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On the Measurement of Negotiation Dialogue Games

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Abstract. Nowadays, multiagent systems became a widely used technology in everyday life. More studies are needed to evaluate these systems from different aspects such as evaluating agent dialogues, the participants to these dialogues, and the protocols governing the dialogues, etc. In this paper, we define new measures for dialogue games from an external agent's point of view. In particular, two measurement sets are proposed: in the first set, we use Shannon entropy to measure the certainty index of the dialogue. This involves i) using Shannon entropy to measure the agent's certainty about each move during the dialogue; and ii) using Shannon entropy to measure the certainty of the agents about the whole dialogue with two different ways. The first way is by taking the average of the certainty index of all moves, and the second way is by determining all possible dialogues and applying the general formula of Shannon entropy. In the second set, we introduce two metrics: i) measuring the goodness of the agents in the real dialogue (i.e. the dialogue that effectively happened between the participants); and ii) measuring the farness of the agents from the right dialogue (i.e. the best dialogue that can be produced by two agents if they know the knowledge bases of each other). Many dialogue game types have been proposed in multiagent systems. In this paper, we focus on one specific type, namely quantitative negotiation such as bargaining.

Keywords. Dialogue games, Multiagent systems, Shannon entropy, Certainty index, Goodness, Farness.

1. Introduction

Autonomous agents and multiagent systems (MAS) provide a technology offering an alternative for the design of intelligent and cooperative systems. Recently, efforts have been made to develop novel tools, methods, and frameworks to establish the necessary standards for wider use of MAS as an emerging paradigm [6]. An increasing interest within this paradigm is on modeling interactions and dialogue systems. In fact, several dialogue systems have been proposed in the literature for modeling *information seeking* dialogues (e.g. [12]), *inquiry* dialogues (e.g. [4]), *persuasion* dialogues (e.g. [2]) and finally *negotiation* dialogues (e.g. [15]). The latter is our concern in this paper. Negotiation is a form of interaction in which a group of agents, with conflicting interests, but a desire to cooperate, try to come to a mutually acceptable agreement on the division of scarce resources [14,3]. Precisely, we focus in this paper on quantitative negotiation dialogue games such as bargaining.

It is worth noticing that in all types of dialogue systems mentioned above, a dialogue game is a normative model of dialogue, which mainly consists of: i) a set of moves (e.g. challenge, assertion, question, etc.); ii) one commitment store for each conversant where the advanced moves are stored; iii) a communication language specifying the locution that will be used by agents during a dialogue for exchanging moves; iv) a protocol specifying the set of rules governing the dialogues; and v) agents' strategies, which are the different tactics used by agents for selecting their moves at each step in a dialogue [1]. A dialogue correctly proceeds as long as the participants conform to the dialogue rules and eventually ends when some termination rules are achieved [1,10,13]. In this paper, we will focus first on the exchanged moves (i.e. the dialogue itself) in terms of the certainty index of selecting the right moves during the dialogue and the certainty index of the whole dialogue. Uncertainty about values of given variables (e.g. the disease affecting a patient in medical applications) can result from some errors and hence from non-reliability (in the case of sensors) or from different background knowledge (in the case of agents). As a result, it is possible to obtain different uncertain pieces of information about a given value from different sources [9]. The second focus of this paper is on the agents' strategies in terms of goodness degree of the agents in the real dialogue (i.e. the dialogue that effectively happened between the participants) and the farness degree of the agents from the right dialogue (i.e. the best dialogue that can be produced by two agents if they know the knowledge bases of each other). For example, in negotiation setting, the best dialogue is the one that, with a minimum number of moves, can achieve the best agreement for both agents if such an agreement exists considering the knowledge bases of both agents.

In this paper, we define new measures for dialogue games from an external agent's point of view. In particular, two measurement sets are proposed: in the first set, we use Shannon entropy to measure the certainty index of the dialogue. This involves i) using Shannon entropy to measure the agent's certainty about each move during the dialogue; and ii) using Shannon entropy to measure the certainty of the agents about the whole dialogue with two different ways. The first way is by taking the average of the certainty index of all moves, and the second way is by determining all possible dialogues and applying the general formula of Shannon entropy. In the second set, we introduce two metrics: i) measuring the goodness of the agent in the real dialogue; and ii) measuring the farness of the agents from the right dialogue. These measures are of great importance since they can be used as guidelines for a protocol in order to generate the best dialogue between autonomous intelligent agents.

The rest of the paper is organized as follows: In Section 2, we present a theoretical background about Shannon entropy and dialogue games. Section 3 presents the first set of our measures using Shannon entropy. In Section 4, we present the second set of our metrics that evaluate the goodness and farness of the participants to the dialogue. Related work is discussed in Section 5. Finally, conclusion and future work are presented in Section 6.

2. Background

In information theory, there exist several ways for defining how to quantify an information. One of the most common used techniques is Shannon entropy. The idea behind Shannon entropy in information theory is based on the amount of randomness that exists in a random event. In fact, Shannon entropy is a measure of the uncertainty associated with a random variable. The more uncertain we are about the content of the message, the more informative it is [5].

Definition 1 [Shannon entropy] *Shannon entropy for a discrete random variable X taking its values from a set of values S (sample space), with probability mass function $P(x)$ is given by equation 1:*

$$H(X) = -\sum_{x \in S} P(x) Log P(x) \tag{1}$$

Shannon entropy $H(X)$ depends on the probability distribution of X rather than the actual values of X. The logarithm in the equation is considered to be of base 2 in the computations. The value of $H(X)$ varies from zero to $Log(|S|)$, where zero means that there is no uncertainty, while $Log(|S|)$ is the maximum value of uncertainty. In this paper, we aim to investigate to what extent we can use Shannon entropy (as one of most well known measures in information theory) in evaluating dialogue games. We place ourselves in the role of an external observer trying to evaluate the dialogue. So, we propose new metrics based on Shannon entropy. The basic idea is to measure how much we are certain about selecting the right move M_i at each step t_i in dialogue D by assuming that there is a set S_i of choices facing the agent at each dialogue step t_i. We call this measure the *certainty index of that move* "$CI(M_i)$", and then we calculate how much we are certain about the whole dialogue using what we call the *certainty index of the dialogue* "$CI(D)$" by taking the average of the certainty index of all moves in this dialogue. Furthermore, we measure the certainty index of the whole dialogue by computing all possible dialogues, using the Cartesian product of all possible moves, and determining the probability of each dialogue, and then applying the general formula of Shannon entropy for the whole dialogue (exactly as what we do in the case of calculating the certainty of the moves). Figure 1 shows a general overview of our approach and the proposed metrics for negotiation dialogue games. We believe that such measures will help in evaluating dialogues and the participants' strategies.

Definition 2 [Moves] *Let $Ag = \{\alpha, \beta, \ldots\}$ be a set of symbols representing agent names that may be involved in a negotiation dialogue D. A move $M_i \in D$ consists of the agent that utters the move: $Speaker(M_i) \in Ag$; the set of agents to which the move is addressed: $Hearer(M_i) \subseteq Ag$; and the content of the move: $Content(M_i)$.*

For simplification reasons, we will consider only two agents involved in the dialogue α and β. Multi-party dialogues is an important issue that we plan to investigate in future work. During a dialogue, several moves may be uttered. Those moves constitute a dialogue D, which is a sequence of moves denoted by $[M_0,M_1,\ldots,M_n]$, where M_0 is the initial move, M_n is the final one, and $|D|$ is the length of the dialogue (i.e. $|D| = n$).

