# The traveling salesman problem

Vašek Chvátal
Department of Computer Science and Software Engineering
Concordia University, Montreal

# The traveling salesman problem:

Given finitely many “cities” along with the cost of travel between any two of them,

find the cheapest way of going through all the cities and coming back to the city you started out from.

# The traveling salesman problem:

Given finitely many “cities” along with the cost of travel between any two of them,

find the cheapest way of going through all the cities and coming back to the city you started out from.

The *symmetric* TSP:
Travel from A to B costs as much as travel from B to A.

# What are the origins of the problem?

# What are the origins of the problem?

Der

**Handlungsreisende**

**wie er sein soll**

und was er zu thun hat, um Aufträge zu erhalten und eines glücklichen Erfolgs in seinen Geschäften gewiß zu sein.

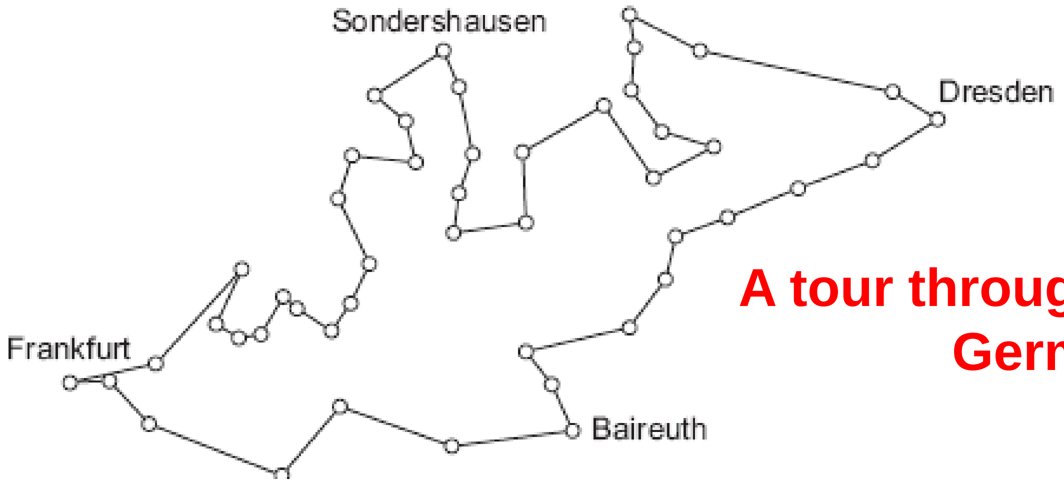
Von

einem alten Commis - Voyageur.

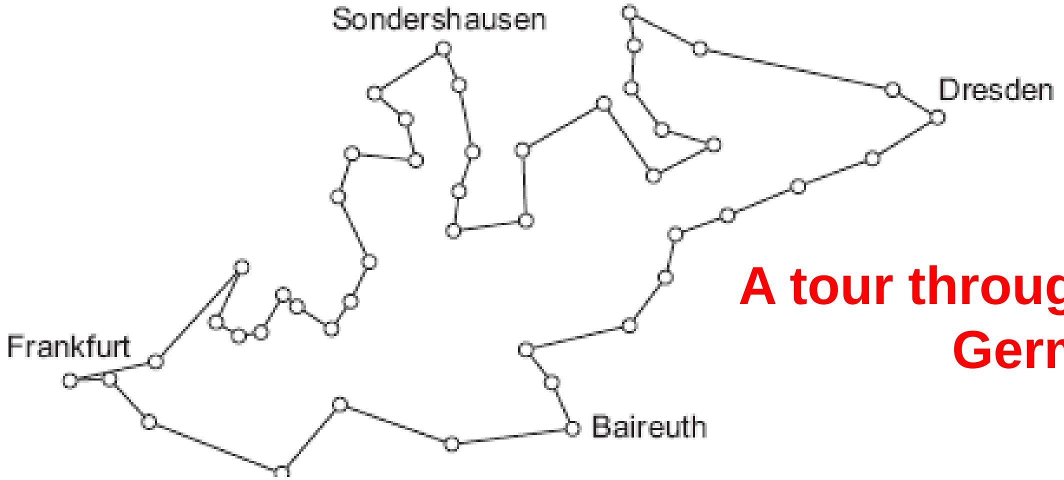
Mit einem Titelkupfer.

Ilmenau 1832,

Druck und Verlag von B. Fr. Voigt.

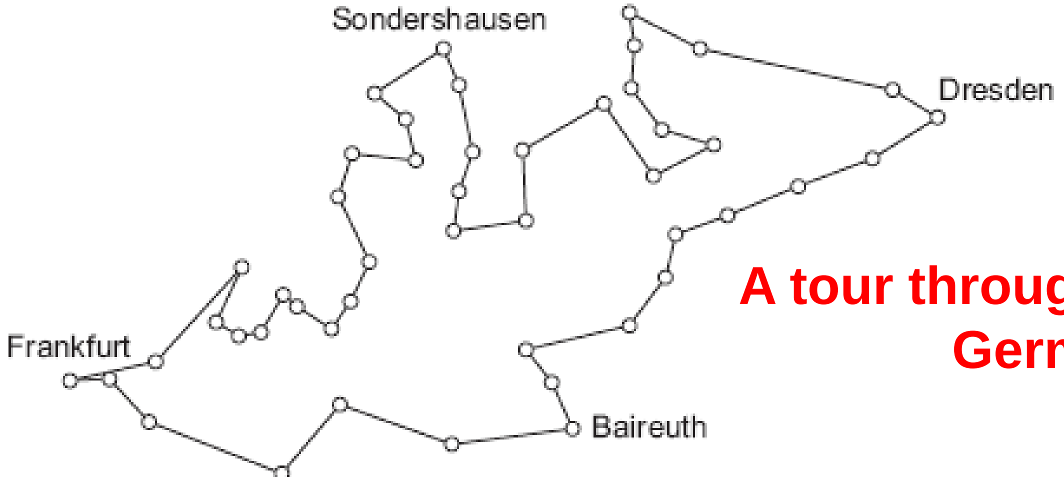



A tour through 47 cities in Germany




**A tour through 47 cities in Germany**

(…) Business leads the traveling salesman here and there, and there is not a good tour for all occurring cases; but through an expedient choice and division of the tour so much time can be won that we feel compelled to give guidelines about this. Everyone should use as much of the advice as he thinks useful for his application. We believe we can ensure as much that it will not be possible to plan the tours through Germany in consideration of the distances and the traveling back and forth, which deserves the traveler's special attention, with more economy.



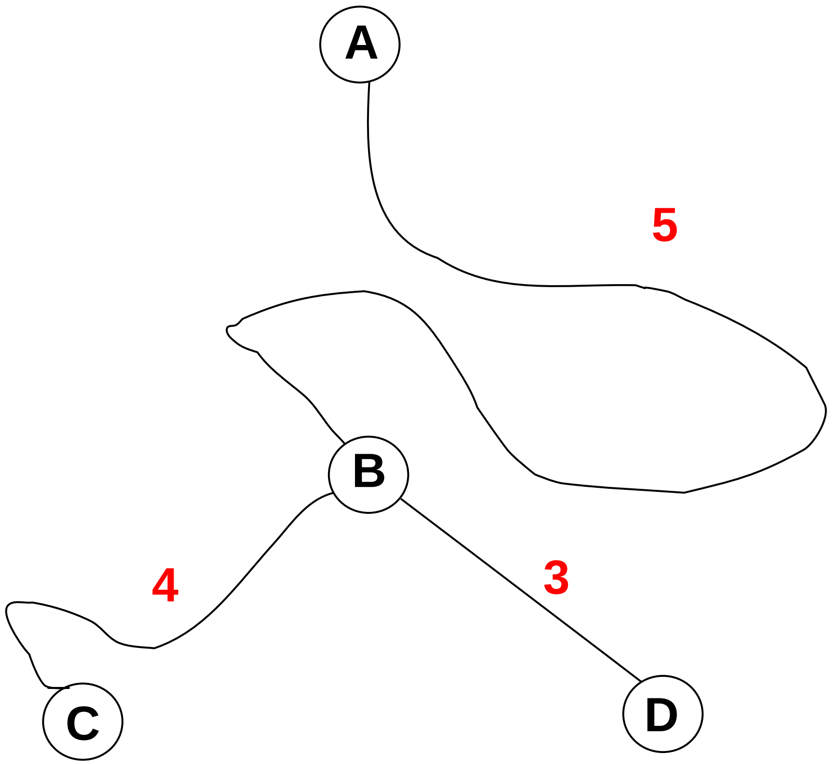

**A tour through 47 cities in Germany**

(…) Business leads the traveling salesman here and there, and there is not a good tour for all occurring cases; but through an expedient choice and division of the tour so much time can be won that we feel compelled to give guidelines about this. Everyone should use as much of the advice as he thinks useful for his application. We believe we can ensure as much that it will not be possible to plan the tours through Germany in consideration of the distances and the traveling back and forth, which deserves the traveler's special attention, with more economy.

