Fall 2011

INDU 6111 Final exam

Solutions

1. [20 points out of 100] Consider the basic feasible solution

$$x_1^* = 2,\ x_2^* = 2,\ x_3^* = 0,\ x_4^* = 0,\ x_5^* = 1,\ x_6^* = 1$$

of the problem

$$\begin{array}{lrcrcrcrcrcrcrcl} \text{maximize} & 3x_1 & + & 2x_2 & + & 4x_3 & + & 3x_4 & + & 4x_5 & + & 5x_6 & & \\ \text{subject to} & 2x_1 & + & 2x_2 & + & 3x_3 & + & 3x_4 & + & 4x_5 & + & 4x_6 & = & 16 \\ & 2x_1 & + & 3x_2 & + & 3x_3 & + & 4x_4 & + & 4x_5 & + & 5x_6 & = & 19 \end{array}$$

$$0 \le x_1, x_2, x_3, x_4, x_5, x_6 \le 2$$

What are the basic variables and what are all the candidates for entering the basis?

Answer: x_5, x_6 are the basic variables x_2, x_3 are all the candidates for entering the basis.

Justification: x_5, x_6 have to be basic since their values are strictly between the bounds. We have

$$\begin{array}{rcl} c^T & = & [3,2,4,3,4,5], \\ [0,1]^T A & = & [2,3,3,4,4,5]. \end{array}$$

Comparing these two vectors, we see that we would like to increase the values of x_1 and x_3 and to decrease the values of x_2 and x_4. However, x_1 is at its upper bound and x_4 is at its lower bound.

2. [20 points out of 100] Starting from the feasible solution

$$x_1^* = 1,\ x_2^* = 0,\ x_3^* = 1,\ x_4^* = 0,\ x_5^* = 1$$

of the problem

$$\begin{array}{lrcrcrcrcrcr} \text{maximize} & 2x_1 & + & x_2 & + & 2x_3 & + & 4x_4 & + & 4x_5 & & \\ \text{subject to} & 2x_1 & + & x_2 & & & + & x_4 & + & x_5 & \le & 4 \\ & & & x_2 & + & 2x_3 & & & + & 3x_5 & \le & 5 \\ & x_1 & - & x_2 & + & x_3 & & & & & \le & 2 \\ & -x_1 & + & x_2 & & & & & & & \le & -1 \\ & & & & & & & \multicolumn{3}{r}{x_1, x_2, x_3, x_4, x_5} & \ge & 0 \end{array}$$

find its optimal solution and justify your answer.

Answer: $x_1^* = 1,\ x_2^* = 0,\ x_3^* = 1,\ x_4^* = 1,\ x_5^* = 1.$

Justification: $y_1^* = 4,\ y_2^* = 0,\ y_3^* = 2,\ y_4^* = 8$ is an optimal solution of the dual problem.

3. [20 points out of 100] Find some values of a, b, c, d such that the system

$$
\begin{array}{rrrcl}
3x & -8y & & \leq & a \\
 & 5y & -3z & \leq & b \\
-5x & & +z & \leq & c \\
7x & +3y & +z & \leq & d
\end{array}
$$

is unsolvable. Justify your answer.

Answer: For instance, $a = -1$, $b = -1$, $c = -1$, $d = -1$. More generally, any choice of a, b, c, d such that $a + b + 2c + d < 0$ is a correct answer.
Justification: The linear combination of the four inequalities with multipliers $1, 1, 2, 1$ (in this order) reads $0 \leq a + b + 2c + d$.

4. [20 points out of 100] Your cat likes three kinds of cans: each can of

- MEEOW costs 40 cents and contains 2 g of protein and 3 mg of cholesterol,
- OOOOH costs 50 cents and contains 3 g of protein and 2 mg of cholesterol,
- PURRR costs 20 cents and contains 2 g of protein and 2 mg of cholesterol.

Find the cheapest menu that has at least 5 g of protein and at most 4 mg of cholesterol and justify your answers.

Answer: 1 can of OOOOH and 1 can of PURRR.
Justification: $y_P = 30$, $y_C = 20$ is an optimal solution of the dual problem,

$$
\begin{array}{lrcrcr}
\text{maximize} & 5y_p & - & 4y_C & & \\
\text{subject to} & 2y_P & - & 3y_C & \leq & 40 \\
 & 3y_P & - & 2y_C & \leq & 50 \\
 & 2y_P & - & 2y_C & \leq & 20 \\
 & & & y_P, y_C & \geq & 0
\end{array}
$$

5. [20 points out of 100] Find a solution of the system

$$
\begin{array}{rcrcrcrcl}
4x_1 & + & 3x_2 & + & 3x_3 & + & 2x_4 & = & 5 \\
 & & & & x_3 & + & x_4 & = & 1 \\
 & & x_2 & + & x_3 & + & 2x_4 & = & 3
\end{array}
$$

$$0 \leq x_1, x_2, x_3, x_4 \leq 1$$

Answer: $x_1^* = 0$, $x_2^* = 1$, $x_3^* = 0$, $x_4^* = 1$.