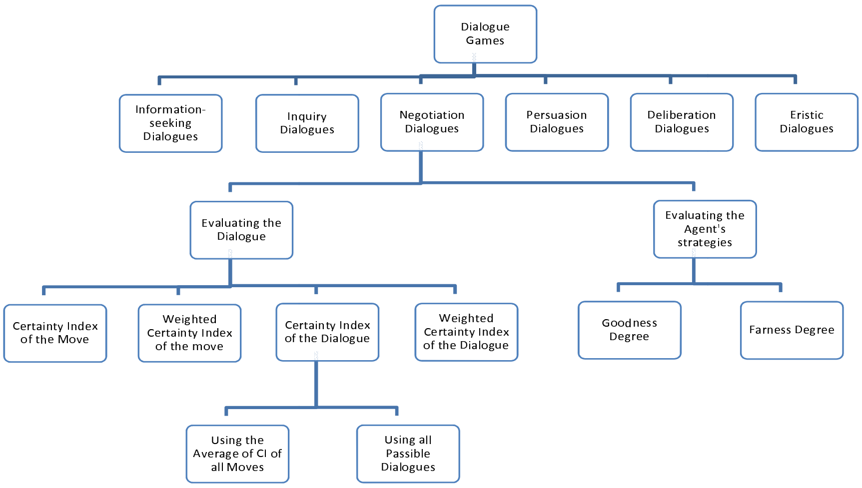

Figure 1. The general form of the approach

Definition 3 [Knowledge and move function] *In negotiation dialogues, the probability of a move at a given step t_i depends on the knowledge the agent has at that step (i.e. the content of the agent's knowledge base at that step). Let κ be a set of agents' knowledge, and δ the set of all possible moves, we define ζ as a function associating a set of knowledge to a set of possible moves and the probability of each move. We call this function knowledge and move function.*

$$\zeta : 2^{\kappa} \longrightarrow 2^{\delta*(0,1]} \tag{2}$$

To allow agents to refer to their dialogue history, a data structure called commitment store "CS" is used to restore utterances that agents utter during the dialogue [7]. Let α and β be two agents ($\alpha \neq \beta$). Also, let Γ_α be α's knowledge base and Γ_β β's knowledge base. $CS_\alpha^{t_i}$ is the commitment store of agent α at step t_i of the dialogue, $CS_\beta^{t_i}$ is the commitment store of agent β at step t_i, and $CS_{\alpha\cup\beta}^{t_i}$ is the commitment store of both agents at step t_i. Suppose that at step t_{i-1}, agent β uttered a move. To utter a move at the next step t_i, agent α should consider his knowledge base and the content of β's commitment store. Let us introduce a new notation: m_i^j as the j^{th} move among the possible moves an agent has at the step t_i and $P(m_i^j)$ the associated probability such that the relationship between M_i and m_i^j is as follows:

$$\forall i\ \exists j : M_i = m_i^j \tag{3}$$

where M_i is the selected move the agent utters at the step t_i, and:

$$\zeta(\Gamma_\alpha \cup CS_\beta^{t_i}) = \{(m_i^j, P(m_i^j)) | m_i^j \in S_i\} \tag{4}$$

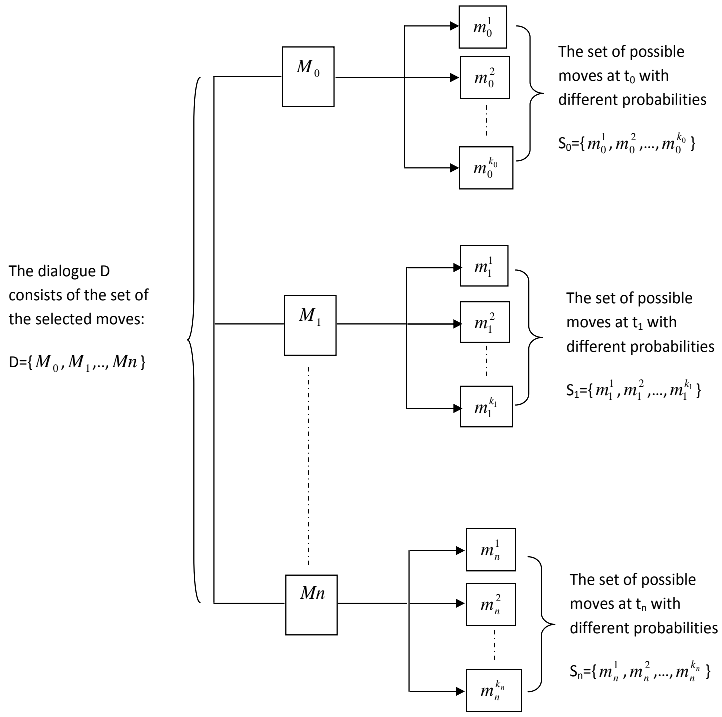

Figure 2. The relationship between the selected move and the set of all possible moves at each dialogue step

where S_i is the set of choices facing the agent at the dialogue step t_i. Figure 2 explains this relationship.

In persuasion and negotiation dialogues, the probability associated with a move corresponds to the probability for the move to be accepted by the addressee. This could be calculated using the risk of a move to be refused as suggested in [11].

3. Applying Shannon Entropy in Dialogue Games

In this section, we will discuss to what extent we can use Shannon entropy in dialogue games. We apply Shannon entropy to measure the certainty index of the agents about their dialogue. To do so, we measure the certainty index of the agents about their moves and the certainty index about the whole dialogue. The following two subsections illustrate these metrics.

3.1. Measuring How Certain the Agent is about its Move

To measure the certainty index of the agent about its move, we calculate Shannon entropy of that move, which means the agent's uncertainty about selecting the right move at a given dialogue step. Here we consider only possible moves, which are moves whose the associated probability is in $(0, 1]$. Then we normalize this value by dividing it by the logarithm of the number of all possible moves at that step. By subtracting the normalized value from one; we get the agent's certainty about selecting the right move at this step.

Definition 4 [Move's entropy] *Let $D = [M_0, M_1, \ldots, M_n]$ be a negotiation dialogue, and suppose that at each dialogue step t_i a set S_i of moves $\{m_i^1, m_i^2, \ldots, m_i^k\}$ are possible, and each one of them is associated with a given probability $P(m_i^j)$, such that $\sum_{m_i^j \in S_i} P(m_i^j) = 1$. The Shannon entropy for a random move M_i taking its values from the set of moves S_i is defined by:*

$$H(M_i) = -\sum_{m_i^j \in S_i} P(m_i^j) Log P(m_i^j) \tag{5}$$

The value of $H(M_i)$ varies from zero to $Log(|S_i|)$, where zero means that there is no uncertainty (i.e. there is only one choice), while $Log(|S_i|)$ means that the uncertainty is at its maximum value (i.e., all moves have the same probability). We further normalize $H(M_i)$ to have a metric that ranges from 0 to 1. We achieve this by dividing $H(M_i)$ by $Log(|S_i|)$. So, our uncertainty about selecting the right move is given by:

$$\mu(M_i) = \begin{cases} 0 & \textit{iff} \quad Log(|S_i|) = 0 \\ H(M_i)/Log(|S_i|) & otherwise \end{cases} \tag{6}$$

Proposition 1 *The uncertainty of a move M_i at a certain step t_i during the dialogue is equal to zero (i.e. $\mu(M_i) = 0$) iff at that step the agent has only one choice.*

Proof
$\mu(M_i) = 0 \Leftrightarrow Log(|S_i|) = 0$

$\Leftrightarrow |S_i| = 1 \quad \square$

So, there is only one move in S_i available to the agent at step t_i. Intuitively, at the beginning steps of a dialogue, the uncertainty is expected to be high as all possible moves have the same probability, then gradually the uncertainty decreases, because agents are rational and they learn from each other when advancing in the dialogue.