The main thing to remember is always to visit as many localities as possible without having to touch them twice. (…)

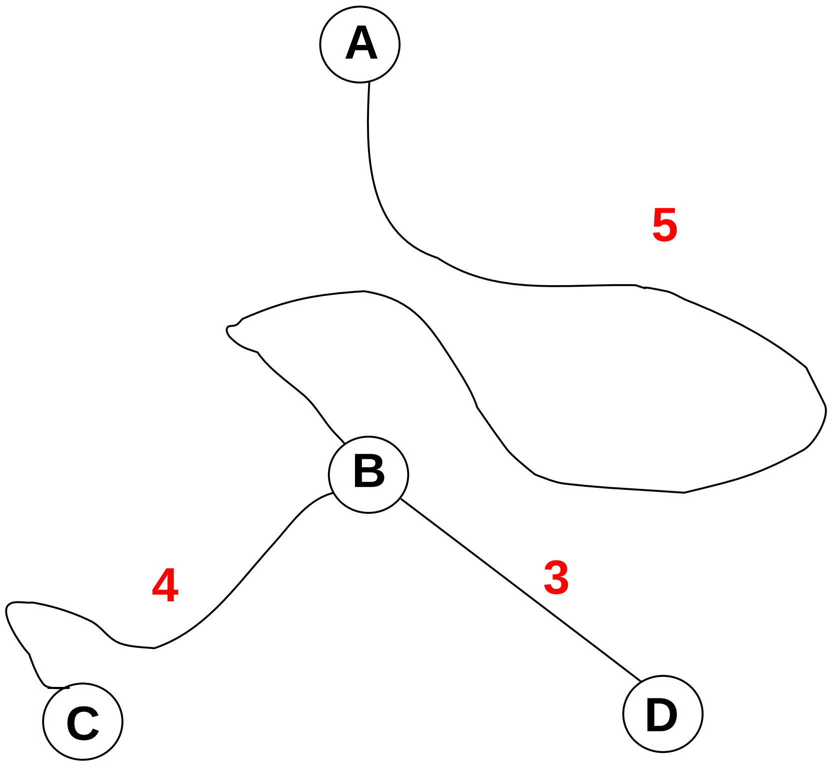
The main thing to remember is always to visit as many localities as possible without having to touch them twice. (…)

The main thing to remember is always to visit as many localities as possible without having to touch them twice. (…)

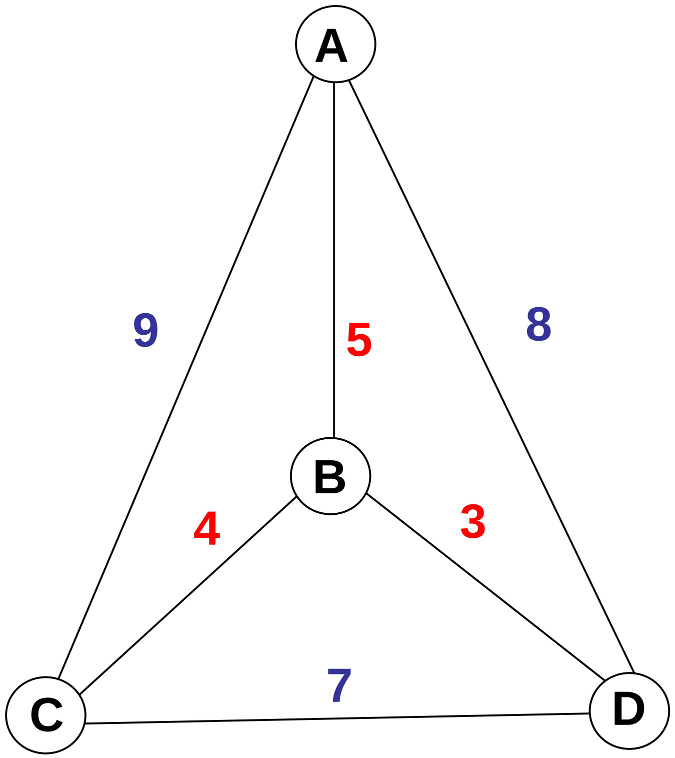



Road map

The main thing to remember is always to visit as many localities as possible without having to touch them twice. (…)

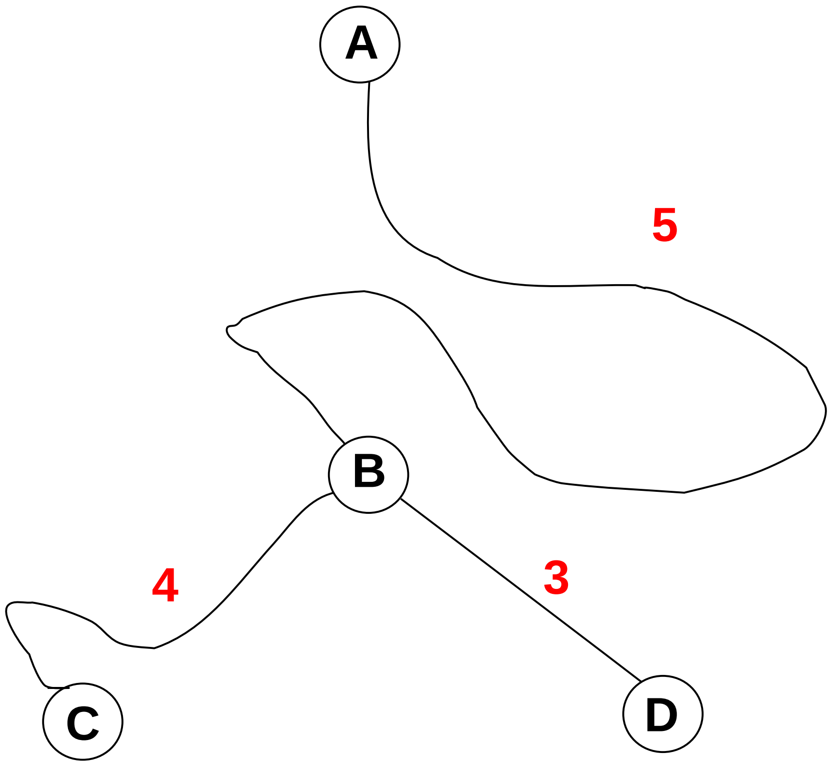



Road map

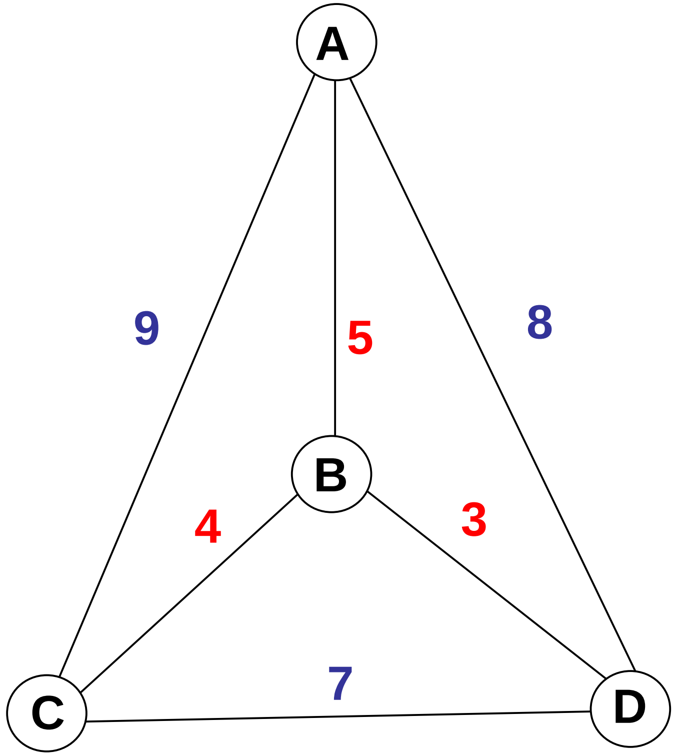



Travel costs

The main thing to remember is always to visit as many localities as possible without having to touch them twice. (…)



