A Review of Routing and Wavelength Assignment Approaches for Wavelength-Routed Optical WDM Networks

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Abstract

This study focuses on the routing and wavelength assignment (RWA) problem in wavelength-routed optical WDM networks. Most of the attention is devoted to such networks operating under the *wavelength-continuity constraint*, in which lightpaths are set up for connection requests between node pairs, and a single lightpath must occupy the same wavelength on all of the links that it spans. In setting up a lightpath, a route must be selected and a wavelength must be assigned to the lightpath. If no wavelength is available for this lightpath on the selected route, then the connection request is blocked. We examine the RWA problem and review various routing approaches and wavelength-assignment approaches proposed in the literature. We also briefly consider the characteristics of wavelength-converted networks (which do not have the wavelength-continuity constraint), and we examine the associated research problems and challenges. Finally, we propose a new wavelength-assignment scheme, called *Distributed Relative Capacity Loss* (DRCL), which works well in distributed-controlled networks, and we demonstrate the performance of DRCL through simulation.

1 Introduction

Wavelength-division multiplexing (WDM) in optical fiber networks has been rapidly gaining acceptance as a means to handle the ever-increasing bandwidth demands of network users [1]. In a wavelength-routed WDM network, end users communicate with one another via all-optical WDM channels, which are referred to as *lightpaths* [2] (Fig. 1). A lightpath is used to support a *connection* in a wavelength-routed WDM network, and it may span multiple fiber links. In the absence of wavelength converters, a lightpath must occupy the same wavelength on all the fiber links through which it traverses; this property is known as the *wavelength-continuity constraint.* Figure 1 illustrates a wavelength-routed network in which lightpaths have been set up between pairs of access nodes on different wavelengths. In the remainder of this work, we will assume that each optical switch is connected to an access node, and we will refer to this integrated unit as a node.

Given a set of connections, the problem of setting up lightpaths by routing and assigning a wavelength to each connection is called the *routing and wavelength assignment* (RWA) problem. Typically, connection requests may be of three types: *static, incremental*, and *dynamic* [3]. With static traffic, the entire set of connections is known in advance, and the problem is then to set up lightpaths for these connections in a global fashion while minimizing network resources such as the number of wavelengths or the number of fibers in the network. Alternatively, one may attempt to set up as many of these connections as possible for a given fixed number of wavelengths. The RWA problem for static traffic is known as the *Static Lightpath Establishment* (SLE) problem. In the incremental-traffic case, connection requests arrive sequentially, a lightpath is established for each connection, and the lightpath remains in the network indefinitely. For the case of dynamic traffic, a lightpath is set up for each connection request as it arrives, and the lightpath is released after some finite amount of time. The objective in the incremental and dynamic traffic cases is to set up lightpaths and assign wavelengths in a manner which minimizes the amount of connection blocking, or which maximizes the number of connections that are established in the network at any time. This problem is referred to as the *Dynamic Lightpath Establishment* (DLE) problem. In this study, we survey the different approaches to solve both the static and the dynamic RWA problems.

The SLE problem can be formulated as a mixed-integer linear program [4], which is NP-complete [2]. To make the problem more tractable, SLE can be partitioned into two subproblems - (1) routing and (2) wavelength assignment - and each subproblem can be solved separately. The work in [5] proposed practical approximation algorithms to solve the SLE problem for large networks, and graph-coloring algorithms were employed to assign wavelengths to the lightpaths once the lightpaths were routed properly. The DLE problem is more difficult to solve, and therefore, heuristics methods are generally employed. Heuristics exist for both the routing subproblem and the wavelength assignment subproblem.

For the routing subproblem, there are three basic approaches which can be found in the literature: *fixed routing*, *fixed-alternate routing*, and *adaptive routing* [6, 7, 8, 9, 10]. Among these approaches, fixed routing is the simplest while adaptive routing yields the best performance. Alternate routing offers a trade-off between complexity and performance. We will briefly discuss these approaches later in this work.

For the wavelength-assignment subproblem, a number of heuristics have been proposed [11, 12, 13, 14, 15, 16, 17]. These heuristics are *Random Wavelength Assignment*, *First-Fit*, *Least-Used/SPREAD*, *Most-Used/PACK*, *Min-Product*, *Least Loaded*, *MAX-SUM*, *Relative Capacity Loss*, *Wavelength Reservation*, and *Protecting Threshold*. We will illustrate these algorithms later in this work and compare them from a complexity and performance standpoint. Currently, the algorithm which offers the best performance is *Relative Capacity Loss* (RCL) [17]; however, RCL is relatively expensive to implement in a distributed-controlled network, and it may introduce some significant control overhead. Here, we introduce a new heuristic called the *Distributed Relative Capacity Loss* (DRCL) algorithm which is based on RCL and which is more efficient in a distributed environment.

The remainder of this work is organized as follows. Section 2 formulates the SLE problem with combined routing and wavelength assignment. Section 3 focuses purely on the routing problem for both static and dynamic traffic. Section 4 discusses and compares various algorithms for static and dynamic wavelength assignment. In this section we also introduce our new DRCL approach for dynamic wavelength assignment in a distributed environment. Section 5 concludes this study.

2 The Routing and Wavelength Assignment (RWA) Problem

2.1 Static Routing and Wavelength Assignment

In this section, we address the static routing and wavelength assignment (RWA) problem, also known as the Static Lightpath Establishment (SLE) problem. In SLE, lightpath requests are known in advance, and the routing and wavelength assignment operations are performed off-line. The typical objective is to minimize the number of wavelengths needed to set up a certain set of lightpaths for a given physical topology. As an alternative to minimizing the number of wavelengths in the network, the dual problem is to maximize the number of connections that can be established (minimize blocking) for a given number of wavelengths and a given set of connection requests. This dual to the SLE problem raises the issue of *fairness*, in that solutions to this problem will tend to establish more short connections which traverse fewer fiber links than long connections which traverse a greater number of links.

SLE with the *wavelength-continuity constraint*, can be formulated as an integer linear program (ILP) in which the objective function is to minimize the flow in each link, which, in turn, corresponds to minimizing the number of lightpaths passing through a particular link. Let λ_{sdw} denote the traffic (number of connection requests) from any source s to any destination d on any wavelength w. We assume that two or more lightpaths may be set up between the same source-destination pair, if necessary, but that each of them must employ a distinct wavelength; hence, $\lambda_{sdw} \leq 1$. Let F_{ij}^{sdw} denote the traffic (number of connection requests) from source s to destination d on link ij and wavelength w. $F_{ij}^{sdw} \leq 1$ since a wavelength on a link can be assigned to only one path. Given a network physical topology, a set of wavelengths, and the traffic matrix Λ in which Λ_{sd} denotes the number of connections needed between source s and destination d, the problem can be formulated as follows (which turns out to be an integer linear program (ILP)):

$$\begin{aligned}
\textbf{Minimize}: &\quad F_{max} && (1)\\
\textit{such that} &&& \\
F_{max} &\geq \sum_{s,d,w} F_{ij}^{sdw} \ \forall \ ij && (2)\\
\sum_i F_{ij}^{sdw} - \sum_k F_{jk}^{sdw} &= \begin{cases} -\lambda_{sdw} & \text{if } s = j \\ \lambda_{sdw} & \text{if } d = j \\ 0 & \text{otherwise} \end{cases} && (3)\\
\sum_w \lambda_{sdw} &= \Lambda_{sd} && (4)\\
F_{ij}^{sdw} &= 0, 1 && (5)\\
\sum_{s,d} F_{ij}^{sdw} &\leq 1 && (6)
\end{aligned}$$

This approach may also be used to obtain the minimum number of wavelengths required for a given set of connection requests by performing a search on the minimum number of wavelengths in the network. For a given number of wavelengths, we can apply the ILP to see if a solution can be found. If a solution is not found, then a greater number of wavelengths is attempted. This procedure is iterated until the minimum number of wavelengths is found.

The problem as formulated above is NP-complete [18]. Section 3 addresses how a simpler form of the problem can be solved by reducing the problem size and by relaxing the integrality constraints, as outlined in [1].

The alternate problem of maximizing the number of established connections for a fixed number of wavelengths and a given set of connection requests can also be formulated as an ILP as follows [4].

The following are defined:

- N_{sd}: Number of source-destination pairs.
- L: Number of links.
- W: Number of wavelengths per link.
- $\mathbf{m} = \{m_i\}, i = 1, 2, \cdots, N_{sd}$: Number of connections established for source-destination pair i.
- ρ: Offered load (total number of connection requests to be routed).
- $\mathbf{q} = \{q_i\}, i = 1, 2, \cdots, N_{sd}$: Fraction of the load which arrives for source-destination pair i (thus, $q_i\rho$ = number of connections to be set up for source-destination pair i). (This is the definition of load for the static case. The definition of load for the dynamic case is different, and will be provided in Section 4.2.)
- P: Set of paths on which a connection can be routed.
- $A = (a_{ij})$: $P \times N_{sd}$ matrix in which $a_{ij} = 1$ if path i is between source-destination pair j, and $a_{ij} = 0$ otherwise.
- $B = (b_{ij})$: $P \times L$ matrix in which $b_{ij} = 1$ if link j is on path i, and $b_{ij} = 0$ otherwise.
- $C = (c_{ij})$: $P \times W$ route and wavelength assignment matrix, such that $c_{ij} = 1$ if wavelength j is assigned to path i, and $c_{ij} = 0$ otherwise.

