Fall 2011

# INDU 6111 Midterm exam: Solutions

1. [25 points/100] True or false? $x_1^* = 0$, $x_2^* = 1$, $x_3^* = 1$, $x_4^* = 0$, $x_5^* = 1$ is the optimal solution of the problem

$$\begin{array}{lrcrcrcrcrcr}
\text{maximize} & 4x_1 & & & + & 4x_3 & - & 2x_4 & - & x_5 & & \\
\text{subject to} & 2x_1 & + & x_2 & - & 2x_3 & + & 3x_4 & & & \leq & 0 \\
 & x_1 & - & x_2 & & & - & x_4 & & & \leq & -1 \\
 & 2x_1 & + & 2x_2 & + & 3x_3 & - & 2x_4 & - & x_5 & \leq & 4 \\
 & 3x_1 & - & x_2 & + & x_3 & + & x_4 & & & \leq & 0 \\
 & & & & & & & \multicolumn{3}{r}{x_1, x_2, x_3, x_4, x_5} & \geq & 0
\end{array}$$

**Solution:** True.
**Justification:** $y_1^* = 0$, $y_2^* = 1$, $y_3^* = 1$, $y_4^* = 1$ is an optimal solution of the dual problem.
**How to get it:** From the Complementary Slackness Theorem. The unique solution of the system $y_1 = 0$,

$$\begin{array}{rcrcrcr}
-y_2 & + & 2y_3 & - & y_4 & = & 0 \\
 & & 3y_3 & + & y_4 & = & 4 \\
 & & -y_3 & & & = & -1
\end{array}$$

satisfies all dual constraints.

2. [25 points/100] Solve the problem

$$\begin{array}{lrrcr}
\text{maximize} & x_1 & -3x_2 & & \\
\text{subject to} & x_1 & -2x_2 & \leq & 3 \\
 & -x_1 & +x_2 & \leq & -1 \\
 & & \multicolumn{3}{l}{x_1, x_2 \geq 0}
\end{array}$$

**The solution:** $x_1 = 3$, $x_2 = 0$.
**One way of getting it:** Set up the auxiliary problem,

$$\begin{array}{lrrrcr}
\text{minimize} & w & & & & \\
\text{subject to} & x_1 & -2x_2 & & \leq & 3 \\
 & -x_1 & +x_2 & -w & \leq & -1 \\
 & & \multicolumn{4}{l}{x_1, x_2, w \geq 0}
\end{array}$$

The first phase begins with its initial dictionary

$$\begin{array}{rcllll} x_3 & = & 3 & -x_1 & +2x_2 & \\ w & = & 1 & -x_1 & +x_2 & +x_4 \end{array}$$

from which a single simplex iteration leads to its optimal dictionary,

$$\begin{array}{rclll} x_1 & = & 1 & +x_2 & +x_4 \\ x_3 & = & 2 & +x_2 & -x_4 \end{array}$$

The second phase begins with its initial dictionary

$$\begin{array}{rclll} x_1 & = & 1 & +x_2 & +x_4 \\ x_3 & = & 2 & +x_2 & -x_4 \\ z & = & 1 & -2x_2 & +x_4 \end{array}$$

from which a single simplex iteration leads to its optimal dictionary,

$$\begin{array}{rclll} x_4 & = & 2 & +x_2 & -x_3 \\ x_1 & = & 3 & +2x_2 & -x_3 \\ z & = & 3 & -x_2 & -x_3 \end{array}$$

3. [25 points/100] True or false? $x_1^* = 1$, $x_2^* = 0$, $x_3^* = 1$, $x_4^* = 0$, $x_5^* = 1$ is the optimal solution of the problem

$$\begin{array}{lrcrcrcrcrcr} \text{maximize} & & & & & 3x_3 & + & 5x_4 & + & 3x_5 & & \\ \text{subject to} & 2x_1 & + & x_2 & & & - & 2x_4 & + & x_5 & \leq & 4 \\ & & & x_2 & + & 2x_3 & + & 2x_4 & + & 3x_5 & \leq & 5 \\ & x_1 & - & x_2 & + & x_3 & - & x_4 & & & \leq & 2 \\ & -x_1 & + & x_2 & & & + & 2x_4 & & & \leq & -1 \\ & & & & & & & & & x_1, x_2, x_3, x_4, x_5 & \geq & 0 \end{array}$$

**Solution:** False

**Justification:** From the Complementary Slackness Theorem. The unique solution of the system $y_1 = 0$,

$$\begin{array}{rcrcrcl} & & y_3 & - & y_4 & = & 0 \\ 2y_2 & + & y_3 & & & = & 3 \\ 3y_2 & & & & & = & 3 \end{array}$$

is $y_1^* = 0$, $y_2^* = 1$, $y_3^* = 1$, $y_4^* = 1$ and it fails to satisfy the dual constraint

$$-2y_1 + 2y_2 - y_3 + 2y_4 \ \geq \ 5.$$

4. [25 points/100] Label each of the following statements "TRUE" or "FALSE". Write out the entire word.

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- The simplex method constructs a degenerate dictionary if it cycles.

  **Answer:** True.

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- There is an unbounded LP problem whose dual is unbounded.

  **Answer:** False

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- There is an infeasible LP problem whose dual is unbounded.

  **Answer:** True.

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- The simplex method constructs a degenerate dictionary only if it cycles.

  **Answer:** False

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- There is an unbounded LP problem whose dual is infeasible.

  **Answer:** True.

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