Problems for CSE 2010 Summer Camp

Monday 7 June

The sole survivor from Sunday:

Let $s_{\max}(n)$ denote the largest number of solutions of the stable marriage problem involving n women and n men. What are the best upper and lower bounds on $s_{\max}(n)$ that you can find?

Two new problems:

1. What is the largest number of integers you can take from $1, 2, \ldots, 2n$ without taking an integer and its multiple?

2. Call a positive integer *simple* if it has no prime divisors other than 2 or 3 (or both 2 and 3): the twenty-one simple integers smaller than 100 are

$$1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 27, 32, 36, 48, 54, 64, 72, 81, 96.$$

It looks as if *every positive integer is the sum of distinct simple integers, none of which divides another:*

$$1 = 1,\ \ 2 = 2,\ \ 3 = 3,\ \ 4 = 4,\ \ 5 = 2 + 3,\ \ 6 = 6,\ \ 7 = 3 + 4,\ \ 8 = 8,\ \ 9 = 9,$$
$$10 = 4 + 6,\ \ 11 = 2 + 9 = 3 + 8,\ \ 12 = 12,\ \ 13 = 4 + 9,\ \ 14 = 6 + 8,\ \ 15 = 6 + 9,$$

and so on. Is this true or false?

(Nan Yang's variation: Is it true that every positive integer, unless itself simple, is the sum of two distinct simple integers, neither of which divides the other?)

Both of these come from Paul Erdős.