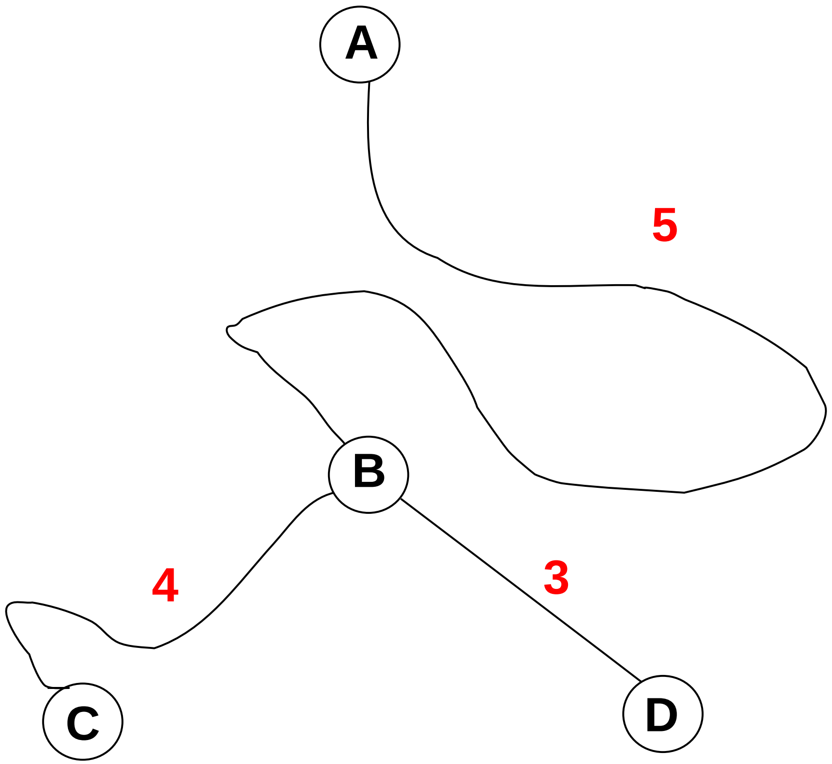

**Road map**




**Travel costs**

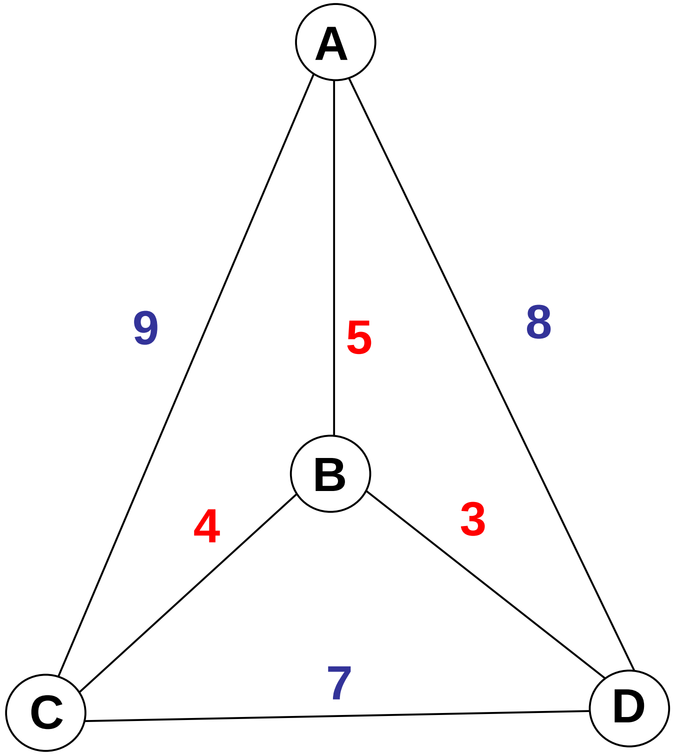
A-B-C-D-A

The main thing to remember is always to visit as many localities as possible without having to touch them twice. (…)




**Road map**

A-B-C-B-D-B-A

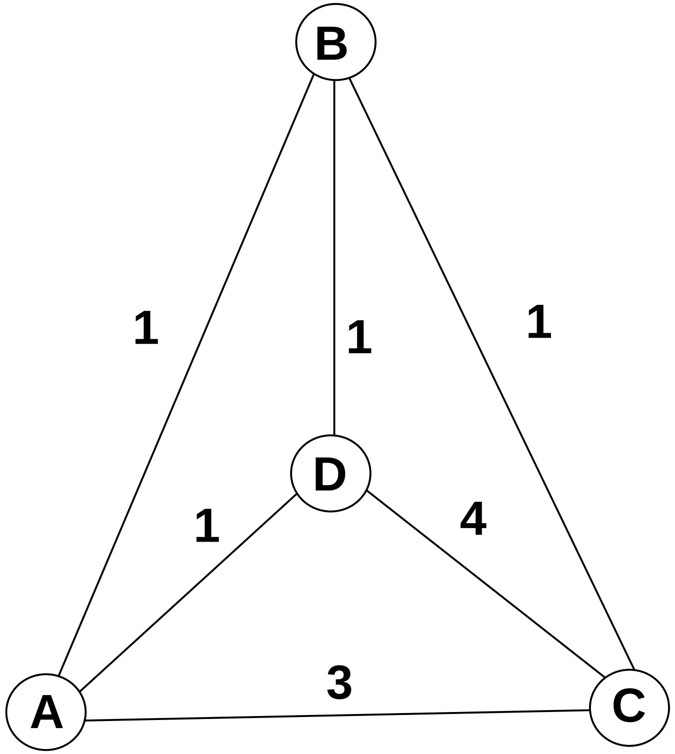



**Travel costs**

A-B-C-D-A

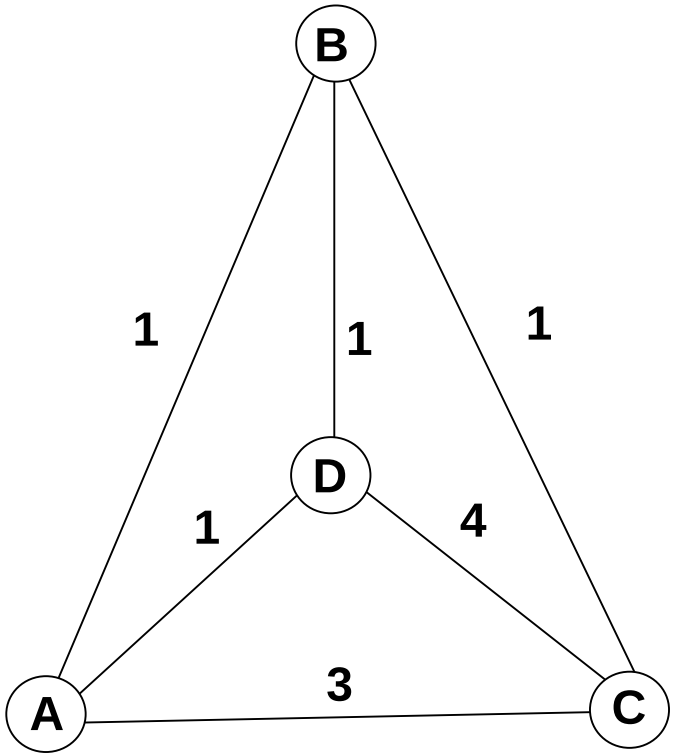
# MISCONCEPTION #23:
# THE TSP IS A PROBLEM IN GEOMETRY

