INSE 6230
Total Quality Project Management

Lecture 4

Project Time Management II.

Project Time Management II

- Shortening a project schedule
  - Project Crashing
- Schedule uncertainty
  - PERT
- Case studies related to project time management

Shortening a Project Schedule

- Three main techniques for shortening schedules
  - **Reduce the duration of activities on the critical path** by adding more resources or changing their scope
  - **Fast tracking** activities by doing them in parallel or overlapping them
    - Can end up lengthening the schedule since starting tasks too soon may increase project risk and result in rework
  - **Crashing** activities by obtaining the greatest amount of schedule compression for the least incremental cost
    - Crash time
      - an amount of time an activity is reduced
    - Crash cost
      - cost of reducing activity time

Project Crashing: Procedure

- Find critical path (CP) by CPM
- Determine Crash Cost per period (week, month) for each activity

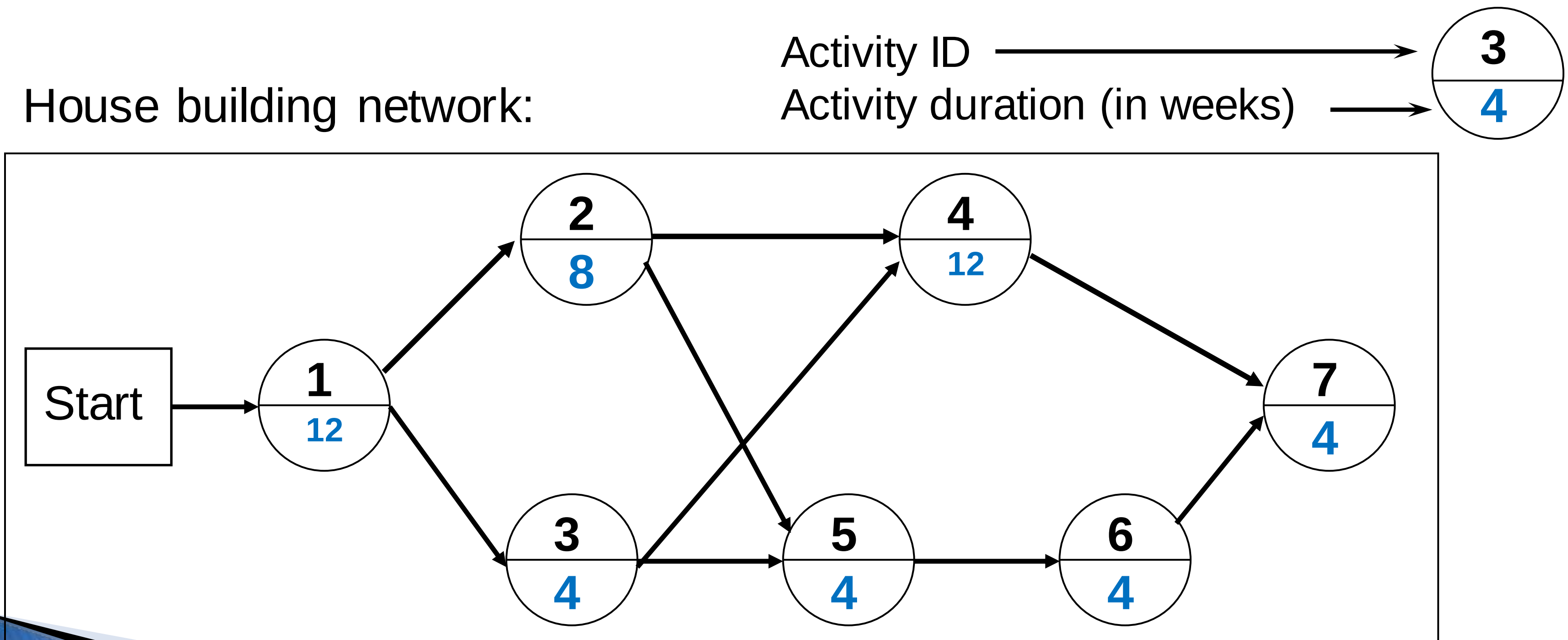
$$\textbf{Crash cost per period} = \frac{\text{Total crash cost}}{\text{Total crash time}}$$

- Find an activity on the CP with the minimum crash cost per period and reduce its duration as much as possible
  - Reduction should fall within its allowable range
  - **Reduction can be carried out only to the point where another path becomes critical!**
    - If the reduction goes beyond the point where another path becomes critical, the costs may be incurred unnecessarily
    - If 2 paths become CP simultaneously, activities on both CPs must be reduced by the same amount
- Continue until you reach the desired project completion time
  - When reducing it is necessary to keep up with all the network paths

