Winter 2012

COMP 6621 Discrete Mathematics of Paul Erdős
Take-home Midterm
Due on Friday March 9

You can e-mail me the midterm as an unzipped pdf file up to 17:45 on March 9 or, if you prefer, hand in the hard copy in class.

1. (3 points) Find the value of $\alpha(5)$ in the prime factorization

$$333! = \prod_p p^{\alpha(p)}.$$

(Just in case the exclamation mark is not clear enough: the left-hand side is the factorial of 333.)

2. (5 points) Find an integer a, as small as you can make it without too much effort, such that $a \to (3)^2_4$.

3. (6 points) Adapt Erdős's proof of the Ramsey number lower bound $r(k,k) > 2^{k/2}$ to get a function f, growing as fast as you can make it, with the following property:

> The integers $1, 2, \ldots, f(k)$ can be coloured red and white so that there is no monochromatic arithmetic progression of k terms.

I will consider answers f_1 and f_2 equally good if $\lim_{k\to\infty} f_1(k)/f_2(k) = 1$.

4. (6 points) **Definitions:** A *truth assignment* is a mapping f that assigns 0 (interpreted as "false") or 1 (interpreted as "true") to each variable in its domain. The *complement* $\overline{x}$ of each such variable x is defined by $f(\overline{x}) = 1 - f(x)$ for all truth assignments f; both x and $\overline{x}$ are called *literals*; if $u = \overline{x}$ then $\overline{u} = x$. A *clause* is a set of distinct literals and a *formula* (in a conjunctive normal form) is a family of clauses. A truth assignment *satisfies* a clause if it maps at least one of its literals to 1; the assignment *satisfies* a formula if and only if it satisfies each of its clauses. A formula is called *satisfiable* if it is satisfied by at least one truth assignment.

Adapt Erdős's proof of the Ramsey number lower bound $r(k,k) > 2^{k/2}$ to get a function g, growing as fast as you can make it, with the following property:

> Every boolean formula in a conjunctive normal form with $g(k)$ clauses and precisely k literals in each clause is satisfiable.

I will consider answers g_1 and g_2 equally good if $\lim_{k\to\infty} g_1(k)/g_2(k) = 1$.