Winter 2012

COMP 6621 Discrete Mathematics of Paul Erdős
Homework Assignment 2
Due on Friday February 3.

You can e-mail me the homework as an unzipped pdf file up to 17:45 on February 3 or, if you prefer, hand in the hard copy in class.

Definitions: Every connected undirected graph induces a metric space on its vertex set, where $dist(u, v)$ is defined as the smallest number of edges in a path from vertex u to vertex v. In an arbitrary metric space, point y is said to lie between points x and z if x, y, z are three distinct points and $dist(x, y) + dist(y, z) = dist(x, z)$. It is customary to write $[xyz]$ for the statement that y lies between x and z. In this notation, a *line* $\overline{uv}$ is defined — for any two distinct points u and v — as

$$\{u, v\} \cup \{p : [puv] \vee [upv] \vee [uvp]\}.$$

Your assignment: Draw all (isomorphism types of) connected undirected graphs on five vertices where no line consists of all five vertices.

Motivation: Chen and Chvátal ("Problems related to a de Bruijn – Erdős theorem", *Discrete Applied Mathematics* **156** (2008), 2101 – 2108) asked the following question, which remains unanswered.

> *True or false? Every metric space on n points where $n \geq 2$ either has at least n distinct lines or else it has a line that consists of all n points.*