Fall 2013

COMP 691D Probabilistic Methods in Computer Science
Term project
Due on November 22
(this deadline is firm: we will discuss the results on November 29 in class)

Choose one of the two following topics. You may work on this project on your own or with another (but no more than one) classmate.

Topic 1

Our textbook claims on page 8 that the result of Bollobás published in 'On generalized graphs', *Acta Math. Acad. Sci. Hungar* 16 (1965) 447–452 has many interesting extensions and applications. Submit a 6–15 page report on at least three (and preferably more) papers containing such extensions or (preferably) applications.

Topic 2

Primitive backgammon is a game for two players, whom we will call 'player $+1$' and 'player -1'. At all times, the current standing of player p is measured by a positive integer $d(p)$, which is known to both players. When it is her turn to play, player p flips a fair coin; heads mean that $d(p)$ gets decremented by 1 and tails mean that $d(p)$ gets decremented by 2; in either case, $d(-p)$ remains unchanged. If this decrement brings $d(p)$ down to 0 or -1, then p wins and the game is over; else it is now the turn of $-p$ to play.

The game is made interesting by the introduction of a *doubling cube,* which shows a power of 2 that is the current stakes; at all times this cube belongs to one of the players or to neither of them. In the beginning, the cube shows 1 and belongs to nobody. Just before flipping the coin, unless her opponent $-p$ already owns the cube, player p may 'offer' the cube to the opponent. Offering the cube means giving the opponent the choice of either accepting or refusing. Accepting means that the cube will now belong to the opponent and the stakes are doubled to the next power of 2; refusing means that the opponent resigns and pays up the current stakes.

Formally, each state of the game is the quintuple $(p, d(+1), d(-1), o, s)$, where p is the player whose turn it is to play, o is either the owner of the cube or 0 (if the cube belongs to nobody), and s are the current stakes. We have $s = 1$ if and only if $o = 0$; accepting the cube means moving from state $(p, d(+1), d(-1), o, s)$ with $o \in \{0, p\}$ to state $(p, d(+1), d(-1), -p, 2s)$.

Let $f(d_1, d_2, o, v)$ be the expected payoff to player $+1$ in state $(+1, d_1, d_2, o, v)$ under the assumption that both players play (= offer, accept, and refuse the cube) optimally. Evaluate this function at the 27 points where $(d_1, d_2) \in \{1, 2, 3\} \times \{1, 2, 3\}$ and (o, v) is $(0, 1)$ or $(+1, 2)$ or $(-1, 2)$. (Optionally, for bonus points, extend these results to a wider range of (d_1, d_2).)