ON THE COMPUTATIONAL COMPLEXITY OF
FINDING A KERNEL
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The purpose of this note is to exhibit a new member of Karp's class of "complete" problems. For the notation and necessary background, the reader is referred to Karp's paper [3].

**THEOREM.** The following problem is complete.

**KERNEL**

**INPUT:** digraph H

**PROPERTY:** there is a set $K \subseteq V$ such that

(i) $(u,v) \in E$ for no $u,v \in K$

(ii) if $u \notin K$ then there is a $v \in K$ with $(u,v) \in E$.

Proof. We will show that SATISFIABILITY $\Leftrightarrow$ KERNEL. The reduction (still in Karp's notation) goes as follows.

$$V = V_1 \cup V_2, E = E_1 \cup E_2 \cup E_3 \cup E_4,$$

$$V_1 = \{\langle \sigma, i \rangle \mid \sigma \text{ is a literal and occurs in } C_i \},$$

$$V_2 = \{1,2,\ldots,p\} \times \{1,2,3\},$$

$$E_1 = \{\langle \langle \sigma, i \rangle, \langle \delta, j \rangle \rangle \mid i = j, \sigma \neq \delta \},$$

$$E_2 = \{\langle \langle \sigma, i \rangle, \langle \delta, j \rangle \rangle \mid i \neq j, \sigma = \overline{\delta} \},$$

$$E_3 = \{\langle \langle i, k \rangle, \langle \sigma, i \rangle \rangle \mid \langle i, k \rangle \in V_2, \langle \sigma, i \rangle \in V_1 \},$$

$$E_4 = \bigcup_{i=1}^{p} \{\langle i,1 \rangle, \langle i,2 \rangle, \langle i,2 \rangle, \langle i,3 \rangle, \langle i,3 \rangle, \langle i,1 \rangle\}.$$ 

**REMARK.** The problem of characterizing digraphs that have a kernel, resp. the problem of finding a kernel in a given digraph attracted attention of several authors [1], [2], [4].

**REFERENCES**

