Fall 2011

INDU 6111 Theory of Operations Research
Homework Assignment 1
Solutions

1. [10 points out of 40] You have 700 kg of rubber, 600 kg of cereal, and 60 kg of tripe lying around. From these ingredients, you can make two kinds of sausages. Each kilogram of JUICY takes 0.2 kg of rubber, 0.4 kg of cereal, and 0.5 kg of tripe for the net profit of 50 cents. Each kilogram of FREEDOM FRANKS takes 0.5 kg of rubber, 0.3 kg of cereal, and 0.25 kg of tripe for the net profit of 70 cents. Formulate the corresponding linear program and show its optimal dictionary.

One solution: The problem can be formulated as

$$\begin{array}{lrcrcr} \text{maximize} & 0.5J & + & 0.7F & & \\ \text{subject to} & 0.2J & + & 0.5F & \leq & 700 \\ & 0.4J & + & 0.3F & \leq & 600 \\ & 0.5J & + & 0.25F & \leq & 60 \\ & & & J, F & \geq & 0 \end{array}$$

Here, the optimal dictionary is

$$\begin{array}{rrrr} F = & 240 & -2J & -4T \\ R = & 580 & +0.8J & +2T \\ C = & 528 & +0.2J & +1.2T \\ z = & 168 & -0.9J & -2.8T \end{array}$$

2. [10 points out of 40] Your workshop can manufacture five kinds of furniture:

- A desk takes 1 unit of wood, 2 units of metal, and 2 hours of work; it brings in a net profit of \$30;
- A chair takes 1 unit of wood, 1 unit of metal, and 1 hour of work; it brings in a net profit of \$20;
- A bedframe takes 1 unit of wood, 2 units of metal, and 1 hour of work; it brings in a net profit of \$30;
- A bookcase takes 1 unit of wood, 1 unit of metal, and 2 hours of work; it brings in a net profit of \$30;
- A coffee table takes 2 units of wood, 2 units of metal, and 1 hour of work; it brings in a net profit of \$25.

You have 40 units of wood, 60 units of metal, and 50 hours of time to work are at your disposal. Formulate the corresponding linear program and show its optimal dictionary.

One solution: The problem can be formulated as

$$\begin{array}{lrrrrrcl}
\text{maximize} & 30D & +20CH & +30BF & +30BC & +25CT & & \\
\text{subject to} & D & +CH & +BF & +BC & +2CT & \leq & 40 \\
 & 2D & +CH & +2BF & +BC & +2CT & \leq & 60 \\
 & 2D & +CH & +BF & +2BC & +CT & \leq & 50 \\
 & & & & & D, CH, BF, BC, CT & \geq & 0.
\end{array},$$

Here, one optimal dictionary is

$$\begin{array}{lcrrrrrr}
CH & = & 10 & +D & -3CT & -3W & +M & +T \\
BF & = & 20 & -D & & +W & -M & \\
BC & = & 10 & -D & +CT & +W & & -T \\
z & = & 1100 & -40D & -30CT & & -10M & -10T
\end{array}$$

and another is

$$\begin{array}{lcrrrrrr}
W & = & \frac{10}{3} & +\frac{1}{3}D & -CT & -\frac{1}{3}CH & +\frac{1}{3}M & +\frac{1}{3}T \\
BF & = & \frac{70}{3} & -\frac{2}{3}D & -CT & -\frac{1}{3}CH & -\frac{2}{3}M & +\frac{1}{3}T \\
BC & = & \frac{40}{3} & -\frac{2}{3}D & & -\frac{1}{3}CH & +\frac{1}{3}M & -\frac{2}{3}T \\
z & = & 1100 & -40D & -30CT & & -10M & -10T
\end{array}$$

3. [10 points out of 40] Show an optimal dictionary for the auxiliary problem arising from the problem

$$
\begin{array}{lrrrrrrl}
\text{maximize} & x_1 & - & x_2 & + & 2x_3 & & \\
\text{subject to} & -x_1 & + & x_2 & + & x_3 & \leq & 1 \\
 & & + & x_2 & - & 2x_3 & \leq & -2 \\
 & x_1 & - & x_2 & - & 2x_3 & \leq & -3 \\
 & x_1 & - & 3x_2 & + & 3x_3 & \leq & 2 \\
 & & & \multicolumn{3}{c}{x_1, x_2, x_3 \geq 0} & &
\end{array}
$$

