, \quad \beta > 1. \tag{9}$$

$\beta = 1.4826$ because the MEAD of a zero-mean, unit variance normal distribution is 0.6745. Fig.4 plots proposed versus [2] edge-strength adaptations for Tukey and Lorentzian $C(\nabla I)$.

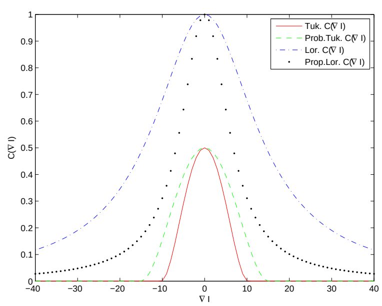
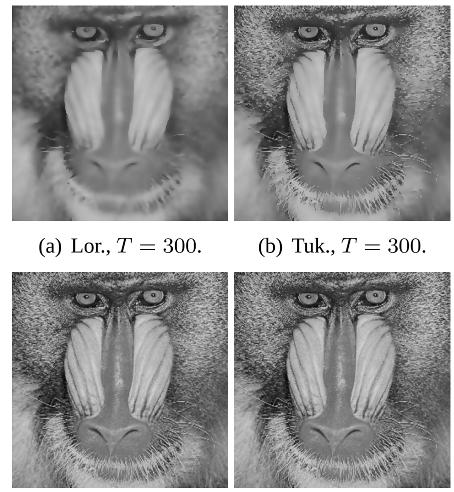


Fig. 4. Proposed edge-strength adaptation (8).

The MAAD performs poor in highly noisy image (see Fig.6) because it is less affected by extremes in the tail than the MEAD (since the data in the tails have less influence on the calculation of the median than they do on the mean [7]). For ATD, the MAAD filters outliers, i.e., high noise is not taken into account to find σ_e which means that σ_e is estimated less than it should be. Also, the MEAD is especially useful in noisy images because the average is "less edge-responsive" than taking the median, i.e., taking the mean gives the edge strength higher value. Thus the MEAD is more adaptive to noise. Fig.7 confirms our observation above for Tukey as well as for Lorentzian edge stopping function. Additionally, computing the MEAN is less complex than the MAAD.

4. APPLICATIONS

Sample results are shown for the 1) *Canal* image that contains both homogeneous and structured areas and also non-Gaussian artifacts such as blur; 2) *Mandril* image that contains mostly high-structured area, and is also widely used in the literature to test stability to high structure; 3) *Lena* that is a moderately challenging image to test stability to noise.



(a) Lor., $T = 300$. (b) Tuk., $T = 300$.

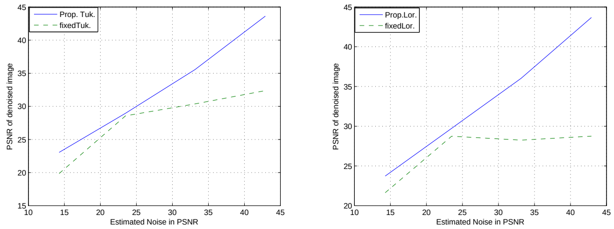
(c) Prop. Lor., auto. T. (d) Prop. Tuk., auto. T.

Fig. 5. Smoothing of *Mandril*.

Structure preservation with proposed stopping time: Fig. 5 shows the performance of [2] edge-stopping functions when applied to high-structure images with and without the proposed function. As can be seen, when using the proposed stopping-time function, image details using both Tukey and Lorentzian are well preserved and no blurring is introduced.

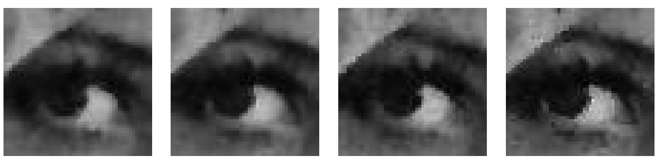
Denoising with proposed stopping time: As discussed earlier, Tukey edge stopping is edge robust (gives much less blurring with high T), but, and because of its robustness, its performance decreases in high noisy images. Lorentzian function denoises well high-noisy but not low-noisy images. Thus both Lorentzian and Tukey adds unwanted artifacts to the processed image. To enable an edge-stopping function to differentiate between noise and edges we need to apply the gradient on the image at multi-levels (higher iteration number). The proposed stopping-time stabilizes both Lorentzian and Tukey functions as shown in Fig. 6 where significant PSNR gain is achieved using the proposed stopping time. Note that good performance with the proposed T was also achieved in blurred (non-Gaussian noisy) images such as *Canal*; results are not shown due to space constraints.

Denoising with proposed edge strength: As confirmed in Figs. 7-9, the use of the MEAD, instead of the MAAD, decreases the residual spurious noise introduced by Tukey function, and this in turn preserves image structures. Moreover, using the proposed σ_e, edges are more clear and smooth. Similarly, when using Lorentzian edge stopping, even though it is more forgiving than Tukey edge stopping, the MEAD improves its performance from the point of view of taking the mean values of the gradient in finding σ_e. This makes the



(a) Tuk.: auto vs. fixed iter. (b) Lor.: auto vs. fixed iter.

Fig. 6. Effect of proposed stopping-time function.



(a) Prop. (b) Lor. (c) Prop. (d) Tuk.

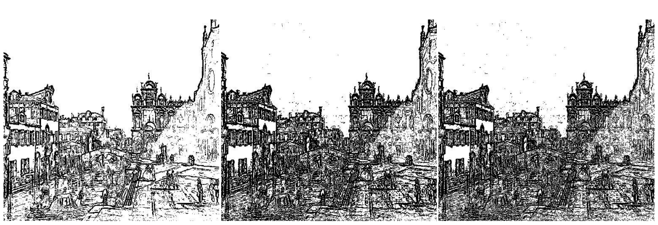
(e) Org. (f) Noisy

Fig. 7. Denoising results, *Lena 23dB*, auto. T.

value of each pixel gradient proportion to its direct neighbors. Therefore, an edge-stopping function using the proposed edge-strength performs better than using [2] estimator.

Note in Fig. 8 the difference between the edge detection which gives better understanding on how much the image was enhanced (more edges means less denoising). As can be seen, the proposed robust scale (or edge strength) parameter provides a compromise between too much blurring (Lorentzian) and to strict (i.e., dense edges detected) smoothing (Tukey).

Fig.9 objectively confirms the superiority of the proposed edge-strength estimator compared to Tukey estimator [2].



(a) Prop. (b) Lor. (c) Tuk.

Fig. 8. Edge detection results, *Canal*, auto. T.

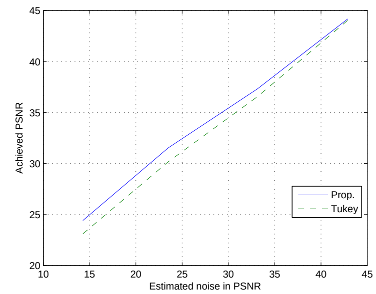


Fig. 9. PSNR gain, *Lena*, auto. T.

5. CONCLUSION

This paper proposed a new noise-adaptive stopping-time function that is fast and automates the iterative anisotropic diffusion process. The paper also proposes a new estimator for the scale parameter of edge strength based on mean absolute deviation. Our adaptive anisotropic diffusion method allows us to effectively reduce the noise and better preserve small structures. The proposed adaptation of anisotropic diffusion gives good objective results in term of PSNR to reduce noise and subjective results in term of significantly less blurring than using related work. Our study shows that estimation of T and σ_e need not be computationally expensive.

6. REFERENCES

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