Proposition 2 *The uncertainty of a move M_i at a certain step t_i during the dialogue is equal to one (i.e. $\mu(M_i) = 1$) iff at that step, all the moves in S_i have the same probability.*

Proof: See Appendix.

Definition 5 [Move's certainty index] *Let $D = [M_0, M_1, \ldots, M_n]$ be a negotiation dialogue, and suppose that at each dialogue step t_i a set S_i of moves m_i^j are possible, and each one of them is associated with a given probability $P(m_i^j)$ such that $\sum_{m_i^j \in S_i} P(m_i^j) = 1$. If Shannon entropy of the move M_i at step t_i is $H(M_i)$, we define the certainty index of the move as follows:*

$$CI(M_i) = \begin{cases} 1 & \textit{iff} \quad Log(|S_i|) = 0 \\ 1 - H(M_i)/Log(|S_i|) & otherwise \end{cases} \tag{7}$$

Using the certainty index, we can see at each dialogue step how much the agent is certain about the move he can play at that time. The following lemmas are straightforward from Propositions 1 and 2.

Lemma 1 *The certainty index of a move M_i at a given step t_i in the dialogue is at its maximum value "1" iff the agent has only one choice at that step.*

Lemma 2 *The certainty index of a move M_i at a given step t_i in the dialogue is at its minimum value "0" iff the agent has more than one move at the same step with equal probabilities.*

By considering the uncertainty and certainty index of dialogue moves, agents should resolve at each dialogue step t_i one of the following equivalent optimization problems.

$$M_i^* = \underset{M_i}{\operatorname{argmin}}\, \mu(M_i) \tag{8}$$

$$M_i^* = \underset{M_i}{\operatorname{argmax}}\, CI(M_i) \tag{9}$$

Theorem 1 *There is an algorithm for solving these optimization problems in a polynomial time.*

Proof: See Appendix.

This theorem is compatible with the intuition that by adding new information in $CS_\beta^{t_i}$, the number of possible choices decreases. However, this is only true when we consider just the next move. When we consider the whole dialogue, the complexity is much higher.

Example 1 *Let us consider a negotiation dialogue D between two agents α and β such that $D = [M_0, M_1, M_2]$, and the number of possible moves at each dialogue step is $|S_i| = 3$. In the following we explain how to measure the certainty index of move M_0:*

$CI(M_0) = 1 - H(M_0)/Log(|S_0|)$

$CI(M_0) = 1 - [(-\sum_{j=1}^{3} P(m_0^j)LogP(m_0^j))/Log(3)]$

$CI(M_0) = 1 + [(P(m_0^1)LogP(m_0^1) + P(m_0^2)LogP(m_0^2) + P(m_0^3)LogP(m_0^3))/Log(3)]$

$CI(M_0) = 1 + [((0.33 * -1.599) + (0.33 * -1.599) + (0.34 * -1.556))/Log(3)]$

Table 1. Measuring the certainty index of the move M_0

Possible Moves	P(m_0^j)	**Log P(**m_0^j**)**	**P(**m_0^j**)Log P(**m_0^j**)**
m_0^1	0.33	-1.599	-0.528
m_0^2	0.33	-1.599	-0.528
m_0^3	0.34	-1.556	-0.529
	H(M_0)=1.58	$\mu(M_0) = 1$	$CI(M_0) = 0$

$CI(M_0) = 1 - 1 = 0$

Table 1 shows the possible choices of the moves that facing agent α at step t_0 to play his first move M_0 in the first column, and their associated probabilities in the second column. From the above calculations, we notice that the certainty index of agent α about selecting the right move at this step is at its minimum value "0" because agent α had different choices of moves with equal probabilities of acceptance from the addressee agent β. This means that agent α was uncertain $\%100$ about which move he should play.
The above calculations and Table 1 are just for the first move M_0, and to obtain the certainty index of the two other moves M_1 and M_2, we can calculate them using the same technique as for M_0.
At step t_1, agent β to play his move M_1 as a reply to agent α, he had different choices of moves with different values of probabilities $(0.05, 0.12, 0.83)$, such that the sum is equal to one. Consequently, agent β was certain about $\%0.50$, $(CI(M_1 = 0.5))$ that he will select the move with higher probability (0.83) of acceptance from agent α. The other two choices that agent β had were with lower probabilities and he was uncertain about them.
At step t_2, agent α to reply to agent β with his move M_2, he had different choices of moves with different values of probabilities $(0.9999, 1E - 8, 1E - 8)$, such that the sum is equal to one. Agent α was then certain almost $\%100$, and the certainty index is at its maximum value "1". This is because all choices that he had are with very low probabilities except one of them was with very high probability.

In order to highlight the quality of information involved in the selected move in terms of its certainty index, we assign a weight to this move. We suppose that the selected moves have different weights reflecting the importance degree of the moves. For example, in some negotiation dialogues, the last moves could be more important than the first moves as they lead to an agreement. Let $\mathcal{M}$ be the set of all moves. The weight function is defined as follows:

$$W : \mathcal{M} \rightarrow \mathbb{N}^* \tag{10}$$

Definition 6 [Weighted certainty index of the move] *Let $D = [M_0, M_1, \ldots, M_n]$ be a negotiation dialogue, and $CI(M_i)$ the certainty index of the move M_i at a step t_i and $W(M_i)$ the weight of that move at that step. We define the weighted certainty index as follows:*

$$W\_CI(M_i) = W(M_i) * CI(M_i) \tag{11}$$

3.2. Measuring how Certain the Agents are about the Dialogue

In the last subsection, we discussed how to measure the agents' certainty about the move at each step during the dialogue. In this subsection, we will discuss how to measure the certainty index of the agents about the whole dialogue. We will analyze this using two methods. The first method is by using the average of certainty index of all moves in the dialogue. The second method is by calculating the possible number of dialogues and the probability of each one, and then apply Shannon entropy as we did for the moves.

3.2.1. Method 1: Using the Average of all Moves

The basic idea is to measure how much each agent is certain about his move at each step in the dialogue. Then we calculate how much the two agents are certain about the "whole dialogue" by taking the average of the certainty index of all moves in the dialogue.