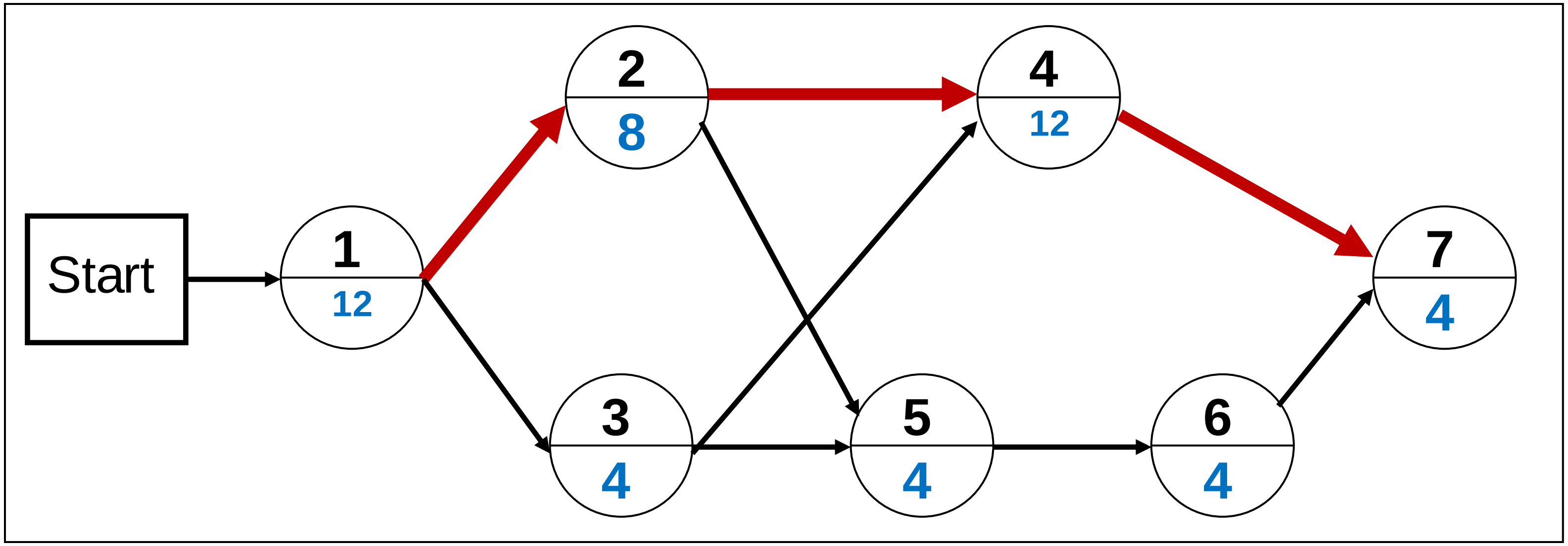
Project Crashing: *Example 1*

In a house building project you are asked to deliver the house in 30 weeks, but you cannot deliver it before 36 weeks based on your original schedule and budget. How much extra cost would need to be incurred to complete the house by this time?

House building network:



Project Crashing: *Example 1* – Critical Path



A: **1-2-4-7**
12 + 8 + 12 + 4 = <u>36 months</u>

B: 1-2-5-6-7
12 + 8 + 4 + 4 + 4 = 32 months

C: 1-3-4-7
12 + 4 + 12 + 4 = 32 months

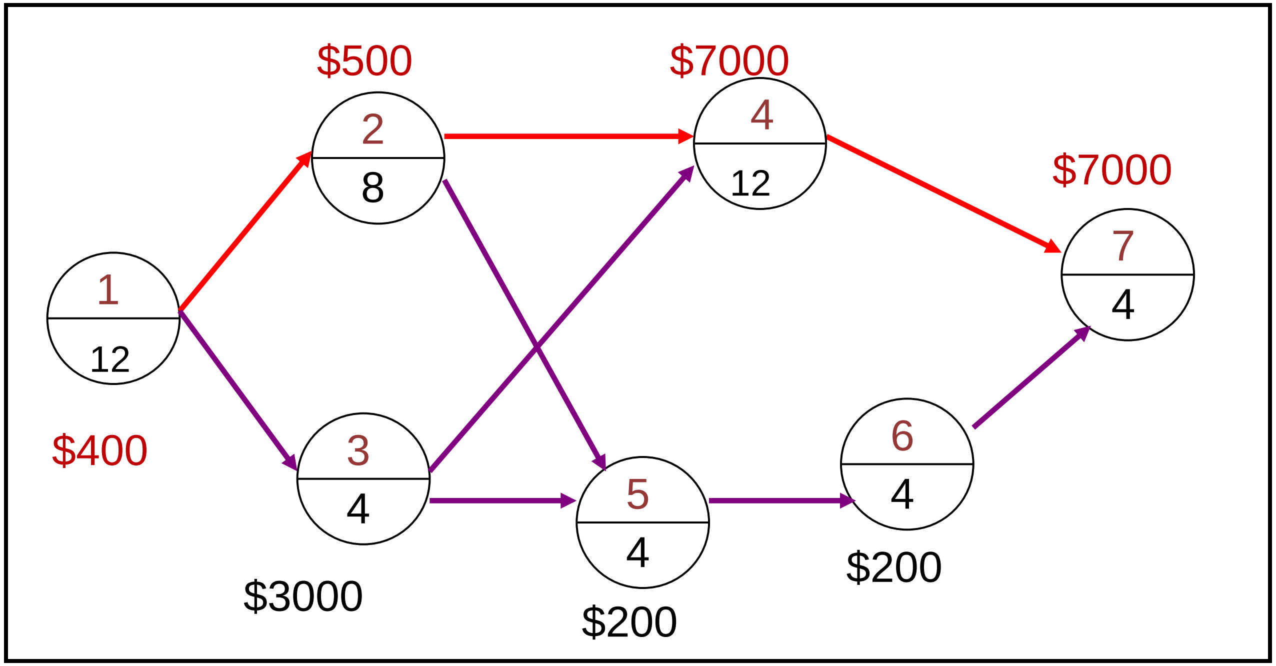
D: 1-3-5-6-7
12 + 4 + 4 + 4 + 4 = 28 months

- Critical path
  - Minimum project completion time at this point is 36 weeks.
  - Can we complete it earlier?

Project Crashing: *Example 1* – Crash Cost

Activity	Normal Time (Weeks)	Crash Time (Weeks)	Normal Cost	Crash Cost	Total Allowable Crash Time (Weeks)	**Crash Cost per Week**	CP
1	12	7	$3,000	$5,000	5	$400	
2	8	5	2,000	3,500	3	500	
3	4	3	4,000	7,000	1	3,000	
4	12	9	50,000	71,000	3	7,000	
5	4	1	500	1,100	3	200	
6	4	1	500	1,100	3	200	
7	4	3	15,000	22,000	1	7,000	
			$75,000	$110,700			

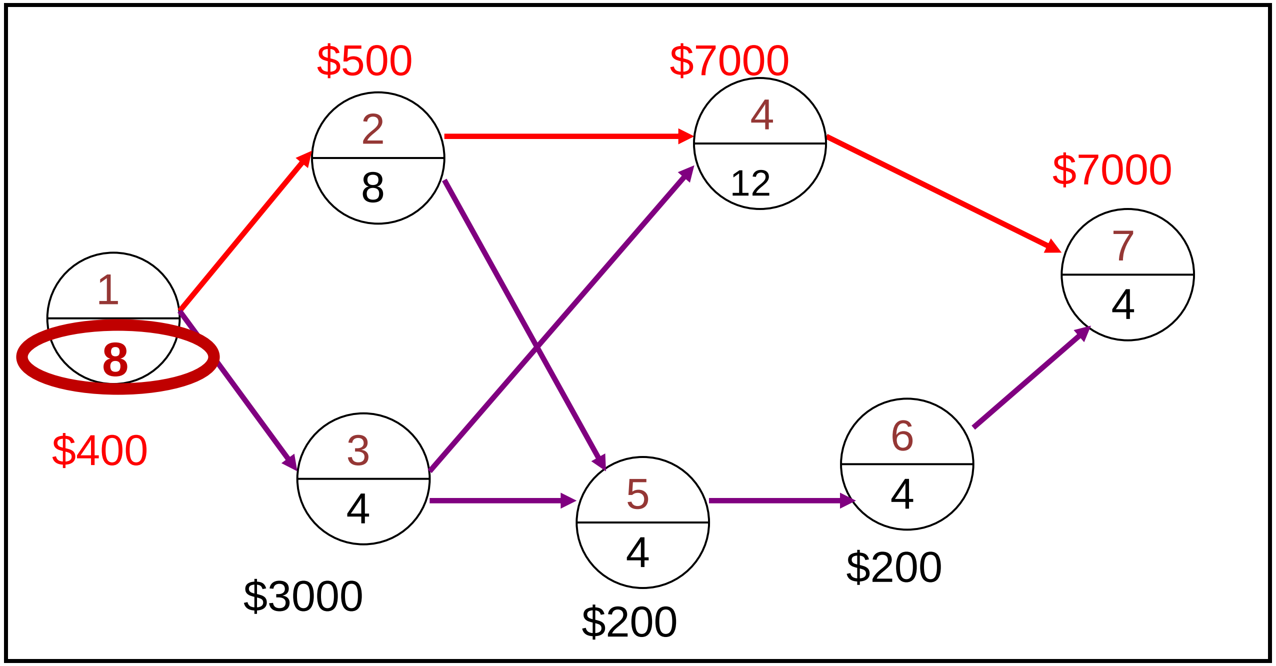
$$\frac{\text{Total crash cost}}{\text{Total crash time}} = \frac{\text{Crash Cost – Normal Cost}}{\text{Normal Time – Crash Time}} = \frac{\$2,000}{5} = \$400 \text{ per week}$$