# MISCONCEPTION #23:
# THE TSP IS A PROBLEM IN GEOMETRY



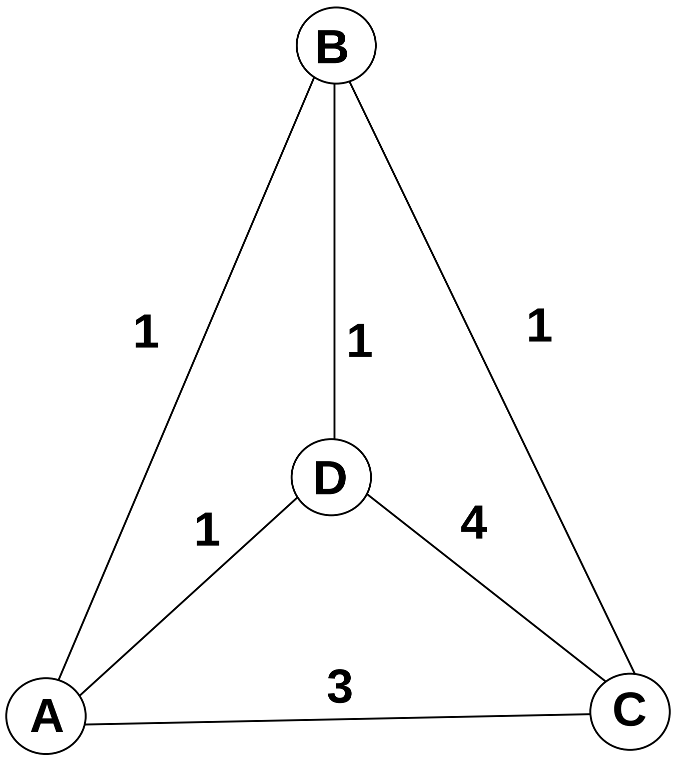

Travel costs

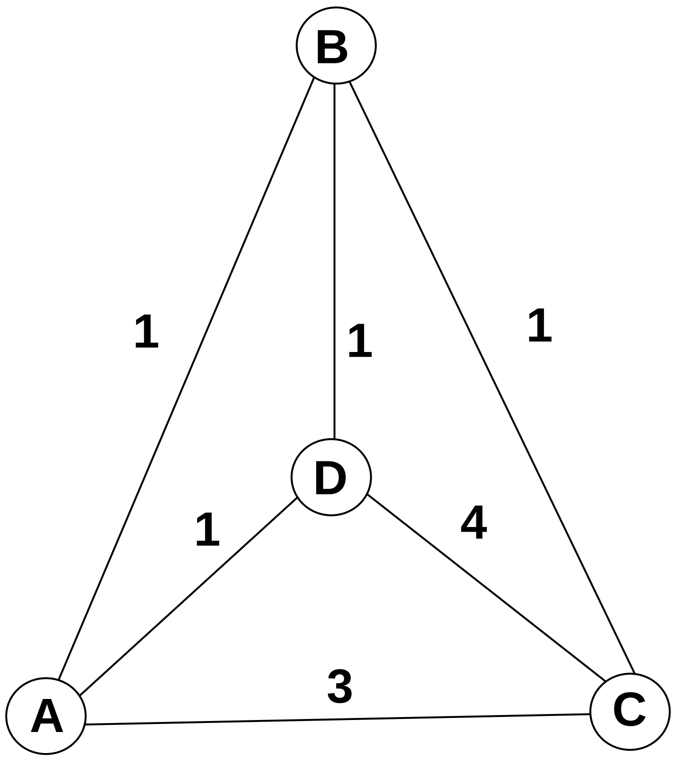
# MISCONCEPTION #23:
# THE TSP IS A PROBLEM IN GEOMETRY

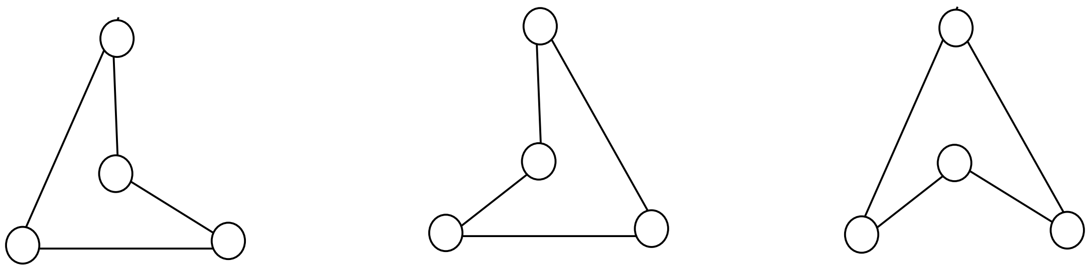


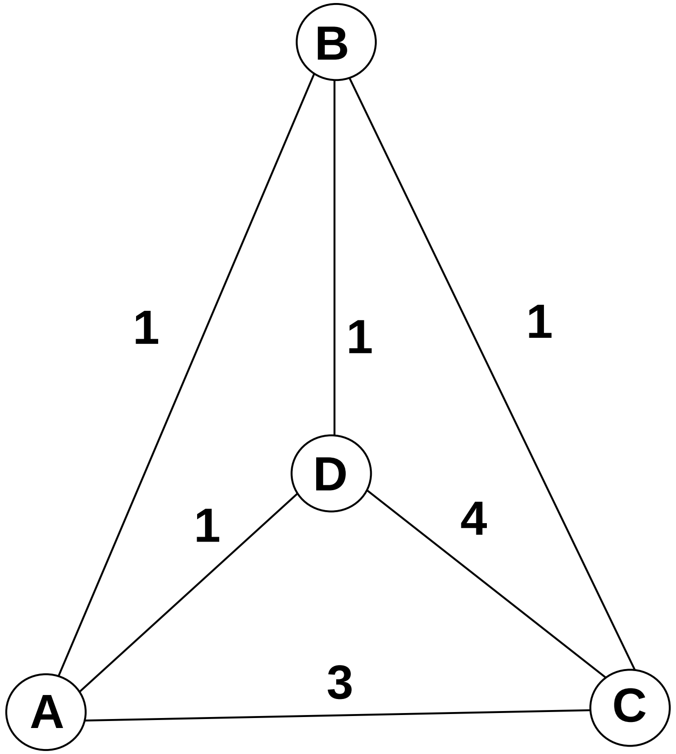

Travel costs

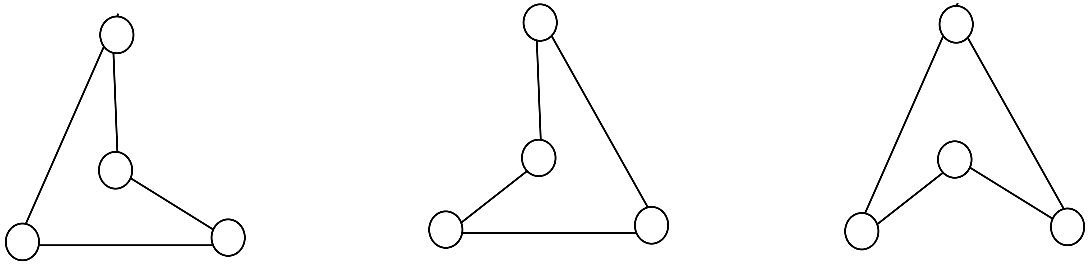
**cost(A,C) exceeds cost(A,B)+cost(B,C)!**