Definition 7 [Agents' certainty about the dialogue] *Let $D = [M_0, M_1, \ldots, M_n]$ be a negotiation dialogue with length $|D| = n + 1$. α and β are the two agents participating to the dialogue, where α utters the even moves and β utters the odd moves. $CI(M_i)$ is the certainty index of the moveM_i at the step t_i. The certainty index of the dialogue "$CI(D)$" is given by:*

$$CI(D) = \sum_{M_i \in D} CI(M_i)/(|D|) \tag{12}$$

Example 2 *Let us consider three negotiation dialogues D_1, D_2 and D_3, such that $D_k = [M_0, M_1, \ldots, M_9]$ $(1 \leq k \leq 3)$. Table 2 shows the possible moves of each dialogue and the certainty index of each move in the dialogue. We suppose that at each dialogue step the agent has different choices, and we calculate the certainty index of each possible move based on equation 7 as we did in example 1. Then we compute the certainty index of each dialogue by applying equation 12. We will calculate the certainty index of D_1, and the same technique can be used to calculate this index for D_2 and D_3:*

$CI(D_1) = \sum_{M_i \in D_1} CI(M_i)/(|D_1|)$

$CI(D_1) = [0.01 + 0.20 + 0.15 + 0.05 + 0.22 + 0.02 + 0.11 + 0.21 + 0.10 + 0.05]/10$

$CI(D_1) = 0.112$

We notice that for the dialogue D_1, the certainty index is low. This is because the certainty indexes of all the moves in this dialogue are low as they range between 0.01 and 0.22. This means the participants were not certain about the dialogue, and even if they achieve an agreement, they do not know whether it is a good agreement or not. Figure 3 shows the certainty indexes of the moves in these three dialogues.

The certainty index of the dialogue D_2 is medium, and that is because of the low certainty of the agent α about his moves during the dialogue, and the high certainty of the agent β about his moves in the same dialogue. So, when we take the average of

Table 2. The certainty index of the dialogues of Example 2

Possible Moves	**CI(M_i) of D_1**	**CI(M_i) of D_2**	**CI(M_i) of D_3**
M_0	0.01	0.01	0.99
M_1	0.20	0.99	0.98
M_2	0.15	0.15	0.95
M_3	0.05	0.95	0.99
M_4	0.22	0.22	0.90
M_5	0.02	0.80	0.90
M_6	0.11	0.11	0.80
M_7	0.21	0.75	0.95
M_8	0.10	0.05	0.90
M_9	0.05	1	1
	CI(D_1)=0.112	CI(D_2)=0.503	CI(D_3)=0.936

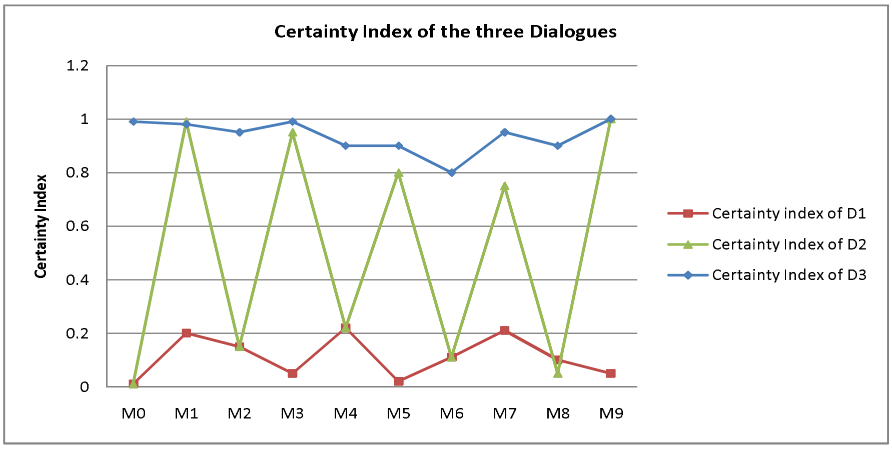

Figure 3. The certainty indexes of the moves in the dialogues of Example 2

the certainty indexes of all the moves, we get a medium certainty index for the whole dialogue. Figure 3 shows the difference between agent α and agent β in the performance during the dialogue, where agent β is doing well compared to agent α.
In the third dialogue D_3, the certainty index is very high. This means the participants were very certain about their moves at each step during the dialogue. We can notice from Figure 3, that the performance of both agents is very good.

Taking the average of the certainty indexes of all the moves gives us an indicator about how certain the agents are about their dialogue. However, it does not allow us to compare different dialogues with the same certainty index. To do so, we define another metric called the *weighted certainty index* of the dialogue by giving weights to the moves and taking the average of the weighted certainty indexes of all the moves.

Definition 8 [Weighted Certainty Index of the Dialogue] *Let $D = [M_0, M_1, \ldots, M_n]$ be a negotiation dialogue. α and β are the two agents participating to the dialogue, where α utters the even moves and β utters the odd moves. $CI(M_i)$ is the certainty index of the*

Table 3. Weighted certainty index of the dialogues of Example 3

Possible Moves	**CI(M_i) of D_1**	**CI(M_i) of D_2**	**CI(M_i) of D_3**	**W(M_i)**
M_0	0.50	0.10	0.90	1
M_1	0.50	0.25	0.75	2
M_2	0.50	0.40	0.60	3
M_3	0.50	0.60	0.40	4
M_4	0.50	0.75	0.25	5
M_5	0.50	0.90	0.10	6
	CI(D_1)=0.50	CI(D_2)=0.50	CI(D_3)=0.50	
	W_CI(D_1)=0.50	W_CI(D_2)=0.64	W_CI(D_3)=0.36	

move M_i at the step t_i, and $W(M_i)$ the weight of M_i. The weighted certainty index of the dialogue is given by:

$$W\_CI(D) = \sum_{M_i \in D} W(M_i) * CI(M_i) / \sum_{M_i \in D} W(M_i) \tag{13}$$

This will help us to compare dialogues with the same certainty index. This is because the average of weighted certainty index of all moves can be different from the average of certainty index of the moves without weights "$CI(D)$".

Example 3 *let us consider the following three negotiation dialogues D_1, D_2 and D_3 (that are different from the dialogues considered in Example 2), such that $D_k = [M_0, M_1, \ldots, M_5]$ $(1 \leq k \leq 3)$. Table 3 shows the certainty index and weight of each move. The certainty index of each move is calculated as in example 1, and the certainty index of each dialogue is calculated based on equation 12, as we did in example 2. By applying equation 13, we obtain the weighted certainty index of each dialogue. In the following we explain how we calculate the weighted certainty index for D_1, and the same technique is used to calculate this index for D_2 and D_3:*

$W\_CI(D_1) = \sum_{M_i \in D_1} W(M_i) * CI(M_i) / \sum_{M_i \in D_1} W(M_i)$

$W\_CI(D_1) = [(0.5 * 1) + (0.5 * 2) + (0.5 * 3) + (0.5 * 4) + (0.5 * 5) + (0.5 * 6)]/21$

$W\_CI(D_1) = 0.50$

We notice that the three different dialogues have the same certainty index, so by calculating just the certainty index of the dialogue we cannot compare such dialogues except if we look at the performance of the participants during the dialogue. However, we can do so by calculating the weighted certainty index of each dialogue, which gives us a good indicator about the goodness of dialogues. In our example, we consider D_2 as the best dialogue, because its weighted certainty index is greater than D_1 and D_3, followed by D_1 and then D_3. We notice in the case of D_2 that both agents have started with low certainty about their moves, then they have started learning from each other till they become more certain, especially in the last two moves.