Project Duration:
36 weeks

Path durations:
1-2-4-7: 36
1-2-5-6-7: 32
1-3-4-7: 32
1-3-5-6-7: 28

- The cheapest activity to crash is Activity 1
- Max allowable time it can be reduced is 5 weeks
  - Can we really reduce it by 5 weeks?
    - Remember, you can reduce only until your CP reaches another path(s), at which point you need to check whether another path(s) did not become critical
    - Your maximum crashing time is thus 4 weeks, because at 32 weeks you reach other 2 paths and you need to recalculate CP



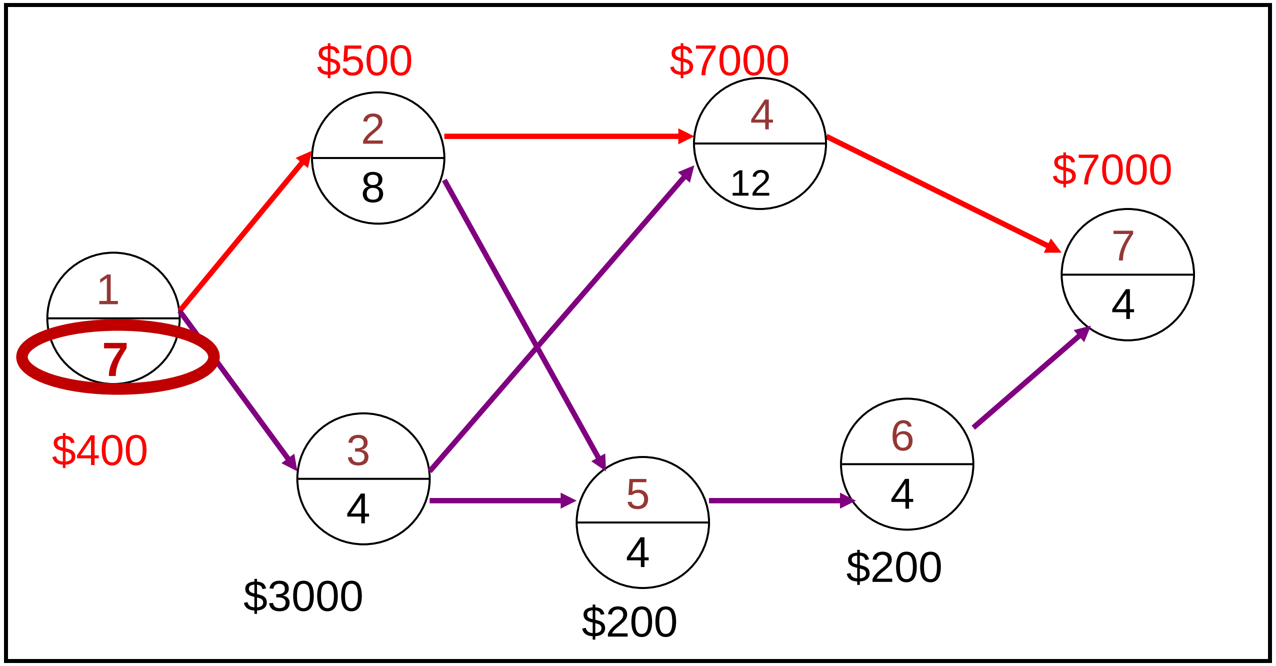
Crash 1 by 4 weeks:

Project Duration:
32 weeks

Path durations:
1-2-4-7: 32
1-2-5-6-7: 28
1-3-4-7: 28
1-3-5-6-7: 24

- Project duration is now 32 weeks
- Additional cost: 4 weeks per $400 = $1,600
- CP is still the same **1-2-4-7**

- We can still continue crashing Activity 1 (still it is cheapest)
- Max allowable time it can still be reduced is 1 week



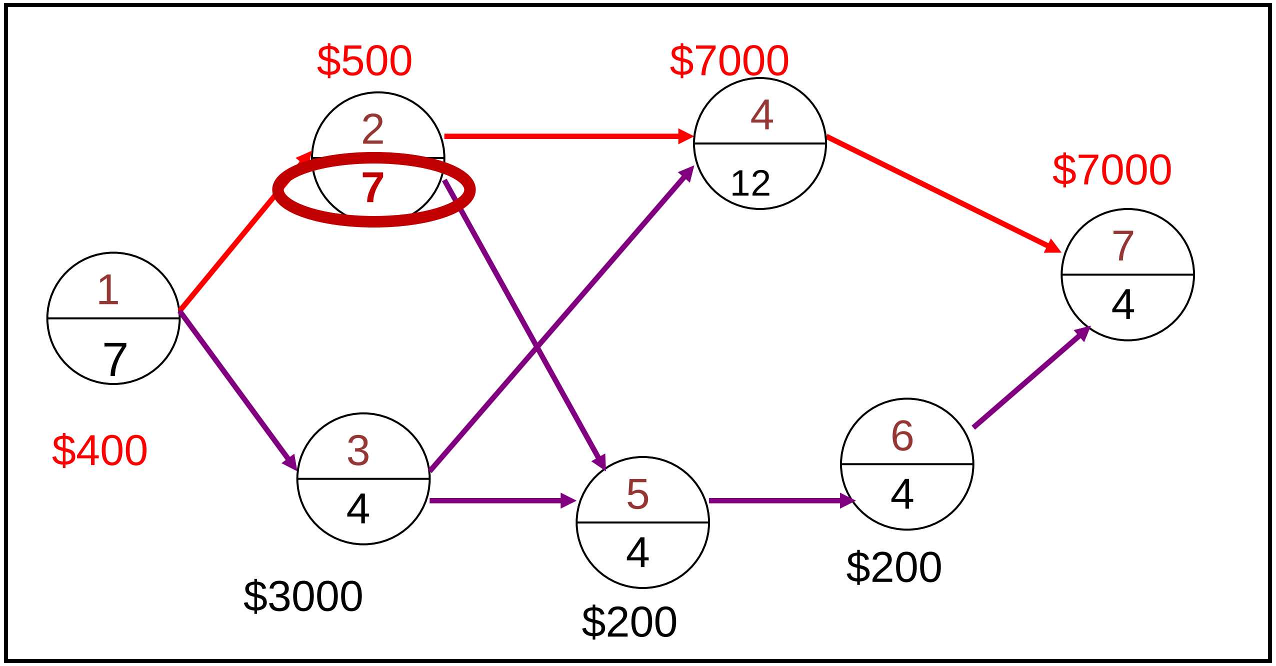
Crash 1 by 1 week:

Project Duration:
31 weeks

Path durations:
1-2-4-7: 31
1-2-5-6-7: 27
1-3-4-7: 27
1-3-5-6-7: 23

- Project duration is now 31 weeks
- Additional cost: 1 week per $400 = $400
- Total crashing cost: $1,600 + $400 = $2,000
- CP is still the same **1-2-4-7**

- Which activity to crash now?
  - Activity 1 cannot be crashed anymore, we reached allowable max
  - Activity 2 is cheapest one to crash now
  - Max allowable time it can be reduced is 3 weeks
  - But we need to crash it by 1 week only (from 31 to 30 weeks) in order to reach the desirable project duration



Crash 2 by 1 week:

Project Duration:
30 weeks

Path durations:
1-2-4-7: 30
1-2-5-6-7: 26
1-3-4-7: 27
1-3-5-6-7: 23

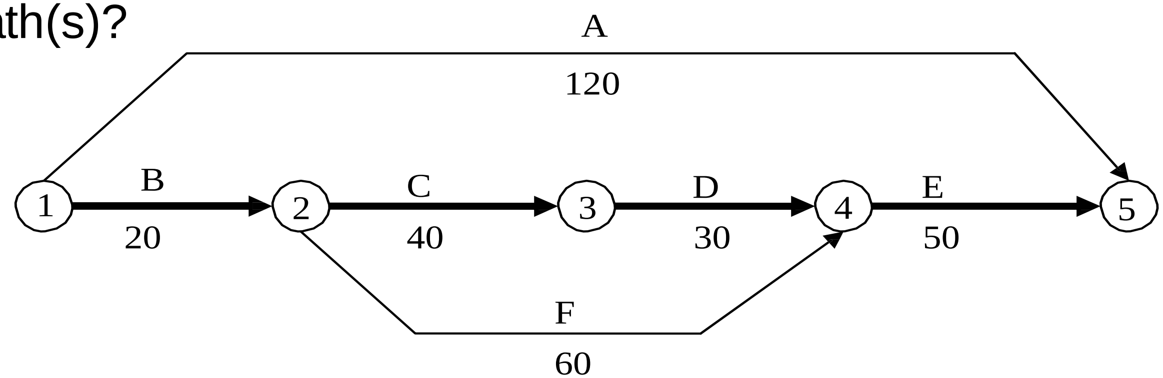
- Project duration is now 30 weeks
- Additional cost: 1 week per $500 = $500
- Total crashing cost: $1,600 + $400 + $500 = $2,500
- CP is still the same **1-2-4-7**

- The project duration has now been reduced to the desirable 30 weeks for a total crashing cost (additional cost) of $2,500

- Following this procedure the network can be crashed to 24 weeks at a total additional cost of $31,500.

Project crashing – *Example 2*

- The network and durations given below show the original schedule for a project. Based on the information in the table you can decrease the durations of activities at an additional expense. The owner wants you to you to finish the project in <u>120 days </u>at the minimum possible cost.
- Determine the project duration and its cost before and after crashing and show the details about the activities which should be crashed. What is (are) the critical path(s)?



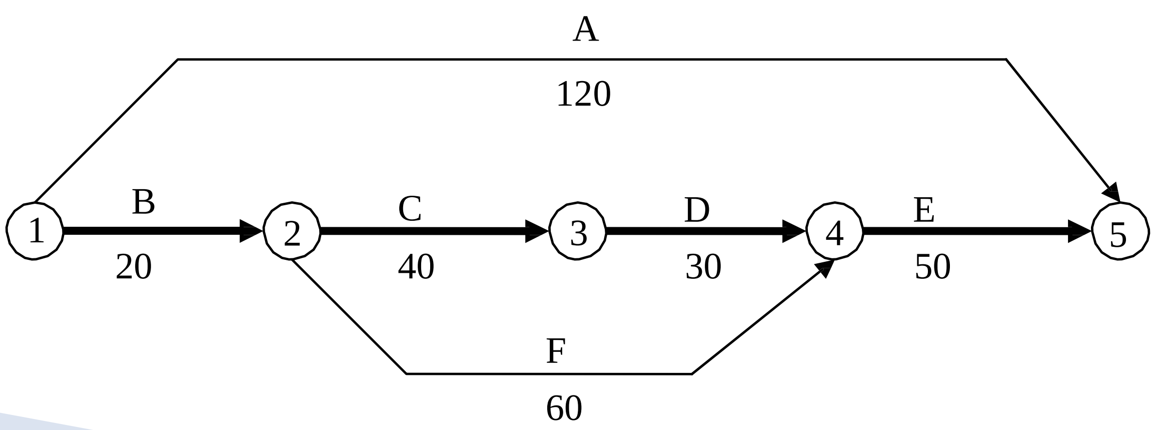
Activity	Normal duration (days)	Crash Duration (days)	Normal Cost	Crash Cost
A	120	100	12000	14000
B	20	15	1800	2800
C	40	30	16000	17000
D	30	20	1400	2000
E	50	35	3600	5400
F	60	40	13500	14500

Project crashing – *Example 2*

Activity	Normal duration (days)	Crash duration (days)	Normal Cost	Crash Cost	**Crash cost per day**
A	120	100	12000	14000	
B	20	15	1800	2800	**200**
C	40	30	16000	17000	**100**
D	30	20	1400	2000	**60**
E	50	35	3600	5400	**120**
F	60	40	3500	14500	
			∑ 38300		

- At this point:
  - The critical path is **B-C-D-E**
  - The cost of the project is $ **38 300**
  - The project duration is **140 days**.