B
1
1
1
D
1
4
3
A
C

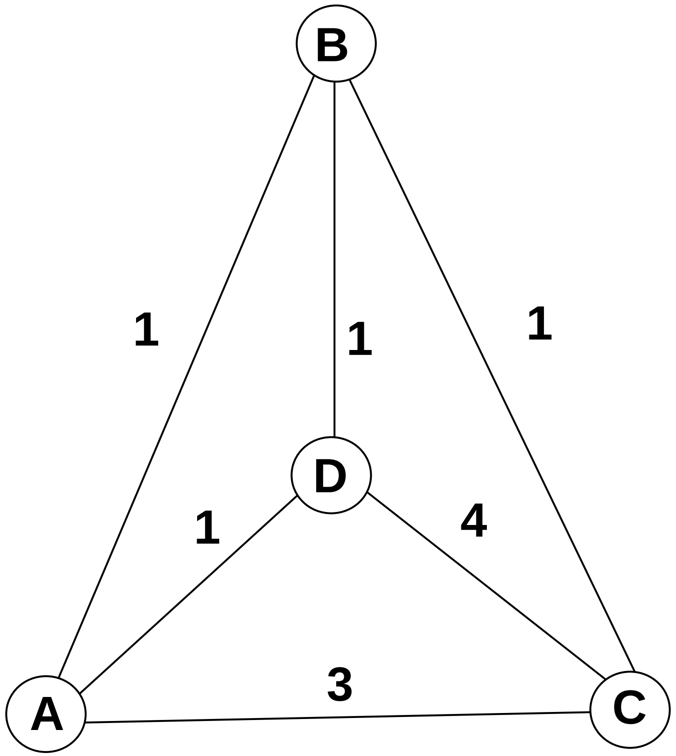

B
1
1
1
D
1
4
3
A
C

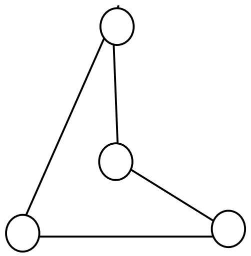




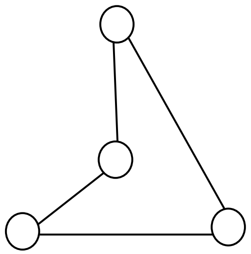



**tour cost 9**

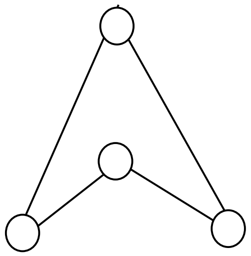


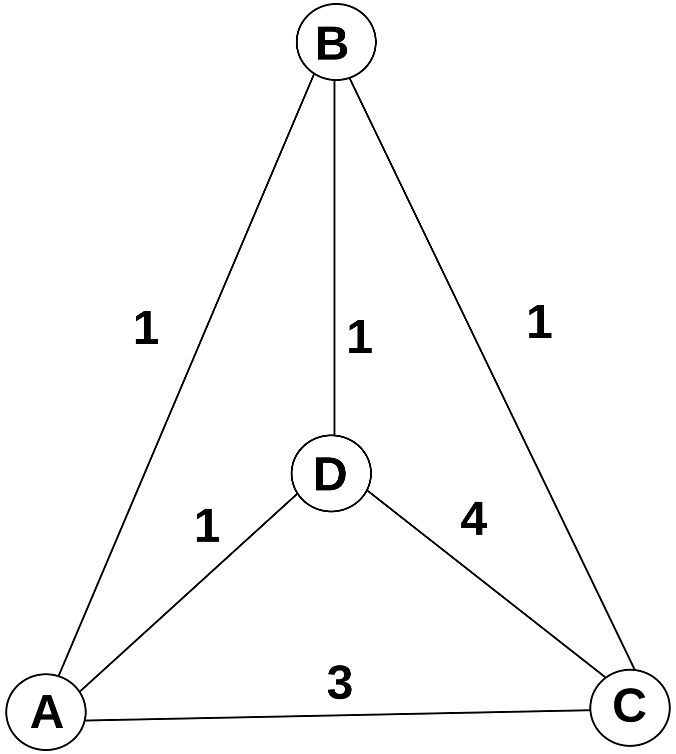



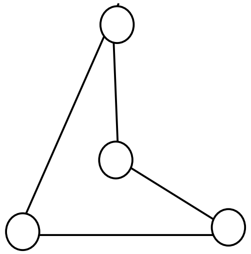
tour cost 9



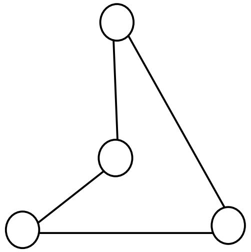
tour cost 6



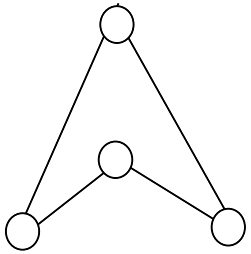


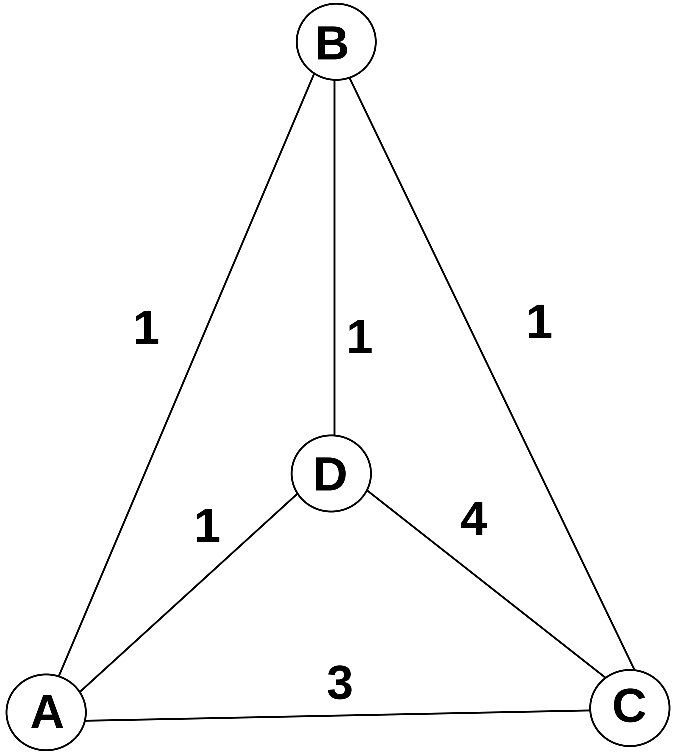
tour cost 9

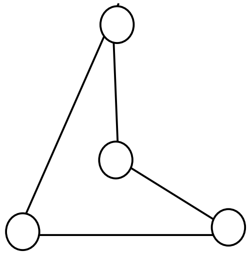


tour cost 6

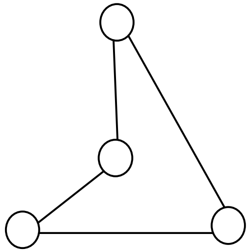


tour cost 7



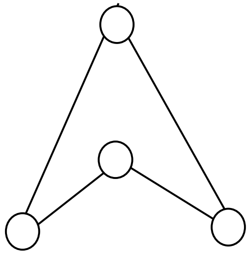



tour cost 9



tour cost 6

OPTIMAL



tour cost 7

# Great terminological confusion

# Great terminological confusion

Mulder, S. A. and Wunsch, D. C. Million city traveling salesman problem solution by divide and conquer clustering with adaptive resonance neural networks. *Neural Networks* **16** (2003), 827-832.

Great terminological confusion

Mulder, S. A. and Wunsch, D. C.  Million city traveling salesman problem solution by divide and conquer clustering with adaptive resonance neural networks. *Neural Networks* **16** (2003), 827-832.

Is x=3 a solution of the equation 2x = 5 ?

Great terminological confusion

Mulder, S. A. and Wunsch, D. C.  Million city traveling salesman problem solution by divide and conquer clustering with adaptive resonance neural networks. *Neural Networks* **16** (2003), 827-832.

Is $x=3$ a solution of the equation $2x = 5$ ?

*Soft computing heuristics*

Great terminological confusion

Mulder, S. A. and Wunsch, D. C.  Million city traveling salesman problem solution by divide and conquer clustering with adaptive resonance neural networks. *Neural Networks* **16** (2003), 827-832.

Is x=3 a solution of the equation 2x = 5 ?

*Soft computing heuristics*

- artificial neural networks

## Great terminological confusion

Mulder, S. A. and Wunsch, D. C. Million city traveling salesman problem solution by divide and conquer clustering with adaptive resonance neural networks. *Neural Networks* **16** (2003), 827-832.

## Is x=3 a solution of the equation 2x = 5 ?

### *Soft computing heuristics*

- artificial neural networks
- genetic algorithms

## Great terminological confusion

Mulder, S. A. and Wunsch, D. C. Million city traveling salesman problem solution by divide and conquer clustering with adaptive resonance neural networks. *Neural Networks* **16** (2003), 827-832.

Is x=3 a solution of the equation 2x = 5 ?

*Soft computing heuristics*

- artificial neural networks
- genetic algorithms
- ant colony optimization

## Great terminological confusion

Mulder, S. A. and Wunsch, D. C. Million city traveling salesman problem solution by divide and conquer clustering with adaptive resonance neural networks. *Neural Networks* **16** (2003), 827-832.

## Is x=3 a solution of the equation 2x = 5 ?

### *Soft computing heuristics*

- artificial neural networks
- genetic algorithms
- ant colony optimization
- particle swarm optimization

Great terminological confusion

Mulder, S. A. and Wunsch, D. C.  Million city traveling salesman problem solution by divide and conquer clustering with adaptive resonance neural networks. *Neural Networks* **16** (2003), 827-832.