Figure 4 shows the diagrams of certainty indexes of the moves in the three dialogues. D_1 is stable and the agents have the same CI from the first move till the last one, and it

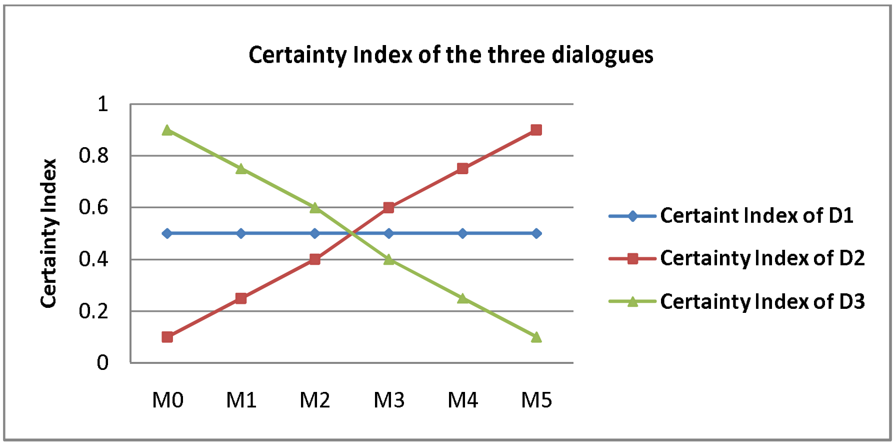

Figure 4. The certainty indexes of the three dialogues of Example 3

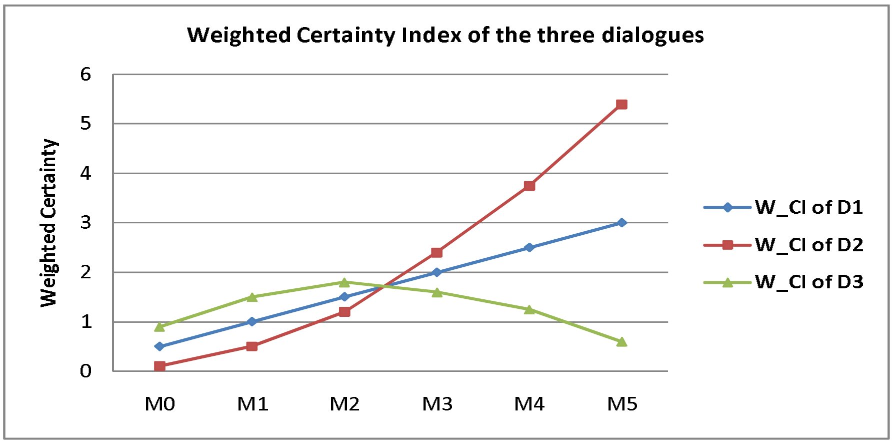

Figure 5. The weighted certainty indexes of the three dialogues of Example 3

is medium so the certainty index of the whole dialogue is medium too. In D_2, the agents have started with very low values of CI for the first moves and then the CI of their moves has started increasing to reach 0.90, which is a very good certainty index to guarantee a good agreement. In D_3, we notice the opposite situation of D_2, where the agents have started very certain about their moves then their certainty has started decreasing, which might result in not achieving an agreement.

Figure 5 shows the weighted certainty indexes of the moves for the three dialogues, and here we we can see the difference between the three dialogues after giving weights to the moves as we discussed before.

3.2.2. Method 2: Using the Probability of all Possible Dialogues

In this subsection, we measure the certainty index of the dialogue by calculating the number of all possible dialogues using the Cartesian product of all possible moves m_i^j at each step t_i in the dialogue. Thus, by knowing the probability of each possible move, we can calculate the probability of each possible dialogue, and then we apply the general formula of Shannon entropy.

Definition 9 [Number of possible dialogues] *Let $D = [M_0, M_1, \ldots, M_n]$ be a negotiation dialogue and α and β two agents participating to it. Suppose that at each step t_i, the move M_i has a set S_i of possible moves $S_i = \{m_i^1, m_i^2, \ldots, m_i^{k_i}\}$, each one with probability $P(m_i^j)$. The number of all possible moves is equal to $\sum_{i=0}^{n} k_i$. The union of all sets of the moves is $\Omega = S_1 \cup S_2 \cup \ldots \cup S_n$, such that there is no intersection between the sets of moves: $S_1 \cap S_2 \cap \ldots \cap S_n = \emptyset$. We calculate the number of all possible dialogues by using the Cartesian product of the sets of moves. So, the number of possible dialogues is $N_D = |S_0 \times S_1 \times \ldots \times S_n|$.*

As explained in Equation 3, each move M_i in a possible dialogue D_l is equal to a possible move m_i^j for a given j. Thus, $P(M_i) = P(m_i^j)$. Knowing this probability, we can calculate the probability of a dialogue D_l as follows:

$$P(D_l) = P(M_0) \times P(M_1) \times \ldots \times P(M_n) \tag{14}$$

Because $\forall i \sum_{j=1}^{k_i} P(m_i^j) = 1$, the sum of the probabilities of all possible dialogues is equal to one (i.e. $\sum_{l=1}^{N_D} P(D_l) = 1$). We can now define the certainty index of the dialogue as we did in section 3 for the moves. First, we measure the uncertainty of the dialogue by the general formula of Shannon entropy:

$$H(D) = -\sum_{l=1}^{N_D} P(D_l) Log(D_l) \tag{15}$$

Then, we normalize it by dividing it by *Log(N_D)* to have a metric between 0 and 1, and finally, we measure the certainty index of the dialogue by subtracting the normalized value from one. So, the certainty index of the dialogue is given by:

$$CI(D) = 1 - H(D)/Log(N_D) \tag{16}$$

Example 4 *Let us consider the negotiation dialogue in example 1, where $n = k_i = 3$ $1 \leq i \leq 3$. The number of all possible dialogues is ($N_D = 27$ dialogues) and the probability of each possible dialogue is computed by the product of probability of its moves. For example for D_1, we take the first possible choice of the moves m_0^1, m_1^1, m_2^1 with their probabilities $0.33, 0.05, 1$ respectively. So, the probability of D_1 is equal to 0.02, and in the same way we compute the probability of all possible dialogues. By applying equation 15, we get the entropy of the dialogue $H(D) = 2.39$, and by applying equation 16, we get the certainty index of the dialogue $CI(D) = 0.497$. Here we notice that the certainty index of the dialogue is medium, and if we compare this method with the previous one in Section 3.2.1, which uses the average of certainty index of all moves, and applying it on the same example (Example 1),we notice that the result is almost the same.*

4. Measures for Agents in Quantitative Negotiation Dialogue Games

In the previous section, we proposed metrics to measure dialogues and dialogue moves. In this section, we focus on the agents playing these moves and participating in these dialogues. In fact, anyone monitors a dialogue between two agents and wants to evaluate the participants to this dialogue may ask these questions: How good are the agents in the real dialogue? and how far are the agents from the right dialogue? To answer these questions, we propose metrics to evaluate the performance of the agents in the dialogue and to calculate the distance between the real dialogue and the right dialogue. In a quantitative negotiation dialogue game such as bargaining, negotiation about meeting time, meeting place, etc., one of the participants (the initiator) gives an initial proposal, which is the most preferred for him, and the other agent gives a counter proposal, which of course is the most preferred for him too. Then, they start negotiating each with his own strategy and according to his knowledge base trying to achieve his goal, which is his initial proposal or closest value to it.