The objective of the routing and wavelength assignment problem is to maximize the number of established connections, $C_0(\rho, \mathbf{q})$. The ILP formulation is as follows:

$$\textbf{Maximize}: \qquad C_0(\rho, \mathbf{q}) = \sum_{i=1}^{N_{sd}} m_i \tag{7}$$

$$\textit{subject to}$$

$$m_i \geq 0, \qquad \text{integer}, \quad i = 1, 2, \cdots, N_{sd} \tag{8}$$

$$c_{ij} \in \{0, 1\} \qquad i = 1, 2, \cdots, P, \quad j = 1, 2, \cdots, W \tag{9}$$

$$C^T B \leq 1_{W \times L} \tag{10}$$

$$\mathbf{m} \leq 1_W C^T A \tag{11}$$

$$m_i \leq q_i \rho, \qquad i = 1, 2, \cdots, N_{sd} \tag{12}$$

Equation (7) gives the total number of established connections in the network. Equation (10) specifies that a wavelength can be used at most once on a given link, where $1_{W \times L}$ is the $W \times L$ matrix whose elements are unity. Equations (11) and (12) ensure that the number of established connections is less than the number of requested connections, where 1_W is the $1 \times W$ matrix whose elements are unity.

2.2 Routing and Wavelength Assignment with Wavelength Conversion

To complete the discussion of RWA, we briefly address wavelength conversion in this section. In a wavelength-routed WDM network, the wavelength-continuity constraint can be eliminated if we are able to use a *wavelength converter* to convert the data arriving on one wavelength on a link into another wavelength at an intermediate node before forwarding it on the next link. Such a technique is feasible and is referred to as *wavelength conversion*. Wavelength-routed networks with this capability are referred to as *wavelength-convertible* networks [19]. If a wavelength converter provides the ability to convert from any wavelength to any other wavelength (such wavelength converters are said to have full-range capacity), and if there is one wavelength converter for each fiber link in every node of the network, then the network is said to have *full wavelength-conversion* capabilities. A wavelength-convertible network with full wavelength-conversion capability at each node is equivalent to a circuit-switched telephone network; thus, only the routing problem needs to be addressed, and wavelength assignment is not an issue.

Notice that a single lightpath in such a wavelength-convertible network can possibly use a different wavelength along each of the links in its path. Thus, wavelength conversion may improve the efficiency in the network by resolving the wavelength conflicts of the lightpaths. Usually, for a given a routing scheme, wavelength conversion provides a lower bound on the achievable blocking probability for a given wavelength-assignment scheme.

Let λ_{sd} denote the traffic (number of connection requests) from any source s to any destination d. Let F_{ij}^{sd} denote the traffic (number of connection requests) from source s to destination d on link ij. The formulation of the RWA problem without the wavelength-continuity constraint is as follows:

$$\textbf{Minimize}: \qquad F_{max} \tag{13}$$

$$\textit{such that}$$

$$F_{max} \geq \sum_{s,d} F_{ij}^{sd} \;\; \forall \;\; ij \tag{14}$$

$$\sum_i F_{ij}^{sd} - \sum_k F_{jk}^{sd} = \begin{cases} -\lambda_{sd} & \text{if } s = j \\ \lambda_{sd} & \text{if } d = j \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

The “dual” version of the RWA problem with wavelength conversion is straightforward and is not shown here.

In many cases, full wavelength conversion in the network may not be preferred and may not even be necessary due to high costs and limited performance gains. It is possible that either a subset of the nodes allow wavelength conversion, a wavelength converter is shared by more than one fiber link, or a node employs converters which can only convert to a limited range of wavelengths. The problems related to designing a wavelength-routed WDM network with limited wavelength conversion are as follows (see [20] for details):

1. **Sparse location of wavelength converters in the network**: As long as wavelength converters remain expensive, it may not be economically viable to equip all the nodes in a WDM network with these devices. The effects of sparse conversion (i.e., having only a few converting switches in the network) on connection-blocking have been examined in [21]. An interesting question is *where* (optimally?) to place these few converters in an arbitrary network and what is the likely *upgrade-path* towards full-fledged convertibility? A heuristic technique for the placement of these sparse converters is presented in [22].

2. **Sharing of converters**: Even among the switches capable of wavelength conversion, it may not be cost-effective to equip all of the output ports of a switch with this capability. Designs of switch architectures have been proposed which allow sharing of converters among the various signals at a switch. It has been shown in [23] that the performance of such a network saturates when the number of converters at a switch increases beyond a certain threshold. An interesting problem is to quantify the dependence of this threshold on the routing algorithm used and the blocking probability desired [20].

3. **Limited-range wavelength conversion**: Four-wave-mixing-based all-optical wavelength converters provide only a limited-range conversion capability. If the range is limited to k, then an input wavelength λ_i can only be converted to wavelengths $\lambda_{max(i-k,1)}$ through $\lambda_{min(i+k,w)}$, where w is the number of wavelengths in the system (indexed 1 through W). Analysis shows that networks employing such devices, however, compare favorably with those utilizing converters with full-range capability, under certain conditions [20, 24]. Limited-range wavelength conversion can also be provided at nodes using opto-electronic conversion techniques [25].

3 Routing

Although combined routing and wavelength assignment is a hard problem, it can be simplified by decoupling the problem into two separate subproblems: the routing subproblem and the wavelength assignment subproblem. In this section, we focus on various approaches to routing connection requests.

3.1 Fixed Routing

The most straightforward approach to routing a connection is to always choose the same fixed route for a given source-destination pair. One example of such an approach is *fixed shortest-path routing*. The shortest-path route for each source-destination pair is calculated off-line using standard shortest-path algorithms, such as Dijkstra's algorithm or the Bellman-Ford algorithm, and any connection between the specified pair of nodes is established using the pre-determined route. In Fig. 2, the fixed shortest-path route from Node 0 to Node 2 is illustrated. This approach to routing connections is very simple; however, the disadvantage of such an approach is that, if resources (wavelengths) along the path are tied up, it can potentially lead to high blocking probabilities in the dynamic case, or may result in a large number of wavelengths being used in the static case. Also, fixed routing may be unable to handle fault situations in which one or more links in the network fail. To handle link faults, the routing scheme must either consider alternate paths to the destination, or must be able to find the route dynamically. Note that, in Fig. 2, a connection request from Node 0 to Node 2 will be blocked if a common wavelength is not available on both links in the fixed route, or if either of the links in the fixed route is cut.

3.2 Fixed-Alternate Routing

An approach to routing which considers multiple routes is *fixed-alternate routing*. In fixed-alternate routing, each node in the network is required to maintain a routing table which contains an ordered list of a number of fixed routes to each destination node. For example, these routes may include the shortest-path route, the second-shortest-path route, the third-shortest-path route, etc. A primary route between a source node s and a destination node d is defined as the first route in the list of routes to node d in the routing table at node s. An alternate route between s and d is any route which does not share any links (is link-disjoint) with the first route in the routing table at s. The term "alternate routes" is also employed to describe all routes (including the primary route) from a source node to a destination node. Figure 3 illustrates a primary route (solid line) from Node 0 to Node 2, and an alternate route (dashed line) from Node 0 to Node 2.

When a connection request arrives, the source node attempts to establish the connection on each of the routes from the routing table in sequence, until a route with a valid wavelength assignment is found. If no available route is found from the list of alternate routes, then the connection request is blocked and lost. In most cases, the routing tables at each node are ordered by the number of fiber link segments (hops) to the destination. Therefore, the shortest path to the destination is the first route in the routing table. When there are ties in the distance between different routes, one route may be selected at random. Fixed-alternate routing provides simplicity of control for setting up and tearing down lightpaths, and it may also be used to provide some degree of fault tolerance upon link failures as will be discussed in Section 3.5. Another advantage of fixed-alternate routing is that it can significantly reduce the connection blocking probability compared to fixed routing. It has also been shown that, for certain networks, having as few as two alternate routes provides significantly lower blocking probabilities than having full wavelength conversion at each node with fixed routing [10].

3.3 Adaptive Routing

In *adaptive routing*, the route from a source node to a destination node is chosen dynamically, depending on the network state. The network state is determined by the set of all connections that are currently in progress. One form of adaptive routing is adaptive shortest-cost-path routing, which is well-suited for use in wavelength-converted networks. Under this approach, each unused link in the network has a cost of 1 unit, each used link in the network has a cost of ∞, and each wavelength-converter link has a cost of c units. If wavelength conversion is not available, then $c = \infty$. When a connection arrives, the shortest-cost path between the source node and the destination node is determined. If there are multiple paths with the same distance, one of them is chosen randomly. By choosing the wavelength-conversion cost c appropriately, we can ensure that wavelength-converted routes are chosen only when wavelength-continuous paths are not available. In shortest-cost adaptive routing, a connection is blocked only when there is no route (either wavelength-continuous or wavelength-converted) from the source node to the destination node in the network. Adaptive routing requires extensive support from the control and management protocols to continuously update the routing tables at the nodes. An advantage of adaptive routing is that it results in lower connection blocking than fixed and fixed-alternate routing. For the network in Fig. 4, if the links (1,2) and (4,2) in the network are busy, then the adaptive-routing algorithm can still establish a connection between Nodes 0 and 2, while both the fixed-routing protocol and the fixed-alternate routing protocols with fixed and alternate paths as shown in Fig. 3 would block the connection.