Is x=3 a solution of the equation 2x = 5 ?

*Soft computing heuristics*

- artificial neural networks
- genetic algorithms
- ant colony optimization
- particle swarm optimization
- simulated annealing

## Great terminological confusion

Mulder, S. A. and Wunsch, D. C. Million city traveling salesman problem solution by divide and conquer clustering with adaptive resonance neural networks. *Neural Networks* **16** (2003), 827-832.

## Is x=3 a solution of the equation 2x = 5 ?

### *Soft computing heuristics*

- artificial neural networks
- genetic algorithms
- ant colony optimization
- particle swarm optimization
- simulated annealing
- tabu search

## Great terminological confusion

Mulder, S. A. and Wunsch, D. C. Million city traveling salesman problem solution by divide and conquer clustering with adaptive resonance neural networks. *Neural Networks* **16** (2003), 827-832.

## Is x=3 a solution of the equation 2x = 5 ?

### *Soft computing heuristics*

- artificial neural networks
- genetic algorithms
- ant colony optimization
- particle swarm optimization
- simulated annealing
- tabu search
- memetic algorithms

## Great terminological confusion

Mulder, S. A. and Wunsch, D. C. Million city traveling salesman problem solution by divide and conquer clustering with adaptive resonance neural networks. *Neural Networks* **16** (2003), 827-832.

## Is x=3 a solution of the equation 2x = 5 ?

### *Soft computing heuristics*

- artificial neural networks
- genetic algorithms
- ant colony optimization
- particle swarm optimization
- simulated annealing
- tabu search
- memetic algorithms
- etc. …

# Origins of the problem

# Origins of the problem

Karl Menger
Vienna 1930

# Origins of the problem

Karl Menger
Vienna 1930

Hassler Whitney
Princeton 1934

# Origins of the problem

Harvard 1931

Karl Menger
Vienna 1930

Hassler Whitney
Princeton 1934

# Origins of the problem

Harvard 1931

Karl Menger
Vienna 1930

Hassler Whitney
Princeton 1934

Merrill Flood
RAND Corporation 1948

# Origins of the problem

Karl Menger
Vienna 1930

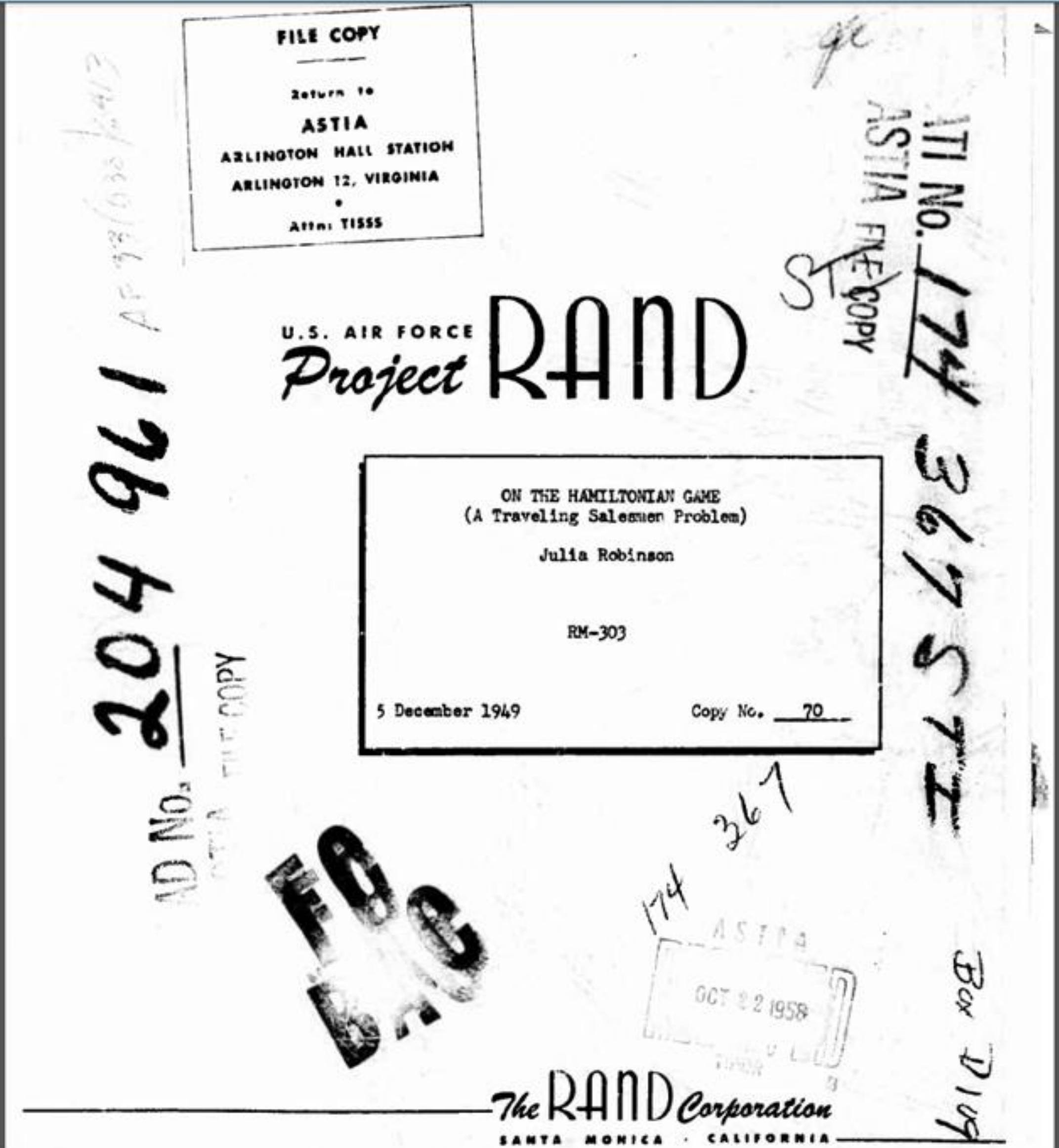
Harvard 1931

Hassler Whitney
Princeton 1934

Princeton

Merrill Flood
RAND Corporation 1948

# Julia Robinson



FILE COPY
Return to
ASTIA
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA
Attn: TISSS

U.S. AIR FORCE
Project RAND

ON THE HAMILTONIAN GAME
(A Traveling Salesmen Problem)

Julia Robinson

RM-303

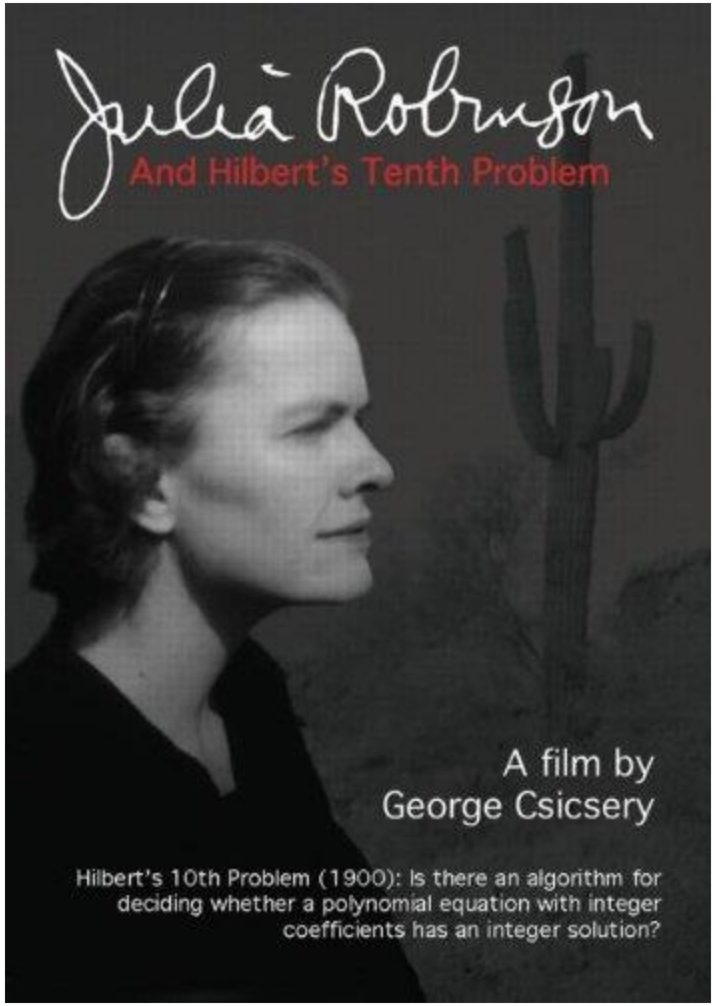
5 December 1949 Copy No. 70

ATI No. 174 367

AD No. 204 961

ASTIA FILE COPY

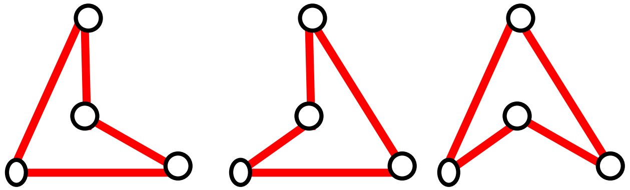
The RAND Corporation
SANTA MONICA · CALIFORNIA