Definition 10 [The achieved agreement] *Let $D = [M_0, M_1, \ldots, M_n]$ be a quantitative negotiation dialogue. The Achieved Agreement $AchAgr$ is the accepted value of the last proposal conveyed by the last move M_n.*

Definition 11 [The best agreement] *Let α and β be two negotiating agents and Γ_α and Γ_β their respective knowledge bases. The best agreement $BestAgr$ between α and β is the agreement that could be achieved in the negotiation dialogue from the evaluator's point of view considering the union of the two participants knowledge bases $\Gamma_\alpha \cup \Gamma_\beta$.*

4.1. Measuring how Good the Agents are in the Real Dialogue

In order to know how good the agents are in their negotiation dialogue (here called the real dialogue), and which agent is doing better than the other, we calculate the difference between the initial value proposed by the agent and the final value achieved in the real dialogue, i.e. the achieved agreement. This difference gives us an indicator about how good the agent is in the real dialogue as it represents the amount of the agent's loss in the negotiation. As mentioned before, the initial proposal (or initial counter proposal) for each agent is supposed to be the most preferred value for him, and each agent tries to achieve this value or the closest value to it. As the achieved value goes far from the initial proposal, the agent's loss value increases and the performance of the agent is considered bad (we can say the ability of the agent to convince the others is weak).

Definition 12 [The agent's loss] *The loss of an agent α in the negotiation is the distance between what is achieved at the end of the dialogue and the initial proposal of the agent α $IP(\alpha)$. It is computed as follows:*

$$Loss(\alpha) = |IP(\alpha) - AchAgr| \tag{17}$$

We take the absolute value to avoid having negative value for the agent who tries to minimize $AchAgr$. For this agent, $AchAgr$ is greater than or equal to his initial proposal. This agent will start with a small value as an initial proposal in attempt to get a good

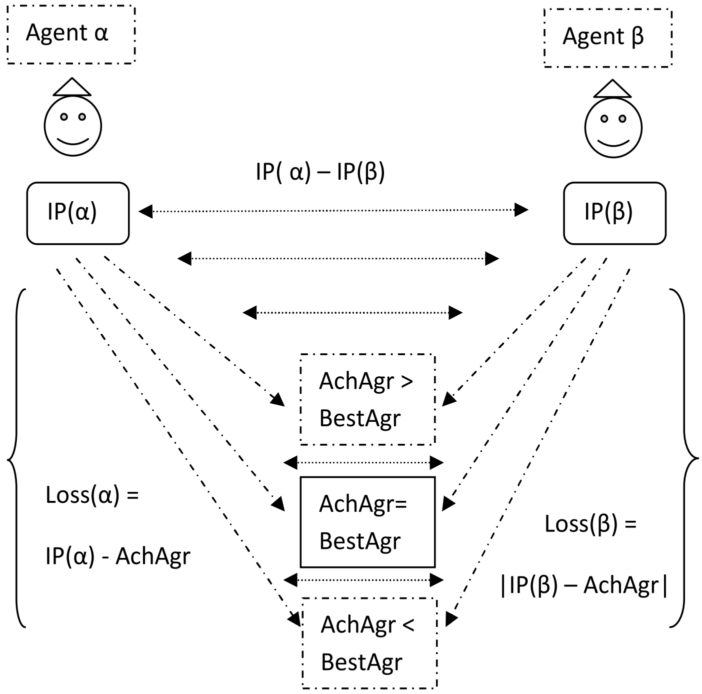

Figure 6. Some metrics of negotiation dialogue games between two agents

deal, then he will increase this value gradually till a possible agreement is achieved. Figure 6 shows the different metrics we consider in a negotiation dialogue game between two agents α and β . These metrics are the initial proposal for each agent, the achieved agreement and it's relation with the best agreement, and the loss function of each agent.

Because agents are rational and self interested, they try to minimize their losses. The final objective of an agent α is then to resolve the following optimization problem considering his knowledge base Γ_α:

$$AchAgr^* = \operatorname*{argmin}_{\Gamma_\alpha} Loss(\alpha) \tag{18}$$

Definition 13 [Goodness degree of the agent] *Let $D = [M_0, M_1, \ldots, M_n]$ be a negotiation dialogue between two agents α and β. We define the goodness degree of an agent $\chi \in \{\alpha, \beta\}$ in the dialogue as follows:*

$$GD(\chi) = 1 - [Loss(\chi)/|IP(\alpha) - IP(\beta)|] \tag{19}$$

We can easily prove the following proposition:

Proposition 3 *Let α and β be two negotiating agents.*

$$GD(\alpha) + GD(\beta) = 1$$

Example 5 *Suppose that we have 2 agents α and β negotiating about car price: α has a car and he wants to sell it and β is interested in buying this car. The first proposal by α was $\$8000$, and the first counter proposal by β was $\$5000$. At the end the car was sold at the price $\$6000$. To find which negotiator is better in this bargaining, we calculate the goodness degree of each one and compare them as follows:*

$Loss(\alpha) = |IP(\alpha) - AchAgr|$

$Loss(\alpha) = |8000 - 6000| = 2000$

$Loss(\beta) = |IP(\beta) - AchAgr|$

$Loss(\beta) = |5000 - 6000| = 1000$

$|IP(\alpha) - IP(\beta)| = |8000 - 5000| = 3000$

$GD(\alpha) = 1 - [Loss(\alpha)/|IP(\alpha) - IP(\beta)|]$

$= 1 - [2000/3000] = 0.33333$

$GD(\beta) = 1 - [Loss(\beta)/|IP(\alpha) - IP(\beta)|]$

$= 1 - [1000/(8000 - 5000)] = 0.66666$

We notice from this example that β is better than α in terms of the goodness degree. The distance from the first proposal to the achieved agreement for α is $\$2000$, which represents the amount of $\alpha's$ loss in the negotiation. For β, this distance is $\$1000$.
The best agreement that α can expect to achieve is $\$8000$. For β, the best expected agreement is $\$5000$. In these two cases, we will have: $GD(\alpha) = GD(\beta) = 1$, which is impossible according to Proposition 3. The worst agreement α can achieve is $\$5000$. For β, the worst case is $\$8000$. In these two cases we will have $GD(\alpha) = GD(\beta) = 0$, which is also impossible according to Proposition 3.

4.2. How Far are the Agents from the Right Dialogue?

In the previous section 4.1, we calculated the goodness degree of the participants to the dialogue based on the achieved agreement in the real dialogue $(AchAgr)$. Now we will consider $BestAgr$, which is the best agreement that could be attained in the dialogue from the evaluator's point of view. In fact, achieving this agreement means that the participants are doing the right dialogue, i.e. the best dialogue that can be produced by the participants if they know the knowledge bases of each other. The idea here is to recompute the goodness degree of the agents based on the best agreement GD_{ba} to see how far the agents are from the right dialogue. Thus, the amount of agent's loss in the dialogue based on the best agreement is:

$$Loss_{ba}(\alpha) = |IP(\alpha) - BestAgr| \tag{20}$$

Then we calculate the goodness degree of the agents in the dialogue using equation 19, where $Loss$ function should be replaced by $Loss_{ba}$.
Now, we will compare the goodness degree of each agent in both cases: in the case of considering the achieved agreement, and in the case of considering the best agreement. The achieved agreement could be less than, equal or greater than the best agreement. We assume here that agent α is trying to minimize $AchAgr$.