Another form of adaptive routing is *least-congested-path* (LCP) routing [6]. Similar to alternate routing, for each source-destination pair, a sequence of routes is pre-selected. Upon the arrival of a connection request, the least-congested path among the pre-determined routes is chosen. The congestion on a link is measured by the number of wavelengths available on the link. Links which have fewer available wavelengths are considered to be more congested. The congestion on a path is indicated by the congestion on the most congested link in the path. If there is a tie, then shortest-path routing may be used to break the tie. An alternate implementation is to always give priority to shortest paths, and to use LCP only for breaking ties.

Both combinations are examined through simulation in [6], and it has been shown that using shortest-path routing first and LCP second works better than using LCP alone.

A disadvantage of LCP is its computational complexity. In choosing the least-congested path, all links on all candidate paths have to be examined. A variant of LCP is proposed in [8] which only examines the first k links on each path (referred to as the source's neighborhood information), where k is a parameter to the algorithm. It has been shown that, when $k = 2$, this algorithm can achieve similar performance to fixed-alternate routing. It is also shown in [8] that LCP performs much better than fixed-alternate routing.

3.4 ILP Formulation for Static Lightpath Establishment

Similar to RWA (Section 2), the routing problem can also be formulated as an ILP in which we minimize the maximum number of lightpaths on any given link. The primary difference between this formulation and the previous formulation is that this formulation does not impose the wavelength-continuity constraint. Instead, wavelength continuity is imposed when actually assigning wavelengths to the lightpaths. Let λ_{sd} denote the traffic (in terms of connection requests) from any source s to any destination d. Let F_{ij}^{sd} denote the traffic (in terms of the number of connections) that is flowing from source s to destination d on link ij. The ILP formulation, which is the same as that for the wavelength-conversion case, can be found in Equations (13) through (15).

This problem is NP-complete [18] but can be approximated successfully by limiting the search space, and by utilizing *randomized rounding* [1]. The search space can be reduced by considering only a limited subset of possible links for a route between a given source-destination pair. The number of constraint equations can be further reduced through the use of randomized rounding. In randomized rounding, the problem is cast as a multicommodity flow problem in which each lightpath corresponds to a single commodity which must be routed from a source to a destination. The flow of a commodity in each link must be either 0 or 1. The problem of minimizing the flow on each link is NP-complete, but the non-integral version of the problem in which the flows of each commodity may take on any value between 0 and 1 can be solved by a suitable linear programming (LP) method. The fractional flows provided by the LP solution must then be converted to to integer flows. This conversion first utilizes (1) path stripping, in which we find a set of possible alternate routes for each lightpath and assign weights to each possible route, and then (2) randomized selection, in which one route is randomly selected for each lightpath according to the weights assigned by path stripping.

This approach to routing the connections is combined with *graph coloring* (described in Section 4.1) to solve the SLE problem, and the corresponding results are very close to the lower bound for the number of wavelengths that are needed to establish a given set of lightpaths.

3.5 Fault-Tolerant Routing

When setting up connections in a wavelength-routed optical WDM network, it is often desirable to provide some degree of protection against link and node failures in the network by reserving some amount of spare capacity [10, 26]. A common approach to protection is to set up two link-disjoint lightpaths (the routes for the lightpaths do not share any common links) for every connection request. One lightpath, called the primary lightpath, is used for transmitting data, while the other lightpath is reserved as a backup in the event that a link in the primary lightpath fails. This approach can be used to protect against any single-link failures in the network (a situation in which any one physical fiber link in the network fails). To further protect against node failures, the primary and alternate paths may also be node-disjoint.

Fixed-alternate routing provides a straightforward approach to handling protection. By choosing the alternate paths such that their routes are link disjoint from the primary route, we can protect the connection from any single-link failures by allocating one of the alternate paths as a backup path.

In adaptive routing, a protection scheme may be implemented in which the backup path is set up immediately after the primary path has been established. The same routing protocol may be used to determine the backup path, with the exception that a link cost is set to ∞ if that link is being used by the primary path on any wavelength. The resulting route will then be link-disjoint from the primary path. An alternative is to implement restoration, in which the backup path is determined dynamically after the failure has occurred. Restoration will only be successful if sufficient resources are available in the network. Note also that, when a fault occurs, dynamic discovery and establishment of a backup path under the restoration approach might take significantly longer than switching over to the pre-established backup path using the protection approach.

The static formulation in Section 3.4 may also be extended to provide for fault protection in the network. The modified formulation would include additional constraint equations requiring that two lightpaths be set up for each connection (one primary lightpath and one backup lightpath), and that the routes for these two lightpaths do not share any links.

For further information regarding protection and restoration, the reader is referred to [10, 26].

4 Wavelength Assignment

In this section, we first study the static wavelength-assignment problem, i.e., given a set of lightpaths and their routes, assign a wavelength to each lightpath such that no two lightpaths share the same wavelength on a given fiber link. One approach to solving this problem is to formulate it as a graph-coloring problem [1].

We then turn to the dynamic wavelength-assignment problem, and discuss 10 wavelength-assignment heuristics. We also introduce a new distributed wavelength-assignment scheme called *Distributed Relative Capacity Loss* (DRCL). These heuristics may also be applied to the static wavelength-assignment problem by ordering the lightpaths and then sequentially assigning wavelengths to the ordered lightpaths.

4.1 The Static Wavelength-Assignment Problem

Once a path has been chosen for each connection, the number of lightpaths traversing any physical fiber link defines the *congestion* on that particular link. Wavelengths must be assigned to each lightpath such that any two lightpaths which are sharing the same physical link are assigned different wavelengths.

Assigning wavelengths to different lightpaths in a manner which minimizes the number of wavelengths used under the wavelength-continuity constraint reduces to the graph-coloring problem, as stated below.

1. Construct an auxiliary graph $G(V, E)$, such that each lightpath in the system is represented by a node in graph G. There is an undirected edge between two nodes in graph G if the corresponding lightpaths pass through a common physical fiber link (see Figs. 5 and 6).

2. Color the nodes of the graph G such that no two adjacent nodes have the same color.

This problem has been shown to be NP-complete, and the minimum number of colors needed to color a graph G (called the chromatic number $\chi(G)$ of the graph) is difficult to determine. However, there are efficient *sequential graph-coloring* algorithms which are optimal in the number of colors used.

In a *sequential graph-coloring* approach, vertices are sequentially added to the portion of the graph already colored, and new colorings are determined to include each newly adjoined vertex. At each step, the total number of colors necessary is kept to a minimum. It is easy to observe that some particular sequential vertex coloring will yield a $\chi(G)$ coloring. To see this, let T_i be the vertices colored i by a $\chi(G)$ coloring of G. Then, for any ordering of the vertices $V(G)$, which has all members of T_i before any member of T_j for $1 \leq i \leq j \leq \chi(G)$, the corresponding sequential coloring will be a $\chi(G)$ coloring.

It is also easy to note that, if $\Delta(G)$ denotes the maximum degree in a graph, then $\chi(G) \leq \Delta(G) + 1$. However, intuitively, if a graph has only a few nodes of very large degree, then coloring these nodes early will avoid the need for using a very large set of colors. This gives rise to the following theorem:

Theorem: Let G be a graph with $V(G) = v_1, v_2, \ldots, v_n$ where $\deg(v_i) \geq \deg(v_{i+1})$ for $i = 1, 2, \ldots, n-1$, and n is the number of nodes in G. Then $\chi(G) \leq \max_{1 \leq i \leq n} \min \{i, 1 + \deg(v_i)\}$. Determination of a sequential coloring procedure corresponding to such an ordering will be termed the *largest-first* algorithm. The proof is straightforward and can be found in [27].

A closer inspection of the sequential coloring procedure shows that, for a given ordering $v_1, v_2, \ldots, v_n$ of the vertices of a graph G, the corresponding sequential coloring algorithm could never require more than k colors where

$$k = \max_{1 \le i \le n} \{1 + \deg_{<v_1, v_2, \ldots, v_n>}(v_i)\}$$

and $\deg_{<v_1, v_2, \ldots, v_n>}(v_i)$ refers to the degree of node v_i in the vertex-induced subgraph denoted by $< v_1, v_2, \ldots, v_n >$. The determination of a vertex ordering that minimizes k was derived in [28] and can be found in the following procedure:

1. For $n = \mid V(G) \mid$, let v_n be chosen to have minimum degree in G.
2. For $i = n-1, n-2, \ldots, 2, 1$, let v_i be chosen to have minimum degree in $< V(G) - v_n, v_{n-1}, \ldots, v_{i+1} >$.