http://www.zalafilms.com/

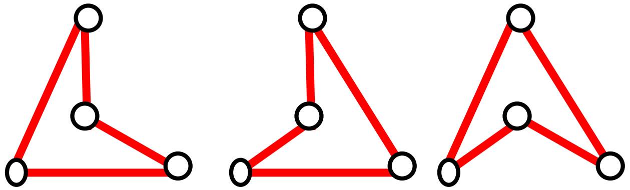
# MISCONCEPTION #7:
# WHAT MAKES THE TSP HARD IS THE LARGE NUMBER OF TOURS

MISCONCEPTION #7:
WHAT MAKES THE TSP HARD IS THE LARGE NUMBER OF TOURS

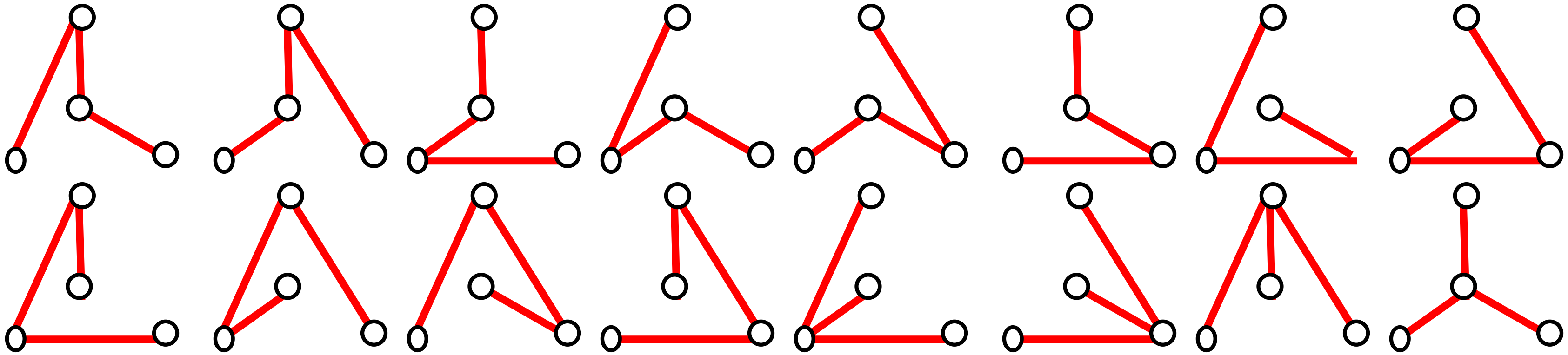


**3 tours through 4 cities**

MISCONCEPTION #7:
WHAT MAKES THE TSP HARD IS THE
LARGE NUMBER OF TOURS

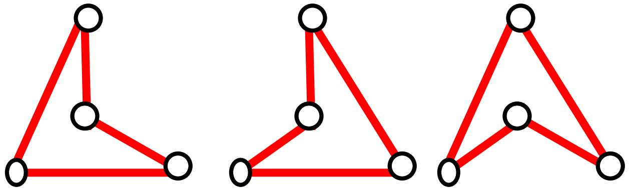


**3 tours through 4 cities**



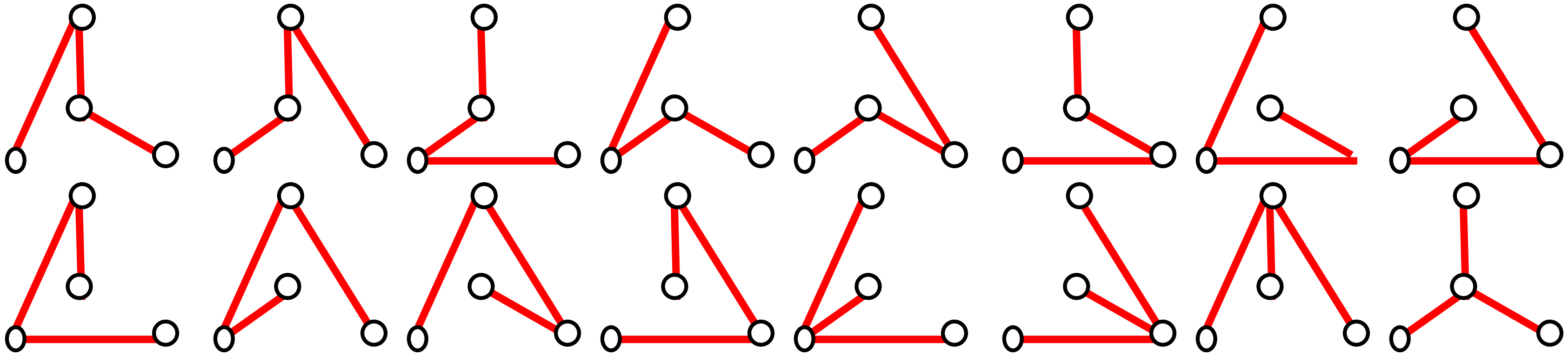
**16 *spanning trees* on 4 cities**

# MISCONCEPTION #7:
# WHAT MAKES THE TSP HARD IS THE LARGE NUMBER OF TOURS



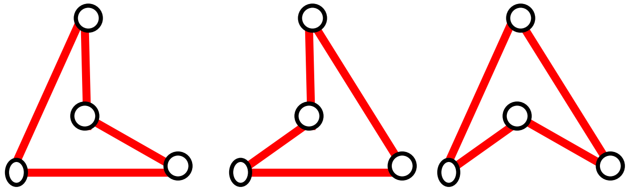
**3 tours through 4 cities**

$$\frac{1}{2}\,(n-1)!$$



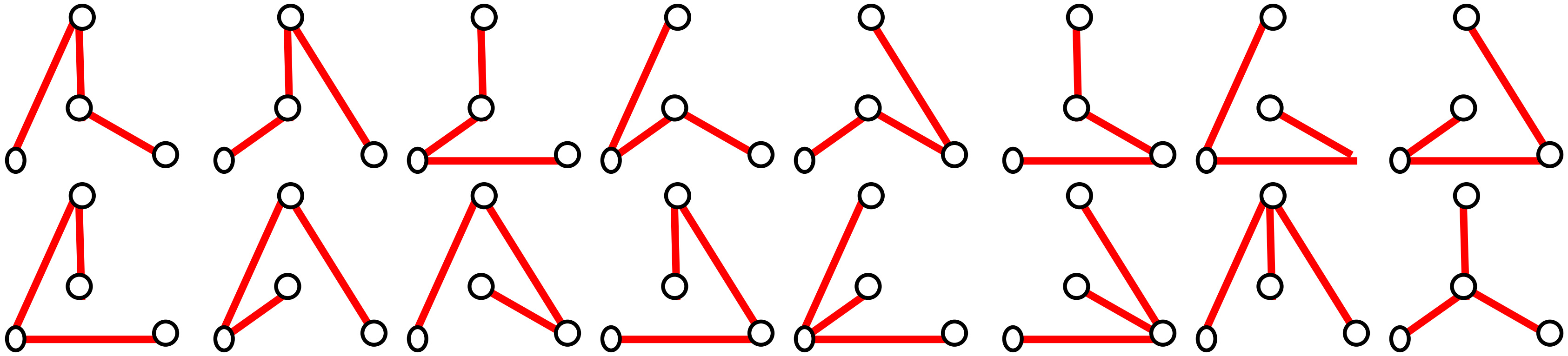
**16 *spanning trees* on 4 cities**

# MISCONCEPTION #7:
# WHAT MAKES THE TSP HARD IS THE LARGE NUMBER OF TOURS



**3 tours through 4 cities**

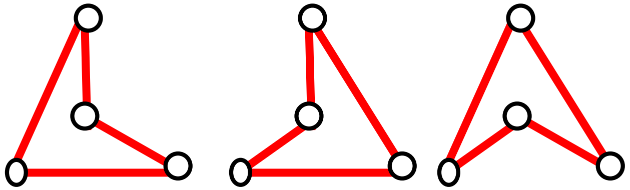
$$\frac{1}{2}(n-1)!$$



**16 *spanning trees* on 4 cities**

$$n^{n-2}$$

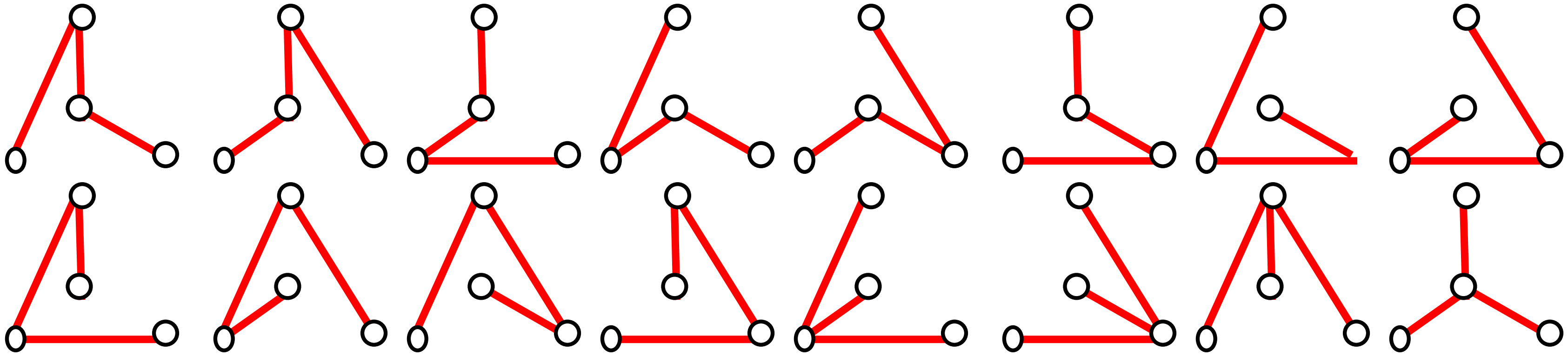