Path durations:
A: 120
B-C-D-E: 140
B-F-E: 130



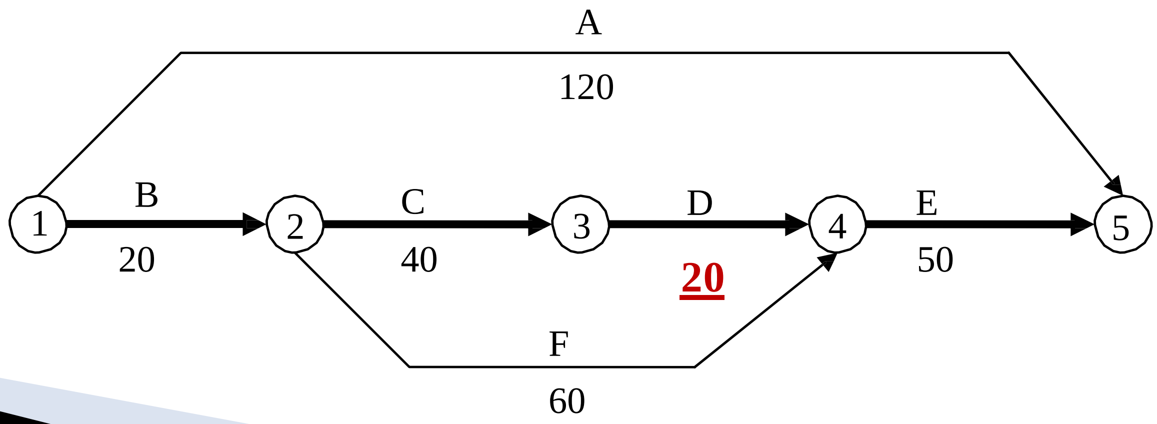
Project crashing – *Example 2*

Activity	Normal duration (days)	Crash duration (days)	Normal Cost	Crash Cost	**Crash cost per day**
A	120	100	12000	14000	
B	20	15	1800	2800	**200**
C	40	30	16000	17000	**100**
D	30	20	1400	2000	**60**
E	50	35	3600	5400	**120**
F	60	40	3500	14500	**550**
			∑ 38300		

- The first activity to crash is activity **D.** It will be crashed by **10** days for an additional cost of **$600**.

We have 2 critical paths now! B-C-D-E and B-F-E (130 days)
Both should be reduced!

Path durations:
A: 120
B-C-D-E: 130
B-F-E: 130

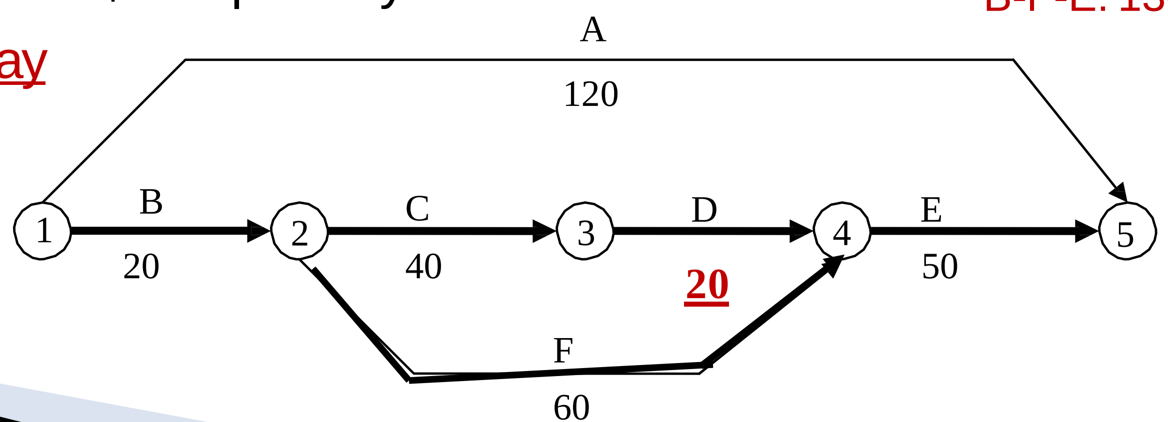


Project crashing – *Example 2*

Activity	Normal duration (days)	Crash duration (days)	Normal Cost	Crash Cost	**Crash cost per day**
A	120	100	12000	14000	
B	20	15	1800	2800	**200**
C	40	30	16000	17000	**100**
D	30	20	1400	2000	**60**
E	50	35	3600	5400	**120**
F	60	40	3500	14500	**550**
			∑ **38300**		

- What activities can we reduce in order to reduce the duration of **BOTH** critical paths?
- Only B: $200 per day
- C and F: $100 + $550 = $650 per day
- D and F: $60 + $550 = $610 per day
- Only E: $120 per day

Path durations:
A: 120
B-C-D-E: 130
B-F-E: 130



Project crashing – *Example 2*

Activity	Normal duration (days)	Crash duration (days)	Normal Cost	Crash Cost	**Crash cost per day**
A	120	100	12000	14000	**100**
B	20	15	1800	2800	**200**
C	40	30	16000	17000	**100**
D	30	20	1400	2000	**60**
E	50	35	3600	5400	**120**
F	60	40	3500	14500	**550**
			∑ 38300		

- The second activity to crash is activity **E**. It will be crashed by **10** days for an additional cost of **$ 1 200**.

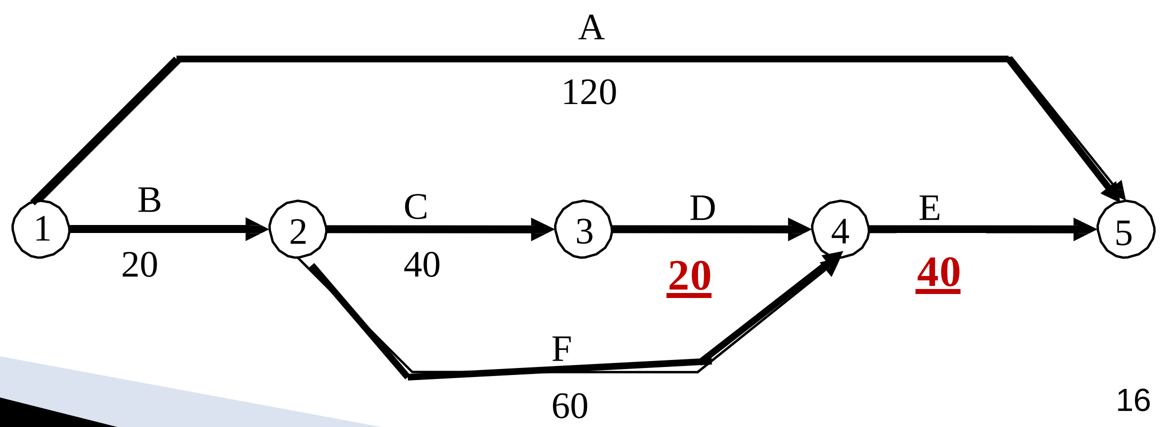
- After crashing, the project cost is **$ 40 100** and the project duration is **120 days.**
- The final critical path(s) are **B-C-D-E**, **B-F-E** and **A**.