For any vertex ordering $v_1, v_2, \ldots, v_n$ determined in this manner, we must have

$$\deg_{<v_1, v_2, \ldots, v_i>}(v_i) = \min_{1 \le j \le i} \deg_{<v_1, v_2, \ldots, v_i>}(v_j)$$

for $1 \le i \le n$, so that such an ordering will be termed a *smallest-last* (SL) vertex ordering. The fact that any smallest-last vertex ordering minimizes k over the $n!$ possible orderings is shown in [28]. Applying SL vertex ordering to the graph in Fig. 6 and using the node index to break ties, we obtain the following ordering: $< 2, 5, 1, 6, 3, 4, 7, 8 >$. Note that this ordering yields 3 wavelengths, which is the minimum number of wavelengths required for the set of lightpaths in Fig. 5.

4.2 Wavelength-Assignment Heuristics

For the case in which lightpaths arrive one at a time (either incremental or dynamic traffic), heuristic methods must be used to assign wavelengths to lightpaths. For the dynamic problem, instead of attempting to minimize the number of wavelengths as in the static case, we assume that the number of wavelengths is fixed (this is the practical situation), and we attempt to minimize connection blocking.

The following heuristics have been proposed in the literature: (1) *Random*, (2) *First-Fit*, (3) *Least-Used/SPREAD*, (4) *Most-Used/PACK*, (5) *Min-Product*, (6) *Least Loaded*, (7) *MAX-SUM*, (8) *Relative Capacity Loss*, (9) *Wavelength Reservation*, and (10) *Protecting Threshold*. These heuristics can all be implemented as on-line algorithms and can be combined with different routing schemes. The first eight schemes attempt to reduce the overall blocking probability for new connections, while the last two approaches aim to reduce the blocking probability for connections which traverse more than one link. In our discussions, we will use the following notation and definitions:

- L: Number of links.
- M_l: Number of fibers on link l
- M: Number of fibers per link if all links contain the same number of fibers.
- W: Number of wavelengths per fiber.
- $\pi(p)$: Set of links comprising path p.
- S_p: Set of available wavelengths along the selected paths p.
- D: L-by-W matrix, where D_{lj} indicates the number of assigned fibers on link l and wavelength j. Note that the value of D_{lj} varies between 0 and M_l.

- Load: For dynamic traffic, the holding time is exponentially distributed with a normalized mean of one unit, and connection arrivals are Poisson; thus, load is expressed in units of Erlangs.

We describe the wavelength-assignment heuristics below.

1. Random Wavelength Assignment (R).
   This scheme first searches the space of wavelengths to determine the set of all wavelengths that are available on the required route. Among the available wavelengths, one is chosen randomly (usually with uniform probability).

2. First-Fit (FF).
   In this scheme, all wavelengths are numbered. When searching for available wavelengths, a lower-numbered wavelength is considered before a higher-numbered wavelength. The first available wavelength is then selected. This scheme requires no global information. Compared to Random wavelength assignment, the computation cost of this scheme is lower because there is no need to search the entire wavelength space for each route. The idea behind this scheme is to pack all of the in-use wavelengths towards the lower end of the wavelength space so that continuous longer paths towards the higher end of the wavelength space will have a higher probability of being available. This scheme performs well in terms of blocking probability and fairness, and is preferred in practice because of its small computational overhead and low complexity. Similar to Random, FF does not introduce any communication overhead because no global knowledge is required.

3. Least-Used (LU)/SPREAD.
   LU selects the wavelength that is the least used in the network, thereby attempting to balance the load among all the wavelengths. This scheme ends up breaking the long wavelength paths quickly; hence, only connection requests which traverse a small number of links will be serviced in the network. The performance of LU is worse than Random, while also introducing additional communication overhead (e.g., global information is required to compute the least-used wavelength). The scheme also requires additional storage and computation cost; thus, LU is not preferred in practice.

4. Most-Used (MU)/PACK.
   MU is the opposite of LU in that it attempts to select the most-used wavelength in the network. It outperforms LU significantly [16]. The communication overhead, storage, and computation cost are all similar to those in LU. MU also slightly outperforms FF, doing a better job of packing connections into fewer wavelengths and conserving the spare capacity of less-used wavelengths.

5. Min-Product (MP).
   MP is used in multi-fiber networks [14]. In a single-fiber network, MP becomes FF. The goal of MP is to pack wavelengths into fibers, thereby minimizing the number of fibers in the network. MP first computes
   $$\prod_{l \in \pi(p)} D_{lj}$$
   for each wavelength j, i.e., $1 \leq j \leq W$. If we let X denote the set of wavelengths j that minimizes the above value, then MP chooses the lowest-numbered wavelength in X.

   As shown in [16], MP does not perform as well as the multi-fiber version of FF in which the fibers, as well as the wavelengths, are ordered. MP also introduces additional computation costs.

6. Least-Loaded (LL).
   The LL heuristic, like MP, is also designed for multi-fiber networks [15]. This heuristic selects the wavelength that has the largest residual capacity on the most-loaded link along route p. When used in single-fiber networks, the residual capacity is either 1 or 0; thus, the heuristic chooses the lowest-indexed wavelength with residual capacity 1. Thus, it reduces to FF in single-fiber networks.

LL selects the minimum indexed wavelength j in S_p that achieves

$$\max_{j \in S_p} \min_{l \in \pi(p)} (M_l - D_{lj}).$$

In [15], it is shown that LL outperforms MU and FF in terms of blocking probability in a multi-fiber network.

7. MAX-SUM ($M\Sigma$).
$M\Sigma$ [12, 16] was proposed for multi-fiber networks but it can also be applied to the single-fiber case. $M\Sigma$ considers all possible paths (lightpaths with their pre-selected routes) in the network and attempts to maximize the remaining path capacities after lightpath establishment. It assumes that the traffic matrix (set of possible connection requests) is known in advance, and that the route for each connection is pre-selected. These requirements can be achieved since the traffic matrix is assumed to be stable for a period of time, and routes can then be computed for each potential path on the fly.

   To describe the heuristic, we introduce the following notation. Let ψ be a network state which specifies the existing lightpaths (routes and wavelength assignments) in the network. In $M\Sigma$, the *link capacity* on link l and wavelength j in state ψ, $r(\psi, l, j)$, is defined as the number of fibers on which wavelength j is unused on link l, i.e.,

$$r(\psi, l, j) = M_l - D(\psi)_{lj},$$

   where $D(\psi)$ is the D matrix in state ψ.

   The path capacity $r(\psi, p, j)$ on wavelength j is the number of fibers on which wavelength j is available on the most-congested link along the path p, i.e.,

$$r(\psi, p, j) = \min_{l \in \pi(p)} r(\psi, l, j).$$

   The *path capacity* of path p in state ψ is the sum of path capacities on all wavelengths, i.e.,

$$R(\psi, p) = \sum_{j=1}^{W} \min_{l \in \pi(p)} c(\psi, l, j).$$

   Let

   - $\Omega(\psi, p)$ be the set of all possible wavelengths that are available for the lightpath which is routed on path p, and
   - $\psi'(j)$ be the next state of the network if wavelength j is assigned to the connection.

   $M\Sigma$ chooses the wavelength j that maximizes the quantity

$$\sum_{p \in P} R(\psi'(j), p),$$

   where P is the set of all potential paths for the connection request in the current state. Once the lightpath for the connection has been established, the network state is updated and the next connection request may be processed. We shall illustrate how this algorithm works with an example later in this section.

8. Relative Capacity Loss (RCL).
RCL was proposed in [17] and is based on $M\Sigma$. $M\Sigma$ can also be viewed as an approach which chooses the wavelength j that minimizes the capacity loss on all lightpaths, which is

$$\sum_{p \in P} (R(\psi, p) - R(\psi'(j), p)),$$

where ψ is the network state before the lightpath is set up.

Since only the capacity on wavelength j will change after the lightpath is set up on wavelength j, $M\Sigma$ chooses wavelength j to *minimize the total capacity loss* on this wavelength, i.e.,

$$\sum_{p \in P} (r(\psi, p, j) - r(\psi'(j), p, j)).$$

On the other hand, RCL chooses wavelength j to minimize the *relative* capacity loss, which can be computed as

$$\sum_{p \in P} (r(\psi, p, j) - r(\psi'(j), p, j))/r(\psi, p, j).$$

RCL is based on the observation that minimizing total capacity loss sometimes does not lead to the best choice of wavelength. When choosing wavelength i would block one lightpath p_1, while choosing wavelength j would decrease the capacity of lightpaths p_2 and p_3, but not block them, then wavelength j should be chosen over wavelength i, even though the total capacity loss for wavelength j is greater than the total capacity loss for wavelength i. Thus, RCL calculates the *Relative Capacity Loss* for each path on each available wavelength and then chooses the wavelength that minimizes the sum of the relative capacity loss on all the paths.

Both $M\Sigma$ and RCL can be used for non-uniform traffic by taking a weighted sum over the capacity losses. RCL has been shown to perform better than $M\Sigma$ in most cases [17]. Illustrative examples comparing the performance of the various wavelength-assignment schemes in terms of connection blocking will be presented later in this work.