Property 1 *If $AchAgr < BestAgr$, this means that α has a good goodness degree in the real dialogue, while β has a bad goodness degree. In other words, in the real dialogue α is doing better than β.*

Property 2 *If $AchAgr = BestAgr$, this means that the two agents are performing the right dialogue from the evaluator's point of view, and they are doing well in the real dialogue.*

Property 3 *If $AchAgr > BestAgr$, this means that α has a bad goodness degree in the real dialogue, while β has a good goodness degree the same dialogue. In other words, in the real dialogue β is performing better than α.*

Example 6 *In example 5, we suppose that the best agreement for that dialogue from the evaluator's point of view is £ 5500, and we recompute the loss amount of each agent:*

$$Loss_{ba}(\alpha) = |IP(\alpha) - BestAgr|$$

$$Loss_{ba}(\alpha) = |8000 - 5500| = 2500$$

$$Loss_{ba}(\beta) = |IP(\beta) - BestAgr|$$

$$Loss_{ba}(\beta) = |5000 - 5500| = 500$$

We notice that the amount of $\alpha's$ loss increases, while that for β decreases, and by recomputing the goodness degree of each agent, we notice that $GD_{ba}(\alpha)$ is less than $GD(\alpha)$, which means α was doing better in the real dialogue. Whereas, $GD_{ba}(\beta)$ is greater than $GD(\beta)$, which means β was not good in the real dialogue.

$$GD_{ba}(\alpha) = 1 - [Loss_{ba}(\alpha)/|IP(\alpha) - IP(\beta)|]$$

$$GD_{ba}(\alpha) = 1 - [2500/3000] = 0.1666$$

$$GD_{ba}(\beta) = 1 - [Loss_{ba}(\beta)/|IP(\alpha) - IP(\beta)|]$$

$$GD_{ba}(\beta) = 1 - [500/3000] = 0.8333$$

We call the distance for an agent α from the real dialogue to the right dialogue farness degree $FD(\alpha)$. It is equal to the difference between the goodness degree of α in the real dialogue and that in the right dialogue.

Definition 14 [Farness degree of an agent] *Let $D = [M_0, M_1, \ldots, M_n]$ be a negotiation dialogue. α and β are the participants to this dialogue. We define the farness degree of $\chi \in \{\alpha, \beta\}$ in the dialogue D as:*

$$FD(\chi) = GD(\chi) - GD_{ba}(\chi) \tag{21}$$

Example 7 *Let us take the results from Example 5 and Example 6, and compute the farness degree of each agent:*

$FD(\alpha) = GD(\alpha) - GD_{ba}(\alpha)$

$FD(\alpha) = 0.3333 - 0.1666 = +0.1667$

$FD(\beta) = GD(\beta) - GD_{ba}(\beta)$

$FD(\beta) = 0.6666 - 0.8333 = -0.1667$

As we see here, the farness degree for α is positive, while for β is negative and these signs "$+, -$" indicate whether the agent is doing better or bad in the real dialogue. If the sign is positive, this means the agent is far from the right dialogue by this value, and he is doing better in the real dialogue, and if it is negative, this means that he is far in the negative side and he is doing bad in the real dialogue.

5. Related Work

To the best of our knowledge, there is no work about using Shannon entropy to measure dialogue games for agent communication. Also, there is no work focussing on measurements for agent communication such those proposed in this paper, namely, move's certainty index, dialogue's certainty index, and agent's certainty, goodness and farness from the best negotiation dialogue. However, some other measurements for dialogue games have been discussed by Amgoud and Florence in [1] for persuasion dialogues, by Hunter in [8] for higher impact argumentation dialogues, and by Yuan et al. in [16] for dialogue strategies.

Amgoud and Florence in [1] have defined a set of quality measures for persuasion dialogue games from an external agent's point of view. They have analyzed already generated dialogues whatever the protocol used and whatever the strategies of the agents are. They have proposed measurements for the quality of exchanging arguments in terms of their persuasive weights and measurements of the behavior of the participants to the dialogue in terms of their coherence, aggressiveness, and the novelty of their arguments. They also proposed metrics for the quality of the dialogue itself in terms of the relevance and usefulness of the exchanged moves. These measures are important to set the foundation of measuring the quality of the dialogue and compare different dialogues on the same subject.

Hunter has defined in [8] a strategy for selecting arguments in a dialogue. He has addressed the need to increase the impact of argumentation by using the pruned argument trees. The basic idea is that an agent selects the arguments that will satisfy the goals of the audience. The agent is thus assumed to maintain two bases: a base containing his own beliefs and another base containing what the agent thinks are the goals of the audience. Amgoud and Florence in [1] and Hunter in [8] have considered only arguments, and they associated with each argumentation system an oriented graph whose nodes are the different arguments, and the edges represent the attack relation between them.

Yuan et al. [16] have proposed some heuristics to measure strategies in order to allow the participants to choose moves in debating settings. However, these measures have been analyzed in a symbolic manner, and no numerical functions have been proposed.

The proposals discussed in this section are thus more concerned with proposing dialogue strategies and analyzing dialogues. However, nothing is said about neither the goodness degree of the agents in the real dialogue nor the farness degree of the agents from the right dialogue that we discussed in this paper. Moreover, we proposed measures for the exchanged moves and the dialogue itself in terms of the certainty index of the agents a bout selecting the moves at each dialogue step and their certainty about the whole dialogue. Considering the best dialogue from an observer's point of view based on the union of agents' knowledge bases is another novelty of this paper.

6. Conclusion and Future Work

In this paper, we presented two sets of metrics for negotiation dialogue games. The first set is based on the concept of Shannon entropy, and the second one is about evaluating agents in quantitative negotiation dialogue games, such as bargaining. In the first set, we used Shannon entropy to assess the agents' certainty about their moves in the negotiation dialogues. We supposed that an external agent is monitoring the dialogue, and he wants to evaluate this dialogue in terms of the agent's certainty about selecting the right move at each step. In fact, at each step, the agent is supposed to have different choices, each choice is associated with a probability, which represents the chance for this move to be accepted from the addressee [11]. In our proposed metrics, the move with the higher certainty index is considered as the best move. We also analyzed the fact that negotiating agents are rational and they always try to perform the actions that will result in the optimal outcome for themselves. We started our work with measuring the certainty index of each move at each step in the dialogue by applying the general formula of entropy, and in order to distinguish between two moves with the same certainty index, we assign weight to each move, which reflects the importance of the move. We also measured the certainty index of the whole dialogue using two methods: 1) the first one is by taking the average of the certainty index of all moves in the dialogue and measuring the weighted certainty index of the dialogue using the weighted certainty index of the moves; and 2) the second method is by computing all possible dialogues and their probabilities and applying Shannon entropy for the dialogue. In the second set, we proposed two metrics: the goodness metric that measures how good the agents are in the real dialogue, and the farness metric that measures how far the agents are in the real dialogue from the right dialogue.

Our plan for future work is to extend the proposed metrics for other dialogue game types such as persuasion, deliberation, inquiry and information seeking. We also plan to analyze argumentation-based dialogues to evaluate agent strategies and analyze them from the optimization perspective. Analyzing the computational complexity of such optimization problems is another direction for future work. Finally, we plan to analyze multi-party dialogues to which many agents can participate. Extending the proposed metrics to this type of dialogues is not straightforward. For example, defining the rules of a multi-party negotiation is much more complicated than two-party dialogues. In fact, multi-party dialogues cannot be simply reduced to many two-party dialogues.