Path durations:

A: 120

B-C-D-E: 120

B-F-E: 120



Dealing with Uncertainty

- There may be a significant amount of uncertainty associated with the actual task durations
- **Ignore the uncertainty**, and schedule the project using the expected or most likely duration for each activity.
  - Drawbacks:
    - Typically results in overly optimistic schedules
    - The use of single activity durations often produces a rigid, inflexible mindset on the part of schedulers and the loss of confidence in the realism of a schedule
- **Include a contingency allowance** in the estimate of activity durations.
    - *E.g.*, an activity with an expected duration of 2 days might be scheduled for a period of 2.2 days, including a 10% contingency.
    - Systematic use of contingency factors can result in more accurate schedules
- Use more elaborate **techniques to deal with uncertainty**
    - PERT
    - Monte Carlo simulation

PERT (Project Evaluation and Review Technique)

PERT is a network analysis technique used to estimate project duration when there is **a high degree of uncertainty about the activity durations**

- A commonly used formal method for dealing with uncertainty in project scheduling.
- Applies the CPM to a weighted duration estimate

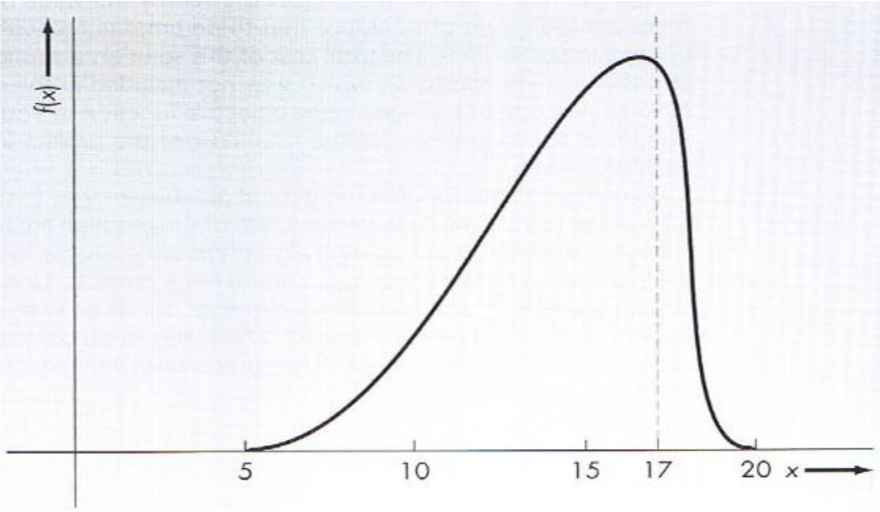
Procedure:

- Calculate the means of durations for each activity based on three point estimates
- Determine critical path using the means
- Find the expected project duration and the variance

PERT: Three point estimates

The mean and variance for each activity duration are typically computed from the three point estimates:

- **optimistic** *($a_{i,j}$)*
- **most likely** *($m_{i,j}$)*
- **pessimistic** *($b_{i,j}$)*



Mean: $$\mu(i,j) = \frac{1}{6}(a_{i,j} + 4m_{i,j} + b_{i,j})$$

Variance: $$\sigma^2(i,j) = \frac{1}{36}(b_{i,j} - a_{i,j})^2$$

Project duration measures:

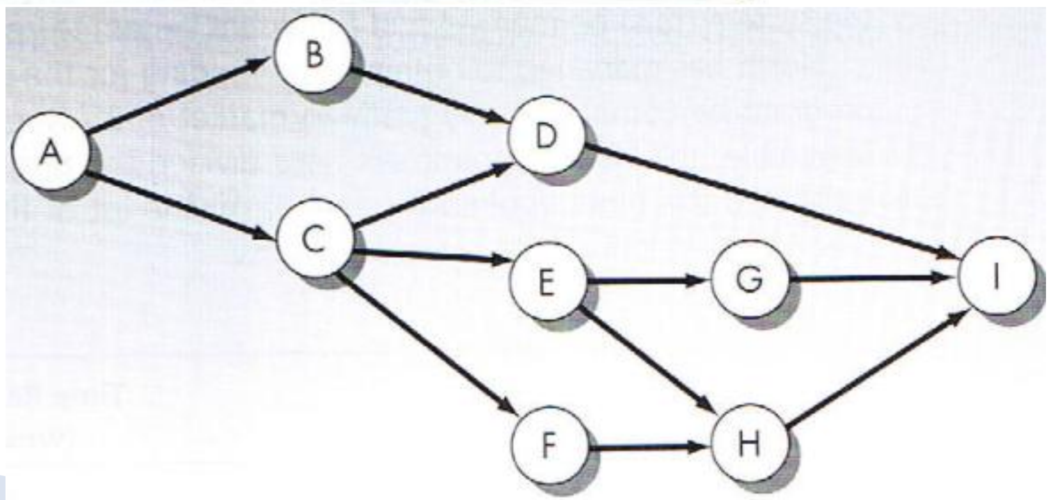
- The **expected project duration *E(T)*** is equal to the sum of the expected durations of the activities along the critical path.
- The **variance σ^2*(T)* in the duration of CP** is calculated as the sum of the variances along the critical path.
- Assuming that activity durations are independent random variables