Thus far, the wavelength-assignment schemes which we have described attempt to minimize the blocking probability. However, considering that longer lightpaths have a higher probability of getting blocked than shorter paths, some schemes attempt to protect longer paths. These schemes are *wavelength reservation* (Rsv) and *protecting threshold* (Thr) [13]. Rsv and Thr differ from other wavelength-assignment schemes in two ways: First, they do not specify which wavelength to choose, but instead specify whether or not the connection request can be assigned a wavelength under the current wavelength-usage conditions. Hence, they can not work alone and must be combined with other wavelength-assignment schemes. Second, other schemes aim at minimizing the overall blocking probability for *all* connection requests, while the Rsv and Thr schemes attempt to protect only the connections which traverse multiple fiber links (multihop connections). Therefore when these two schemes are used, the overall blocking probability performance in the network may be higher, but a greater degree of fairness can be achieved, in that connections which traverse multiple fiber links will not have significantly higher blocking probabilities than connections which traverse only a single fiber link.

9. Wavelength Reservation (Rsv).
   In Rsv, a given wavelength on a specified link is reserved for a traffic stream, usually a multihop stream. For example, in Fig. 2, wavelength λ_1 on link (1,2) may be reserved only for connections from Node 0 to Node 3; thus, a connection request from Node 1 to Node 2 cannot be set up on λ_1 link (1,2), even if the wavelength is idle. This scheme reduces the blocking for multihop traffic, while increasing the blocking for connections which traverse only one fiber link (single-hop traffic) [13].

10. Protecting Threshold (Thr).
    In Thr, a single-hop connection is assigned a wavelength only if the number of idle wavelengths on the link is at or above a given threshold [13].

The above wavelength-assignment schemes can work on-line since they make use of the current network state information. It is straight-forward to show that they can also work off-line for static network traffic by handling the static set of lightpaths sequentially. An additional issue when applying these heuristics to the static problem is how to order the lightpaths when assigning wavelengths. Approaches similar to those in Section 4.1 may be applied.

4.2.1 Illustrative Example

We use an example to illustrate how the above wavelength-assignment schemes work in a single-fiber network. This example was borrowed from [17] and was initially used to illustrate the $M\Sigma$ and RCL heuristics.

Consider a six-link segment of a single-fiber network spanning a tandem sequence of seven nodes (numbered 0 through 6) with a current wavelength-usage pattern as shown in Fig. 7. If we want to set up a lightpath P_1: (2, 4), we observe that four wavelengths (λ_0 through λ_3) are available.

In the Random scheme, any of the four wavelengths can be chosen with equal probability.

If First-Fit is used, λ_0 will be assigned. λ_0 will also be assigned in Min-Product and Least-Loaded, since they reduce to First-Fit in single-fiber networks.

Since λ_0, λ_1, and λ_3 are each used on two out of the six links in the network and λ_2 is used only on one link, Least-Used will choose λ_2 and Most-Used will choose either λ_0, λ_1, or λ_3 with equal probability.

When performing the calculations using $M\Sigma$ and RCL, a pre-determined traffic matrix which specifies a set of connections and their paths must be assumed. We consider the case in which there are only three other potential paths which share common links with P_1, and that no other paths need to be considered. These paths are P_2: (1, 5), P_3: (3, 6), and P_4: (0, 3). The total capacity loss (for $M\Sigma$) and total relative capacity loss (for RCL) are calculated in Table 1 and Table 2, respectively. For $M\Sigma$, we observe that setting up lightpath P_1 on wavelength λ_0 will block path P_4 on λ_0, setting up P_1 on λ_1 will block P_3, setting up P_1 on λ_2 will block both P_2 and P_3, and setting up P_1 on λ_3 will block P_2. Choosing λ_2 will result in the highest total capacity loss, or the highest amount of blocking for possible future calls; thus, any of λ_0, λ_1, and λ_3, which have equal total capacity loss, may be chosen by $M\Sigma$. However, note that, by choosing λ_0, path P_4 will be blocked on all wavelengths, whereas if we chose λ_1 or λ_3, each of P_2, P_3, and P_4 would still have at least one wavelength on which they would not be blocked.

RCL attempts to improve on $M\Sigma$ by also taking into consideration the number of available alternate wavelengths for each potential future connection. We observe that path P_2 may choose either of two wavelengths, λ_2 or λ_3; thus, if P_1 is established on either of these wavelengths, then the relative capacity loss for P_2 is $\frac{1}{2}$. Similarly, P_3 has two wavelengths on which a connection can be established; therefore, its relative capacity loss on these wavelengths is also $\frac{1}{2}$. However, a connection on P_4 can only be established on wavelength λ_0; thus, its relative capacity loss is 1 for wavelength λ_0. Summing the relative capacity loss for each wavelength over all paths yields the total relative capacity loss on a given wavelength. Choosing the wavelength with the smallest total relative capacity loss results in either λ_1 or λ_3 being chosen, but not λ_0 or λ_2.

Since Wavelength Reservation and Protecting Threshold must work together with other protecting schemes, their operation is not discussed here.

4.2.2 Analysis of Wavelength-Assignment Algorithms

Approximate analysis has been done for some of the wavelength-assignment algorithms [8, 29, 30], wherein performance issues such as blocking probabilities are studied and validated by simulation. Among these studies, all three types of routing approaches have been analyzed but only two types of wavelength-assignment heuristics, Random and First-Fit, have been analyzed. The work in [29] studied the blocking probability of fixed routing and least-congested routing combined with random wavelength assignment. In [30], the authors gave an analytical model for fixed routing and alternate routing with First-Fit wavelength assignment. The work in [8] analyzed the performance of least-congested routing with random wavelength assignment. Interested readers are referred to these papers.

wavelength	Capacity loss on each path			Total capacity loss on each wavelength
	P_2: (1, 5)	P_3: (3, 6)	P_4: (0, 3)	
λ_3	1	0	0	1
λ_2	1	1	0	2
λ_1	0	1	0	1
λ_0	0	0	1	1

Table 1: The calculation in $M\Sigma$.

wavelength	RCL on each path			Total RCL loss on each wavelength
	P_2: (1, 5)	P_3: (3, 6)	P_4: (0, 3)	
λ_3	0.5	0	0	0.5
λ_2	0.5	0.5	0	1
λ_1	0	0.5	0	0.5
λ_0	0	0	1	1

Table 2: The calculation in RCL.

wavelength	RCL from source to each destination					Total capacity loss on each wavelength
	(2, 0)	(2, 1)	(2, 3)	(2, 5)	(2, 6)	
λ_3	0	1/3	1/4	1/3	0	11/12
λ_2	0	1/3	1/4	1/3	1/2	17/12
λ_1	0	0	1/4	1/3	1/2	13/12
λ_0	1	1/3	1/4	0	0	19/12

Table 3: The calculation in DRCL.

4.2.3 Simulation Results

We compare the first eight wavelength-assignment heuristics via simulation. Fixed routing is used in all simulations as required by $M\Sigma$ and RCL. Simulations are carried out on the network shown in Fig. 8, and each link in the network contains of M fibers, and each fiber supports W wavelengths. Results are shown in Figs. 9 through 11 for different values of M and W.

A distributed link-state control protocol [31] is used in our simulations, and the results depend on the propagation delays in the network. This approach differs significantly from that in [16], in which centralized control is used and no propagation delays are assumed. In the link-state protocol, each node has full information regarding the network state. When a lightpath is set up, the appropriate information is broadcast to all nodes which then update their state information. Since it takes a certain amount of time for the information to reach all of the nodes, some nodes may make routing and wavelength-assignment decisions based on outdated information if connection requests are arriving at a high rate. These decisions based on outdated information can lead to higher blocking probabilities; thus, heuristics which rely on more state information may potentially have higher blocking probabilities if the propagation delay is high and connection-arrival rates are high.

In the single-fiber case (Fig. 9), MU is found to achieve the best performance under low load while $M\Sigma$ and RCL work well when the load is high ($\geq$ 50 Erlangs) with the other approaches not that far behind. When the number of fibers per link is two ($M = 2$), MU, MP, and RCL perform well under low load, while LL and $M\Sigma$ offer better performance under a higher load (Fig. 10). When the number of fibers per link is four, ($M = 4$), LL appears to give the best performance (Fig. 11). In each case, we observe that the difference among the various heuristics is not too significant.

4.2.4 Computational Complexity

We now address the issue of computational complexity for the various heuristics. Random and FF are the simplest in terms of computational complexity and their running times are on the order of $O(W)$, where W is the number of wavelengths in the network.

LU and MU are more complex than Random and FF. Considering a single-fiber network with W wavelengths and L links, LU and MU will run in $O(WL)$. For a multi-fiber network with M fibers on each link, these heuristics will run in $O(WLM)$.

MP and LL are both usable in multi-fiber networks. Let K denote the number of nodes in such a network. MP calculates $\prod_{l \in \pi(p)} A_{lj}$ for all W wavelengths and then chooses the wavelength that minimizes the product. The number of links on a path is bounded by $O(N)$. Hence, the computation in MP will take $O(KW)$. The calculation in LL is similar and also takes $O(KW)$.

