INDU 6111 Theory of Operations Research
Homework Assignment 3
Due on Wednesday November 30 in class.
Late homeworks will not be accepted.

Please, print out the Expectation of Originality Form
(available from a link on our class web page),
fill it in, sign it, and attach it to your homework.

1. [10 points out of 40] Consider the basic feasible solution

$$x_1^* = 0,\ x_2^* = 0,\ x_3^* = 1,\ x_4^* = 1,\ x_5^* = 2,\ x_6^* = 2$$

of the problem

$$\begin{array}{lrcrcrcrcrcrcl} \text{maximize} & 3x_1 & + & 6x_2 & + & 6x_3 & + & 7x_4 & + & 7x_5 & + & 10x_6 & & \\ \text{subject to} & 2x_1 & + & 2x_2 & + & 3x_3 & + & 3x_4 & + & 4x_5 & + & 4x_6 & = & 22 \\ & 2x_1 & + & 3x_2 & + & 3x_3 & + & 4x_4 & + & 4x_5 & + & 5x_6 & = & 25 \end{array}$$

$$0 \le x_1, x_2, x_3, x_4, x_5, x_6 \le 2$$

What are the basic variables and what are all the candidates for entering the basis?

2. [10 points out of 40] Find a solution of the system

$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & + & 2x_3 & - & x_4 & = & 1 \\ -x_1 & + & x_2 & - & 3x_3 & + & 2x_4 & = & 1 \\ 3x_1 & - & 4x_2 & + & 8x_3 & - & 5x_4 & = & -1 \end{array}$$

$$x_1, x_2, x_3, x_4 \ge 0$$

3. [10 points out of 40] Write down the dual of the problem

$$\begin{array}{lrcrcrcr} \text{maximize} & & & & & x_3 & & \\ \text{subject to} & x_1 & + & x_2 & & & = & 3 \\ & x_1 & - & 3x_2 & - & x_3 & \ge & 0 \\ & 3x_1 & - & 5x_2 & + & x_3 & \le & 0 \end{array}$$

$$x_1 \ge 0,\ x_2 \ge 0$$

and solve both problems.

4. [10 points out of 40] Illustrate Theorem 9.4 on the system

$$\begin{array}{rrrcr} x & +3y & +z & \le & 4 \\ -x & -y & +3z & \le & -3 \\ -3x & +2y & & \le & 0 \\ & 2y & -z & \le & 1 \\ x & -2y & -z & \le & -1 \end{array}$$

and certify that the smaller system is inconsistent.