PERT: *Example 1*

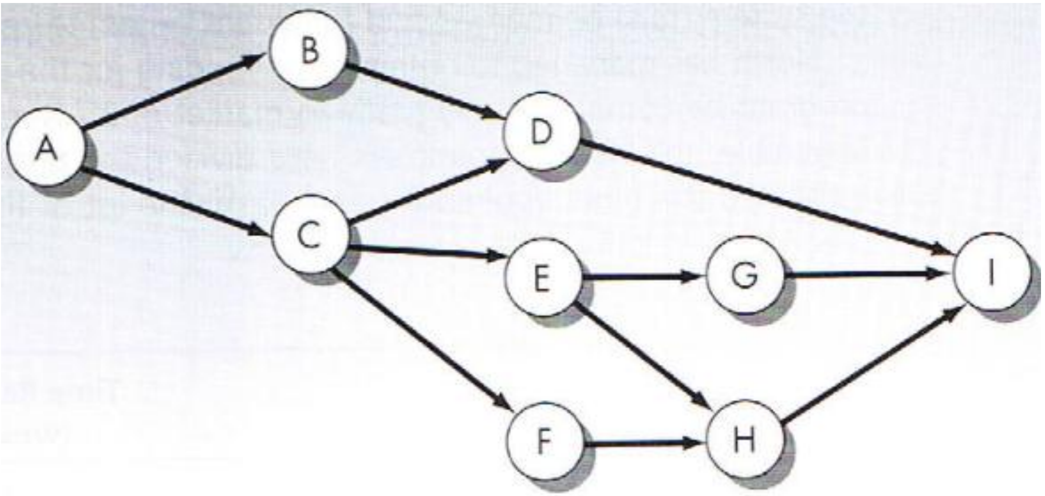
Task	Time Required (weeks)	Immediate Predecessors
A. Perform market survey	3	
B. Design graphic icons	4	A
C. Develop flowchart	2	A
D. Design input/output screens	6	B, C
E. Module 1 coding	5	C
F. Module 2 coding	3	C
G. Module 3 coding	7	E
H. Module 4 coding	5	E, F
I. Merge modules and graphics and test program	8	D, G, H

Calculate CPM – 2 options:

- On the network
- Through ES,EF,LS and LF



PERT: *Example 1*



Task	Time Required (weeks)
A.	3
B.	4
C.	2
D.	6
E.	5
F.	3
G.	7
H.	5
I.	8

Option 1: Calculate CP on the network:

ABDI: 21 weeks

ACDI: 19 weeks

ACEGI: **25 weeks** ⟵ The critical path ***A – C – E – G – I***

ACEHI: 23 weeks

ACFHI: 21 weeks

Completion time **without considering uncertainty** is 25 weeks

PERT: *Example 1*

Option 2: Calculate CP through ES, EF, LS & LF:

Activity	Time	Immediate Predecessors	ES	EF	LS	LF	Total Slack
A	3	—	0	3	0	3	0
B	4	A	3	7	7	11	4
C	2	A	3	5	3	5	0
D	6	B, C	7	13	11	17	4
E	5	C	5	10	5	10	0
F	3	C	5	8	9	12	4
G	7	E	10	17	10	17	0
H	5	E, F	10	15	12	17	2
I	8	D, G, H	17	25	17	25	0

The critical path: ***A – C – E – G – I***

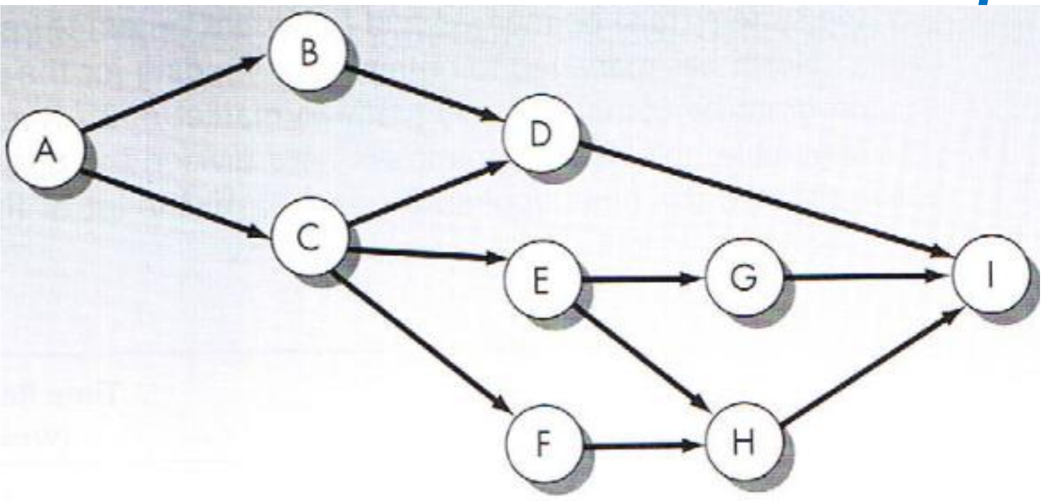
Completion time without considering uncertainty: 3+2+5+7+8= <u>25 weeks</u>

PERT: *Example 1*

If we can obtain three point estimates we can incorporate uncertainty into the project duration calculations

Activity	Min (a)	Most Likely (m)	Max (b)	$\mu = \frac{a+4m+b}{6}$	$\sigma^2 = \frac{(b-a)^2}{36}$
A	2	3	4	3	0.11
B	2	4	10	4.67	1.78
C	2	2	2	2	0
D	4	6	12	6.67	1.78
E	2	5	8	5	1.00
F	2	3	8	3.67	1.00
G	3	7	10	6.83	1.36
H	3	5	9	5.33	1.00
I	5	8	18	9.17	4.69

PERT: *Example 1*



Task	$\mu = \frac{a + 4m + b}{6}$
A.	3
B.	4.67
C.	2
D.	6.67
E.	5
F.	3.67
G.	6.83
H.	5.33
I.	9.17

Calculate CP while considering uncertainty:

ABDI: 23.5 weeks

ACDI: 20.84 weeks

ACEGI: **26 weeks** ← The critical path ***A – C – E – G – I***

ACEHI: 24.5 weeks

ACFHI: 23.17 weeks

Completion time **with considering uncertainty,** i.e. expected project duration, is 26 weeks

PERT: *Example 1*

Activity	Min (a)	Most Likely (m)	Max (b)	$\mu = \frac{a + 4m + b}{6}$	$\sigma^2 = \frac{(b-a)^2}{36}$
A	2	3	4	3	0.11
B	2	4	10	4.67	1.78
C	2	2	2	2	0
D	4	6	12	6.67	1.78
E	2	5	8	5	1.00
F	2	3	8	3.67	1.00
G	3	7	10	6.83	1.36
H	3	5	9	5.33	1.00
I	5	8	18	9.17	4.69

In order to determine the variance in duration of the critical path you can calculate variances only for activities on the critical path.

