A de Bruijn - Erdős theorem in metric spaces?

A corollary of the Sylvester-Gallai theorem asserts that every noncollinear set of n points in the plane determines at least n distinct lines; this theorem has been generalized by De Bruijn and Erdős (On a combinatorial problem, *Proc. Akad. Wet. Amsterdam* **51** (1948), 1277-1279). Chen and Chvátal (Problems related to a de Bruijn–Erdős theorem, *Discrete Appl. Math.* **156** (2008), 2101–2108) suggested another conceivable generalization:

> *True or false? Every finite metric space* (X, d) *where no line consists of the entire ground set* X *determines at least* $|X|$ *distinct lines.*

Here, the *line* $\overline{uv}$ in a metric space is defined as the union of $\{u, v\}$ and all the pins containing $\{u, v\}$, where a *pin* is any three-point set $\{x, y, z\}$ such that $d(x, y) + d(y, z) = d(x, z)$.

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