$M\Sigma$ and RCL are relatively expensive. If we consider the number of paths in a single-fiber network which share common links with a given path, the worst case running time is on the order of $O(K^2)$. To calculate the capacity on each such paths, all the links along that path have to be examined for the minimal number of available wavelengths. The number of links on a path is bounded by $O(K)$. Hence, in the worst case, we have $O(WK^2)$ cells in the table we used to calculate $M\Sigma$ or RCL and filling each cell takes at most $O(K)$. The overall computation cost will be $O(WK^3)$, which is very expensive.

Rev and Thr will both take constant time.

4.3 Our Proposal: Distributed Relative Capacity Loss (DRCL)

There are additional costs in implementing algorithms LU, MU, MP, LL, $M\Sigma$, and RCL which involve global knowledge of the network state in a distributed-controlled network. Information on the network state must be exchanged frequently to ensure accurate calculations, similar to what must be done in implementing the link-state routing protocol. $M\Sigma$ and RCL perform well but are difficult and expensive to implement in a distributed environment. Furthermore, $M\Sigma$ and RCL both require fixed routing, which makes it difficult to improve network performance. In order to implement an effective wavelength-selection policy in a distributed adaptive-routing environment, two problems have to be solved:

- how is information of network state exchanged? and
- how can we reduce the amount of calculation upon receiving a connection request?

To speed up the wavelength-assignment procedure, each node in the network stores information on the capacity loss on each wavelength so that only a table lookup and a small amount of calculation is required upon the arrival of a connection request. To maintain a valid table, the related values should be updated as soon as the network state has changed. To simplify the computation, we propose an algorithm called *Distributed Relative Capacity Loss* (DRCL). The routing is implemented using the Bellman-Ford algorithm [32]. In Bellman-Ford, each node exchanges routing tables with its neighboring nodes and updates its own routing table accordingly. We introduce an RCL table at each node and allow the nodes to exchange their RCL tables as well. The RCL tables are updated in a similar manner as the routing tables. Each entry in the RCL table is a triple of (wavelength w, destination d, $rcl(w,d)$). When a connection request arrives and more than one wavelength is available on the selected path, computation is carried out among these wavelengths. Similar to the manner in which $M\Sigma$ and RCL consider a set of potential paths for future connections, DRCL considers all of the paths from the source node of the arriving connection request to every other node in the network, excluding the destination node of the arriving connection request. DRCL then chooses the wavelength that minimizes the sum of $rcl(w,d)$ over all possible destinations d. The $rcl(w,d)$ at node s is calculated as follows:

- If there is no path from node s to node d on wavelength w, then $rcl(w,d) = 0$; otherwise,
- If there is a direct link from node s to node d, and the path from s to d on wavelength w is routed through this link (note that it is possible for a direct link to exist between two nodes, but for the path to be routed around this link), then $rcl(w,d) = 1/k$, where k is the number of available wavelengths on this link through which s can reach d; otherwise,
- If the path from node s to node d on wavelength w starts with node n (n is s's next node for destination d on wavelength w), and there are k wavelengths available on link $s \rightarrow n$ through which s can reach d, then $rcl(w,d)$ at node s is set to $\max(1/k, rcl(w,d)$ at node $n)$.

Table 3 shows the computation carried out by DRCL for the same example given in Fig. 7 and Section 4.2.1. If we are attempting to set up a connection on path (2,4), we must then calculate the RCL for each of the paths (2, 0), (2, 1), (2, 3), (2, 5), and (2, 6) on each wavelength. The path (2, 0) can only be established on one possible wavelength, λ_0; thus, its RCL on wavelength λ_0 is 1, and its RCL on the other wavelengths is 0. Path (2, 1) can be established on one of three wavelengths, yielding RCL values of $\frac{1}{3}$ for each of these wavelengths. Path (2, 3) can be established on any of the four wavelengths, leading to RCL values of $\frac{1}{4}$ for all wavelengths. Path (2, 5) can be established on three wavelengths, giving RCL values of $\frac{1}{3}$, and path (2, 6) can be established on two wavelengths, yielding RCL values of $\frac{1}{2}$. Note that these RCL values can be calculated at Node 2 only using the RCL tables from the adjacent nodes 1 and 3. Also, the calculations for RCL can be done prior to the arrival of a connection request, reducing the time required to select a wavelength and set up the lightpath. When a connection request arrives, DRCL simply has to sum the RCL values for each wavelength over all destinations excluding the destination of the connection request itself. The wavelength which yields the lowest total relative capacity loss is then selected. In the above example, wavelength λ_3 will be chosen (see Table 3).

DRCL works well with adaptive routing for distributed-controlled networks because it distributes the computation load among all the network nodes, and each node can thus utilize information from other nodes. We have compared the performance of DRCL with other schemes through simulation in a single-fiber network (shown in Fig. 8) and our results are shown in Fig. 12. Specifically, DRCL's performance is compared to those of RCL (with fixed routing) and FF (with fixed and adaptive routing) in Fig. 12. Note

Problems		Approaches	On/Off line	Comments	References
Static RWA		ILP formulation	off line	NP-complete	[1, 4]
Routing		ILP formulation	off line	NP-complete	[1, 4]
		fixed routing alternate routing	on/off line	in order of increasing performance & complexity	[6, 8, 9, 10]
		adaptive routing	on line		
WA	WA (connections and routes are known)	graph coloring	off line	NP-complete	[1, 2]
	WA + fixed routing	LU (SPREAD) Random MP (multi-fiber) FF MU (PACK) LL (multi-fiber) $M\Sigma$ RCL	on/off line	heuristics approximately in order of increasing performance	[11, 12, 14, 15, 16, 17]
	Other WA algorithms	Rsv Thr	on line	must be combined with other WA algorithms	[13]

Table 4: Summary of RWA schemes.

that RCL cannot be implemented with adaptive routing. We observe that DRCL slightly outperforms FF (with adaptive routing) in the reasonable region (the region in which the network performs well in terms of blocking probability, which is 45-65 Erlangs in this network), and they both perform better than RCL and FF with fixed routing. Overall, it appears that the routing scheme has much more of an impact on the performance of the system than the wavelength-assignment scheme. This "routing is more significant" conclusion is consistent with the findings in previous studies [9]. Therefore, it is important to first decide on a good routing mechanism, and then to choose a wavelength-assignment scheme which can easily be implemented in conjunction with the selected routing mechanism.

5 Conclusion

In this work, we have provided an overview of the various approaches that can be used to route and assign wavelengths to connections in a wavelength-routed optical WDM network. The combined routing and wavelength assignment problem can be formulated as an integer linear program (ILP), which is NP-complete. While the use of simplifying assumptions and problem-size reduction may allow the ILP to be solved for small networks, it may be more practical to decouple the RWA problem into its routing component and its wavelength-assignment component for larger networks.

The static routing problem, in which the set of connections which need to be routed is known in advance, may also be formulated as an ILP which is NP-complete. A more traditional approach to routing is to use shortest-path algorithms; however, relying on a single fixed shortest path may lead to high blocking probabilities. It has been shown in the literature that techniques such as fixed-alternate routing and adaptive routing provide significant benefits over fixed-shortest-path routing, and often, these routing approaches even provide improved performance over wavelength conversion [9]. In networks which require protection, having either fixed-alternate or adaptive routing is a requirement [10, 26].

A significant amount of work has been done in the literature to address the issue of wavelength assignment. Table 4 gives an overview of some of the proposed wavelength-assignment algorithms. Graph-coloring heuristics can be applied to the static case in which all connections and their routes are known in advance. For dynamic traffic in which connection requests arrive one at a time, there are a number of possible heuristics. It is found that more complicated heuristics, such as Max-Sum and RCL, provide lower blocking probability than simpler heuristics; however, the difference in performance among the various heuristics is not very large (as was shown in Figs. 9 through 11). Also, these two heuristics rely on fixed routing and cannot be directly applied within an adaptive-routing environment.

We introduced a new wavelength-assignment algorithm, called Distributed Relative Capacity Loss (DRCL), which is based on RCL. The proposed approach is well suited for a distributed-controlled network in which adaptive routing is utilized, and it was shown that DRCL performs as well as the other wavelength-assignment heuristics in an adaptive-routing environment. Since routing decisions play a significant role in determining the blocking performance of a network, it is critical to choose a wavelength-assignment scheme, not based solely on its blocking performance relative to other wavelength-assignment schemes, but also based on its compatibility with the chosen routing protocol.

Topics of our ongoing investigation include the comparison of RCL and DRCL in several other aspects, such as bandwidth requirement of control messages and computation overhead.