The solution: The auxiliary problem is

$$
\begin{array}{lrrrrrrrl}
\text{minimize} & w_2 & + & w_3 & & & & & \\
\text{subject to} & -x_1 & + & x_2 & + & x_3 & & \leq & 1 \\
 & & + & x_2 & - & 2x_3 & -w_2 & \leq & -2 \\
 & x_1 & - & x_2 & - & 2x_3 & -w_3 & \leq & -3 \\
 & x_1 & - & 3x_2 & + & 3x_3 & & \leq & 2 \\
 & & & \multicolumn{4}{c}{x_1, x_2, x_3, w_2, w_3 \geq 0} & &
\end{array}
$$

and its optimal dictionary is

$$
\begin{array}{rcrrrrrr}
x_1 & = & 3.5 & +4.5x_4 & & +3x_6 & +0.5x_7 & -3w_3 \\
x_2 & = & 2.5 & +2.5x_4 & & +2x_6 & +0.5x_7 & -2w_3 \\
x_3 & = & 2 & +x_4 & & +x_6 & & -w_3 \\
w_2 & = & 0.5 & +0.5x_4 & +x_5 & & +0.5x_7 & \\
w & = & 0.5 & +0.5x_4 & +x_5 & & +0.5x_7 & +w_3
\end{array}
$$

Since artificial variables may be deleted as soon as they become nonbasic, the dictionary

$$
\begin{array}{rcrrrrr}
x_1 & = & 3.5 & +4.5x_4 & & +3x_6 & +0.5x_7 \\
x_2 & = & 2.5 & +2.5x_4 & & +2x_6 & +0.5x_7 \\
x_3 & = & 2 & +x_4 & & +x_6 & \\
w_2 & = & 0.5 & +0.5x_4 & +x_5 & & +0.5x_7 \\
w & = & 0.5 & +0.5x_4 & +x_5 & & +0.5x_7
\end{array}
$$

is also acceptable as the optimal dictionary of the auxiliary problem.

4. [10 points out of 40] Show a complete sequence of feasible dictionaries constructed by the simplex method in the process of solving the problem

$$\begin{array}{ll} \text{maximize} & 3x_1 - 3x_2 \\ \text{subject to} & 2x_1 - x_2 \leq 5 \\ & x_2 \leq 3 \\ & x_1 - 2x_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{array}$$

Draw the feasible region and point out the sequence of its vertices visited by the simplex method.

The solution:
First dictionary:

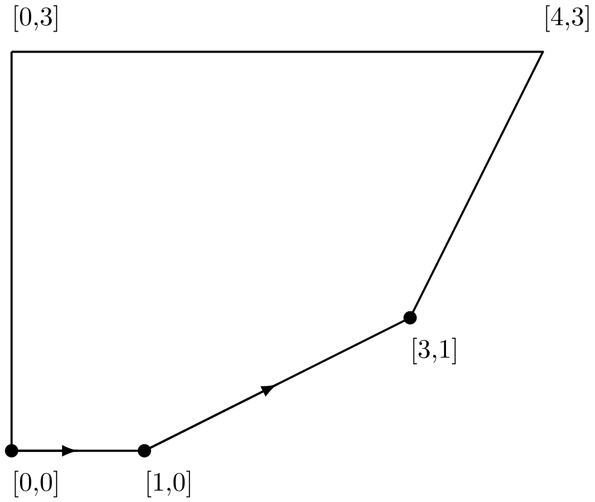
$$\begin{array}{rcrrr} x_3 & = & 5 & -2x_1 & +x_2 \\ x_4 & = & 3 & & -x_2 \\ x_5 & = & 1 & -x_1 & +2x_2 \\ z & = & & 3x_1 & -3x_2 \end{array}$$

Second dictionary:

$$\begin{array}{rcrrr} x_1 & = & 1 & +2x_2 & -x_5 \\ x_3 & = & 3 & -3x_2 & +2x_5 \\ x_4 & = & 3 & -x_2 & \\ z & = & 3 & +3x_2 & -3x_5 \end{array}$$

Third dictionary:

$$\begin{array}{rcrrr} x_2 & = & 1 & +\frac{2}{3}x_5 & -\frac{1}{3}x_3 \\ x_1 & = & 3 & +\frac{1}{3}x_5 & -\frac{2}{3}x_3 \\ x_4 & = & 2 & -\frac{2}{3}x_5 & +\frac{1}{3}x_3 \\ z & = & 6 & -x_5 & -x_3 \end{array}$$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