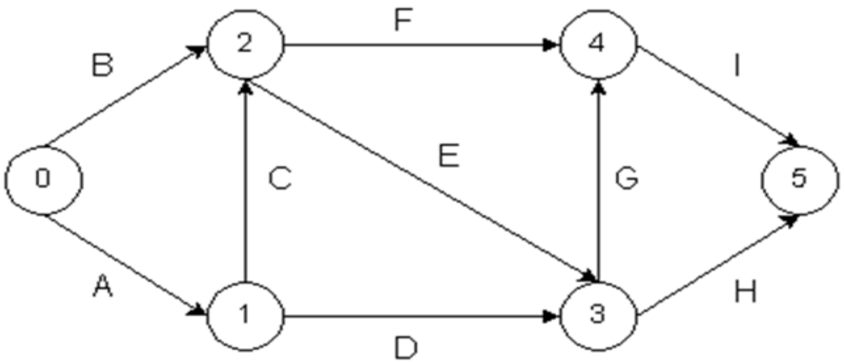
The critical path: ***A – C – E – G – I***

Variance: $\mathrm{Var}(T) = 0.11 + 0 + 1.0 + 1.36 + 4.69 = 7.16.$

Standard deviation: $\sigma = \sqrt{7.16} = 2.68.$

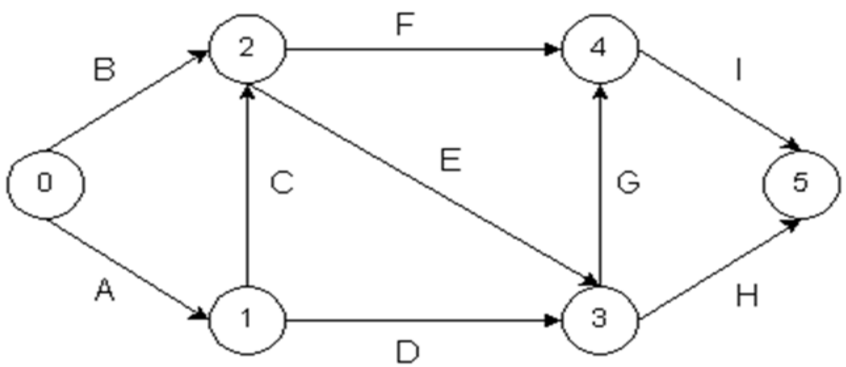
PERT: *Example 2*

- The network diagram below represents a project consisting of 9 activities, the durations of which are uncertain. The activity most likely, optimistic and pessimistic estimates are indicated in the table below
- Determine the completion time of the project without considering uncertainty, the expected project duration based on the three point estimates, the corresponding variance and the critical path.



Activity	Optimistic Duration (weeks)	Most Likely Duration (weeks)	Pessimistic Duration (weeks)
A	3	4	5
B	2	3	5
C	6	8	10
D	5	7	8
E	6	9	14
F	10	12	14
G	2	2	4
H	4	5	8
I	4	6	8

PERT: *Example 2*



Activity	Optimistic Duration (weeks)	Most Likely Duration (weeks)	Pessimistic Duration (weeks)
A	3	**4**	5
B	2	**3**	5
C	6	**8**	10
D	5	**7**	8
E	6	**9**	14
F	10	**12**	14
G	2	**2**	4
H	4	**5**	8
I	4	**6**	8

Calculate CP without considering uncertainty
For the calculations consider only most likely durations

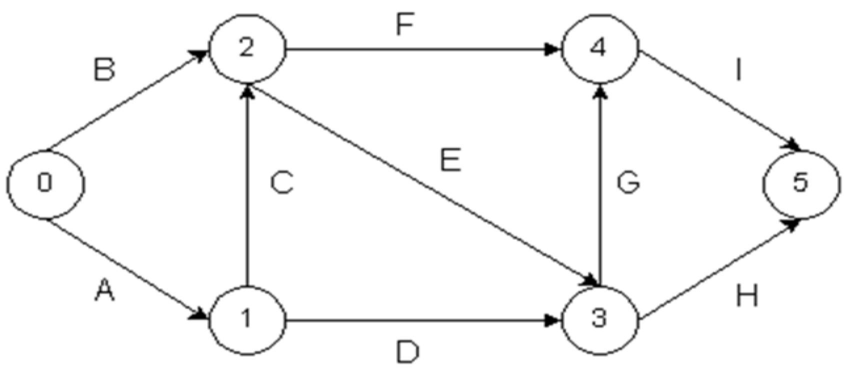
BFI: 21 weeks
BEGI: 20 weeks
BEH: 17 weeks
ACFI: **30 weeks**
ACEH: 26 weeks
ACEGI: 29 weeks
ADGI: 19 weeks
ADH: 16 weeks

Critical path is **ACFI**

Project duration without considering uncertainty is 30 weeks

$$\mu(i,j) = \frac{1}{6}(a_{i,j} + 4m_{i,j} + b_{i,j})$$

PERT: *Example 2*



Activity	Optimistic Duration (weeks)	Most Likely Duration (weeks)	Pessimistic Duration (weeks)	μ
A	3	4	5	4
B	2	3	5	3.16
C	6	8	10	8
D	5	7	8	6.83
E	6	9	14	9.33
F	10	12	14	12
G	2	2	4	2.33
H	4	5	8	5.33
I	4	6	8	6

Calculate CP while considering uncertainty

First compute μ, **i.e.** the means for the activity durations

BFI: 21.16 weeks
BEGI: 20.8 weeks
BEH: 17.82 weeks
ACFI: **30 weeks**
ACEH: 26.66 weeks
ACEGI: 29.66 weeks
ADGI: 19.16 weeks
ADH: 16.16 weeks

Critical path is **ACFI**

Project duration while considering uncertainty is 30 weeks

PERT: *Example 2*

$$\sigma^2(i,j) = \frac{1}{36}(b_{i,j} - a_{i,j})^2$$

Activity	Optimistic Duration (weeks)	Most Likely Duration (weeks)	Pessimistic Duration (weeks)	μ	σ^2
A	3	4	5	4	**0111**
B	2	3	5	3.16	
C	6	8	10	8	**0.444**
D	5	7	8	6.83	
E	6	9	14	9.33	
F	10	12	14	12	**0.444**
G	2	2	4	2.33	
H	4	5	8	5.33	
I	4	6	8	6	**0.444**

The critical path is **A – C – F – I**

In order to determine the variance in duration of the critical path calculate variances for activities on the critical path.

Variance: $\boldsymbol{\sigma^2 = 0.111+0.444+0.444+0.444 = 1.444}$

Expected project duration based on the three point estimates is **30 weeks** and the corresponding variance is **1.444**

Problems with using PERT method

- The procedure focuses on *a single critical path*, when many paths might become critical due to random fluctuations.
  - As a consequence, the PERT method typically *underestimates* the actual project duration.
- Three point estimations *involve more work* than CPM
- *Subjective* time estimates
- It is assumed that the activity durations are independent random variables.
  - In practice, the *durations are often correlated* with each other

Next Lecture

- Project Cost Management