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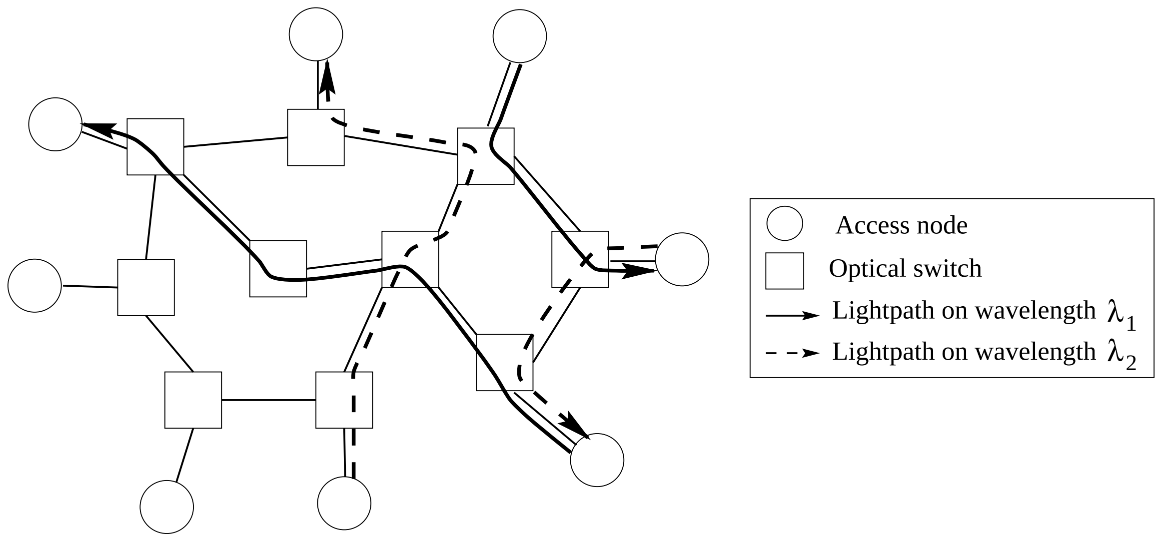

Figure 1: A wavelength-routed optical WDM network with lightpath connections.

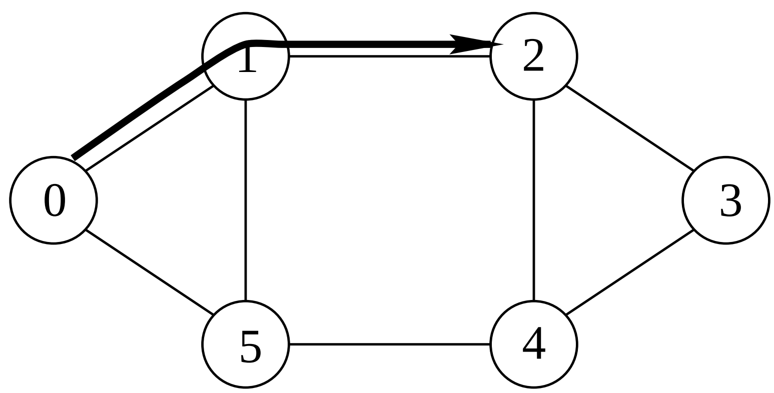

Figure 2: Fixed shortest-path route from Node 0 to Node 2.

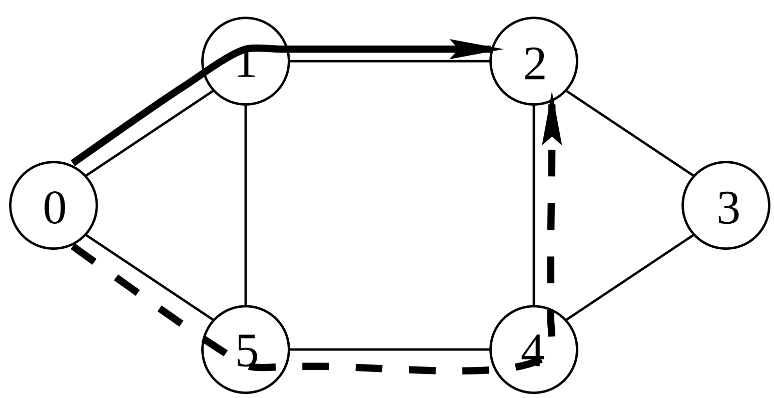

Figure 3: Primary (solid) and alternate (dashed) routes from Node 0 to Node 2.

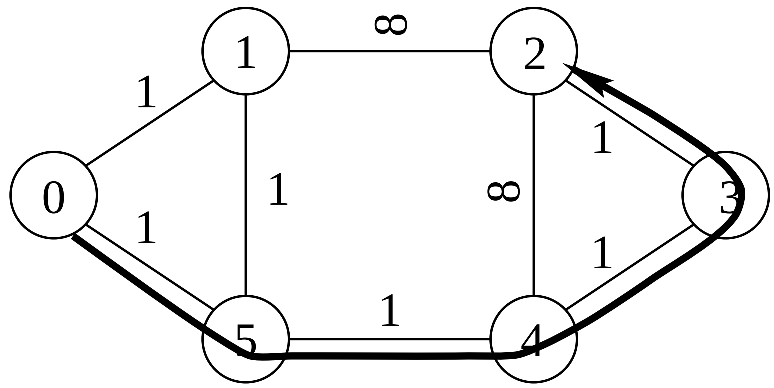

Figure 4: Adaptive route from Node 0 to Node 2.

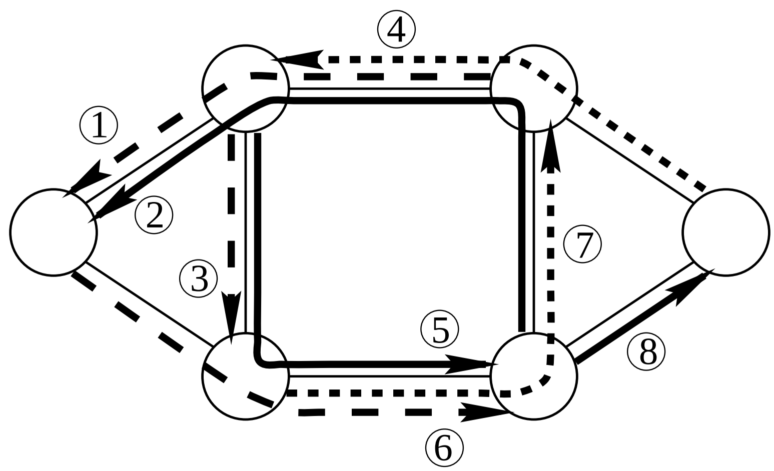

Figure 5: A network with eight routed lightpaths.

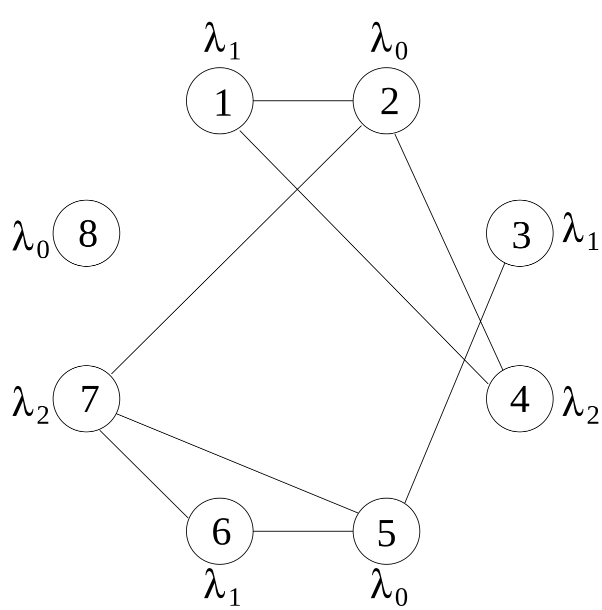

Figure 6: The auxiliary graph, $G(V,E)$, for the lightpaths in the network shown in Fig. 5.

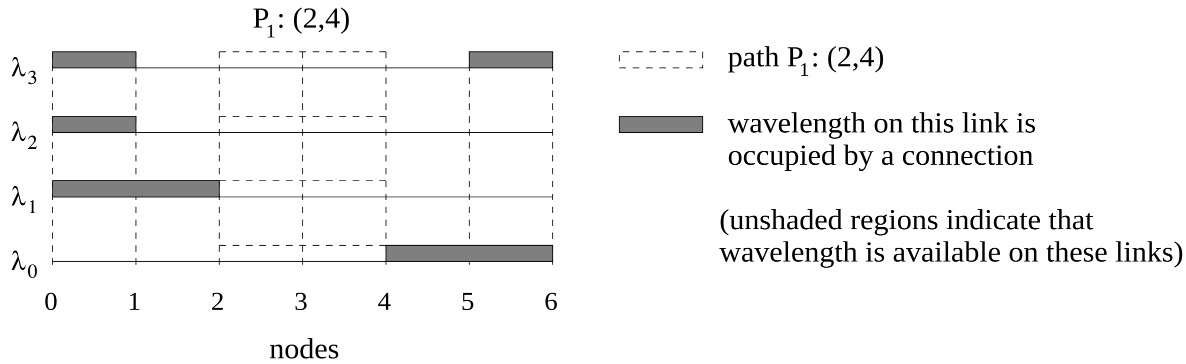

Figure 7: Wavelength-usage pattern for a network segment consisting of six fiber links in tandem.