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References

[1] L. Amgoud and F. D. de Saint-Cyr. Measures for persuasion dialogs: A preliminary investigation. In *2nd International Conference on Computational Models of Arguments*, pages 13–24, Toulouse, France, 2008. IOS Press.

[2] L. Amgoud, N. Maudet, and S. Parsons. Modelling dialogues using argumentation. In *Proceedings of the Fourth International Conference on MultiAgent Systems*, pages 31–38, Boston, USA, 2000.

[3] J. Bentahar and J.-J. C. Meyer. A new quantitative trust model for negotiating agents using argumentation. *International Journal of Computer Science and Applications (IJCSA)*, 4(2):1–21, 2007.

[4] E. Black and A. Hunter. A generative inquiry dialogue systems. In *International Joint Conference on Autonomous Agents and MultiAgent Systems*, pages 963–970, New York, NY, USA, 2007. ACM Press.

[5] T. M. Cover and J. A. Thomas. *Elements of Information Theory*. John Wiley and Sons, LTD, New yourk, NY, USA, 1991.

[6] M. Dastani, J. Dix, and A. E. Fallah-Seghrouchni. In *First International Workshop on Programming Multi-Agent Systems*, Melbourne, Australia, 2004. Springer.

[7] C. L. Hamblin. *Fallacies*. Methuen London, LTD, Methuen London, UK, 1970.

[8] A. Hunter. Towards higher impact argumentation. In *Proceedings of the National Conference on Artificial Intelligence*, pages 275–280, San Jose, California, 2004. AAAI.

[9] I. Jenhani, N. B. Amor, Z. Elouedi, S. Benferhat, and K. Mellouli. Information affinity: A new similarity measure for possibilistic uncertain information. In *Proceedings of the 9th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU)*, pages 840–852, Hammamet, Tunisia, 2007. Springer.

[10] N. Maudet. Negotiating dialogue games. *Autonomous Agents and Multi-Agent Systems*, 7(3):229–233, 2003.

[11] M. Mbarki, J. Bentahar, and B. Moulin. Specification and complexity of strategic-based reasoning using argumentation. *ArgMAS*, Lecture Notes in Computer Science (4766):142–160, 2007.

[12] S. Parsons, M. Wooldridge, and L. Amgoud. Properties and complexity of some formal inter-agent dialogues. *Journal of Logic and Computation*, 13(3):347–376, 2003.

[13] H. Prakken. Relating protocols for dynamic dispute with logics for defeasible argumentation. *Synthese*, 127(1-2):187–219, 2001.

[14] I. Rahwan, R. Sarvapali, J. Nicholas, P. Mcburney, S. Parsons, and L. Sonenberg. Argumentation-based negotiation. *The Knowledge Engineering Review*, 18(4):343–375, 2003.

[15] K. P. Sycara. Persuasive argumentation in negotiation. *Theory and Decision*, 28(3):203–242, 1990.

[16] T. Yuan, D. Moore, and A. Grierson. An assessment of dialogue strategies for a human computer debating system, via computational agents. In *Proceedings of ECAI 2004 Workshop on Computational Models of Natural Argument*, pages 17–24, 2004.

Appendix

Proof of Proposition 2

Let us first prove the direct implication $\Rightarrow$. We assume that all the moves at a certain step t_i have the same probability, and prove that the uncertainty is equal to one. Without loos of generality, we assume that $|S_i|$=*k*. So we have:

$\mu(M_i) = H(M_i)/Log(k)$

$= -\sum_{m_i^j \in S_i, j=1}^{k} P(m_i^j)LogP(m_i^j)/Log(k)$

$= -k[(1/k)Log(1/k)]/Log(k)$

$= -[Log(1/k)]/Log(k)$

$= Log(k)/Log(k)$

$= 1$

Let us now prove the inverse implication $\Leftarrow$. We assume that the uncertainty is equal to one, and prove that all moves have the same probability. So we have:

$\mu(M_i) = 1$

$\Rightarrow 1 = H(M_i)/Log(k)$

$\Rightarrow 1 = -\sum_{m_i^j \in S_i, j=1}^{k} P(m_i^j)LogP(m_i^j)/Log(k)$

$\Rightarrow -Log(k) = \sum_{m_i^j \in S_i, j=1}^{k} P(m_i^j)LogP(m_i^j)$

$\Rightarrow Log(1/k) = \sum_{m_i^j \in S_i, j=1}^{k} P(m_i^j)LogP(m_i^j)$

$\Rightarrow Log(1/k) = P(m_i^1)LogP(m_i^1) + P(m_i^2)LogP(m_i^2) + \ldots + P(m_i^k)LogP(m_i^k)$

By taking the exponential of both sides of the equation, we obtain:

$\exp^{Log(1/k)} = \exp^{P(m_i^1)LogP(m_i^1)+P(m_i^2)LogP(m_i^2)+\ldots+P(m_i^k)LogP(m_i^k)}$

$\Rightarrow 1/k = \exp^{P(m_i^1)LogP(m_i^1)} * \exp^{P(m_i^2)LogP(m_i^2)} * \ldots * \exp^{P(m_i^k)LogP(m_i^k)}$

$\Rightarrow 1/k = \exp^{LogP(m_i^1)^{P(m_i^1)}} * \exp^{LogP(m_i^2)^{P(m_i^2)}} * \ldots * \exp^{LogP(m_i^k)^{P(m_i^k)}}$

$\Rightarrow 1/k = P(m_i^1)^{P(m_i^1)} * P(m_i^2)^{P(m_i^2)} * \ldots * P(m_i^k)^{P(m_i^k)}$

$\Rightarrow 1/k = \Pi_{m_i^j \in S_i, j=1}^{k} P(m_i^j)^{P(m_i^j)}$

Because 1 is the maximum uncertainty, the solution of this equation can be obtained by resolving the following optimization problem:

$$Max[\Pi^{k}_{m_i^j \in S_i, j=1} P(m_i^j)^{P(m_i^j)}]$$

$$subject\ to:$$

$$\begin{cases} \sum^{k}_{m_i^j \in S_i, j=1} P(m_i^j) = 1 \\ 0 < P(m_i^j) \leq 1 \quad \forall\, 1 \leq j \leq k \end{cases}$$

Using the nonlinear programming techniques, we can easily find the solution of this problem, which is: $\forall\, 1 \leq j \leq k\ P(m_i^j) = 1/k$. $\square$

Proof of Theorem 1

Because these problems are equivalent, we consider only one of them, for example the maximization one. Without loss of generality, we assume that agent α should solve this problem. The algorithm is as follows: 1) α should calculate the probability of each possible move m_i^j $(1 \leq j \leq k)$ using the *knowledge and move function*: $\zeta(\Gamma_\alpha \cup CS_\beta^{t_i})$ considering his knowledge base Γ_α and β's commitment store $CS_\beta^{t_i}$; 2) take the move with the highest probability. Because Γ_α and $CS_\beta^{t_i}$ are bounded at each step t_i, this calculation is clearly polynomial and searching the maximum probability is polynomial, so we are done. $\square$