**4-bit Carry Ripple Adder**

Assume you want to add two operands A and B where

A = A3 A2 A1 A0

B = B3 B2 B1 B0

For example:

\[
\begin{align*}
A &= 1011 + \\
B &= 1101 \\
\hline
A + B &= 11000 = C_{out} S_3 S_2 S_1 S_0
\end{align*}
\]

From the example above it can be seen that we are adding 3 bits at a time sequentially until all bits are added. A full adder is a combinational circuit that performs the arithmetic sum of three input bits: augends \(A_i\), addend \(B_i\) and carry in \(C_{in}\) from the previous adder. Its results contain the sum \(S_i\) and the carry out, \(C_{out}\) to the next stage.

Fig. 1 4-bit adder
So to design a 4-bit adder circuit we start by designing the 1-bit full adder then connecting the four 1-bit full adders to get the 4-bit adder as shown in the diagram above. For the 1-bit full adder, the design begins by drawing the Truth Table for the three input and the corresponding output SUM and CARRY. The Boolean Expression describing the binary adder circuit is then deduced. The binary full adder is a three input combinational circuit which satisfies the truth table below.

![Diagram of Full Adder](image)

Fig.2. Diagram and Truth Table of Full Adder

The Boolean equations of a full adder are given by:

\[
S_{out} = ABC + AB'C' + A'B'C + BA'C' \\
=(AB'+BA')C +AB+A'B') C' \\
S_{out} = A \oplus B \oplus C \\
C_{out} = AB + AC + BC \\
C_{out} = AB + C (A \oplus B)
\]

The circuit diagram is shown in Fig.3 and the simulation results is shown in Fig.4
Fig. 3. The Gate level Diagram of Full Adder

Fig. 4 1 bit full adder simulation results

To design a 4-bit adder now we cascade 4 1-bit adders as shown in Fig. 5

Fig. 5 4-bit Ripple Carry adder
As is seen from Fig. 5 and Fig. 6 the carry ripples through the 4 full adders to appear at the output, while the sums are available after 2 XOR delay.

Fig. 5 Carry Propagation

The carry propagation is shown in Fig. 5 as a block and as a path through the circuit in Fig. 6

Fig. 6 carry propagation through the circuit