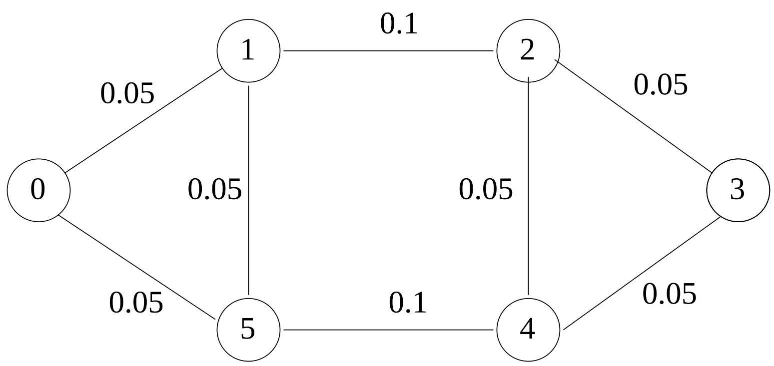

Figure 8: Simulation network.

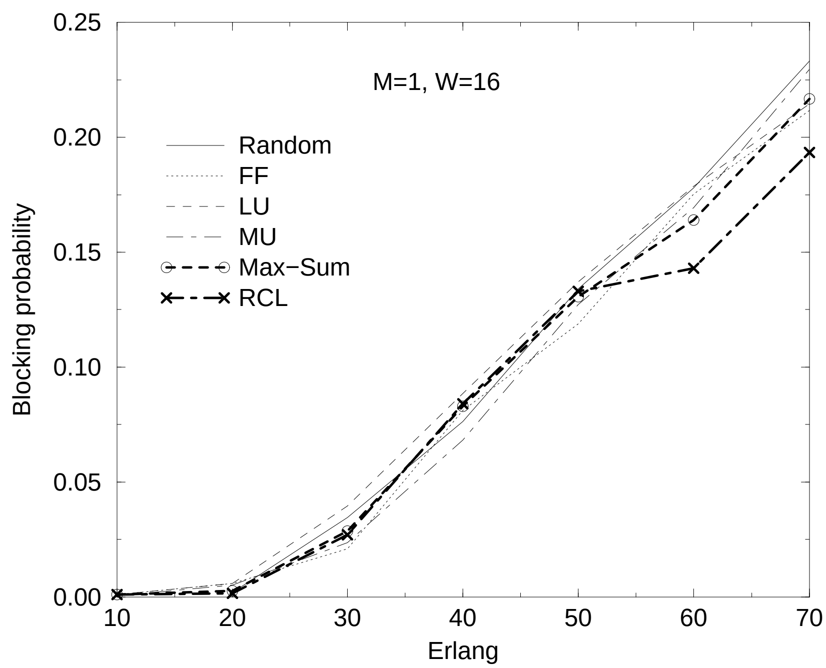

Figure 9: Comparison of Random, FF, LU, MU, Max-Sum, and RCL for a single-fiber network with 16 wavelengths.

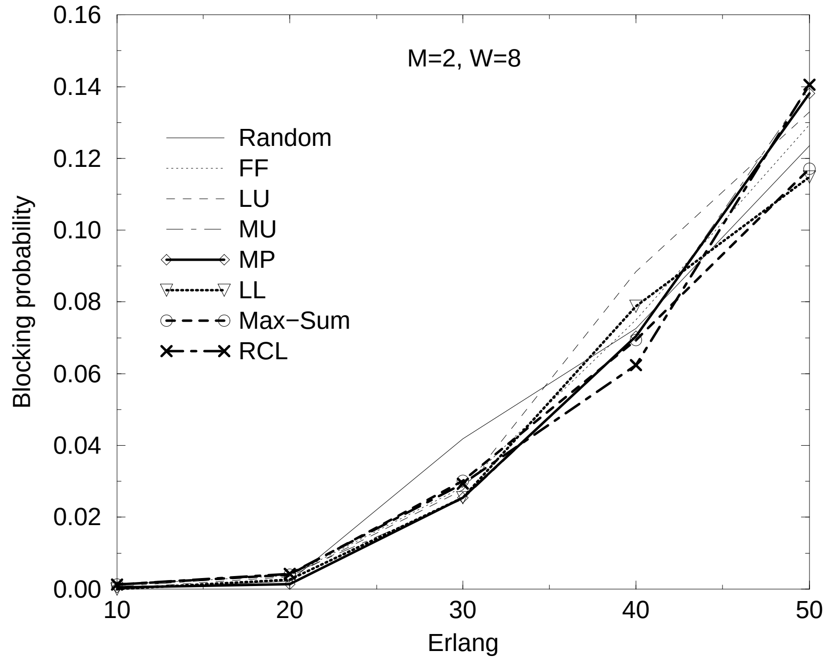

Figure 10: Comparison of Random, FF, LU, MU, MP, LL, Max-Sum, and RCL for a two-fiber network with eight wavelengths.

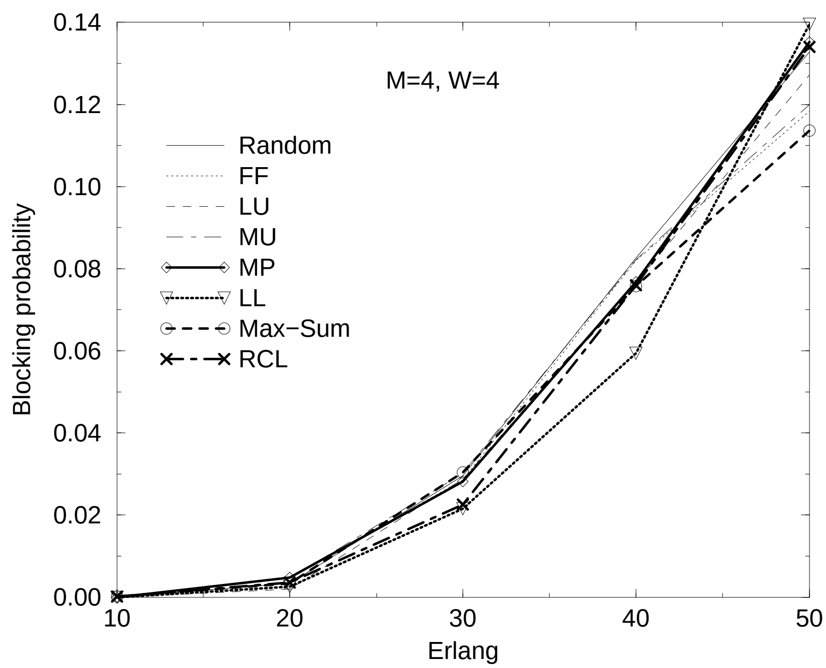

Figure 11: Comparison of Random, FF, LU, MU, MP, LL, Max-Sum, and RCL for a four-fiber network with four wavelengths.

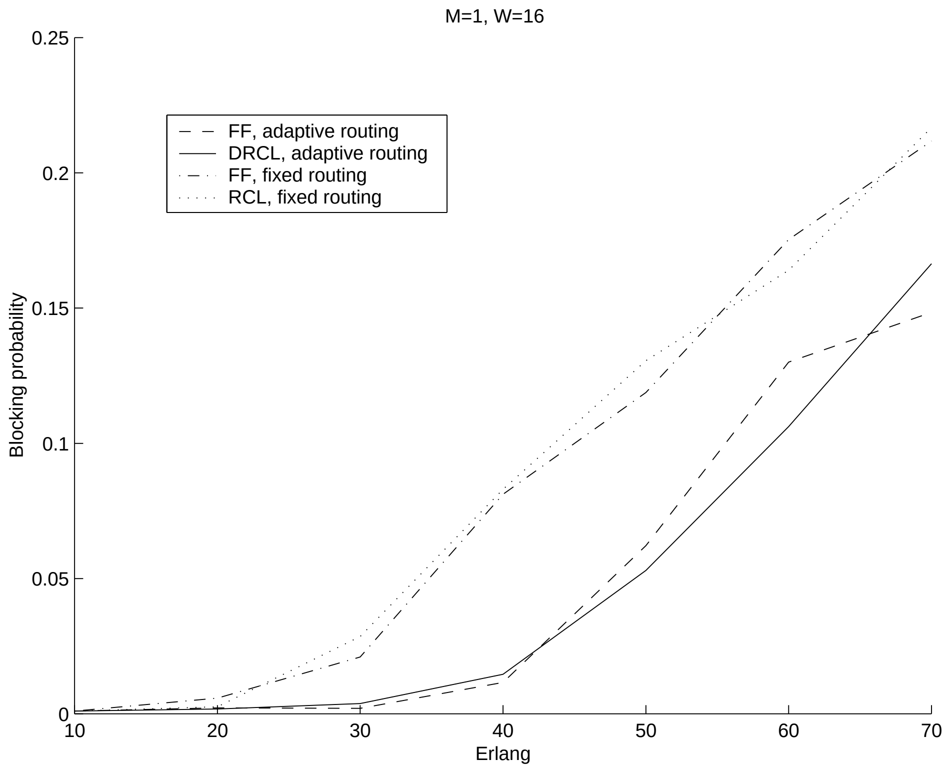

Figure 12: Comparison of DRCL, FF with adaptive routing, RCL (which can only be implemented with fixed routing), and FF with fixed routing.