# MISCONCEPTION #7:
# WHAT MAKES THE TSP HARD IS THE LARGE NUMBER OF TOURS



**3 tours through 4 cities**

$$\frac{1}{2}(n-1)!$$

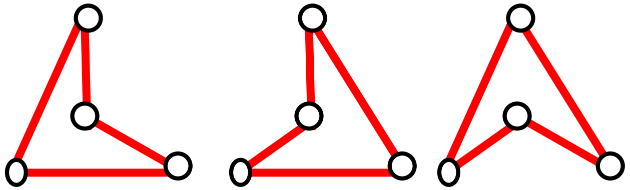
$$= n^{n-2}\, e^{-n} O(n^{3/2})$$



**16 *spanning trees* on 4 cities**

$$n^{n-2}$$

MISCONCEPTION #7:
WHAT MAKES THE TSP HARD IS THE LARGE NUMBER OF TOURS

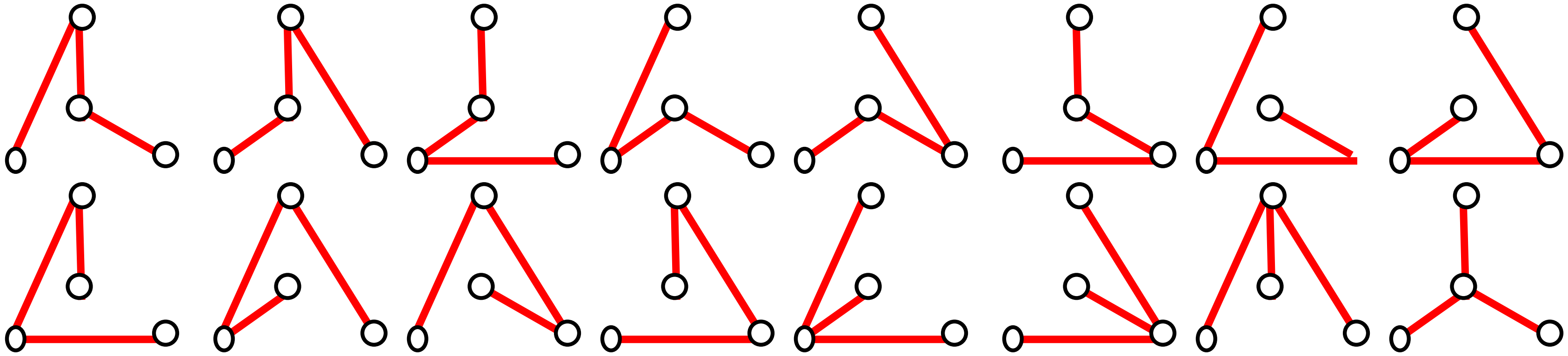


**3 tours through 4 cities**

$$\frac{1}{2}(n-1)!$$

$$= n^{n-2}\, e^{-n} O(n^{3/2})$$

THE MINIMUM-COST SPANNING TREE PROBLEM IS EASY!



$$n^{n-2}$$

**16 *spanning trees* on 4 cities**

# WHAT MAKES THE TSP SO HARD?

# WHAT MAKES THE TSP SO HARD?

**This question stimulated the study of *computational complexity* and, in particular, the development of the *theory of NP-completeness***

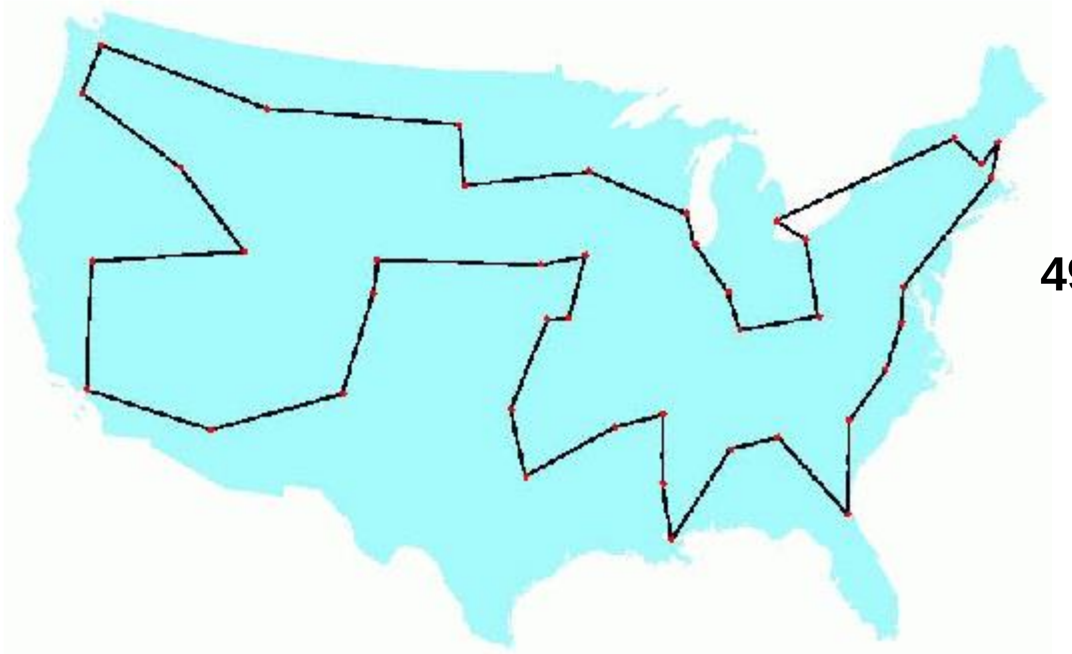
# WHAT MAKES THE TSP SO HARD?

**This question stimulated the study of *computational complexity* and, in particular, the development of the *theory of NP-completeness***

Stephen A. Cook, *The complexity of theorem-proving procedures*. In Proc. 3rd Annual ACM Symposium on the Theory of Computing, 1971, pp.151—158.

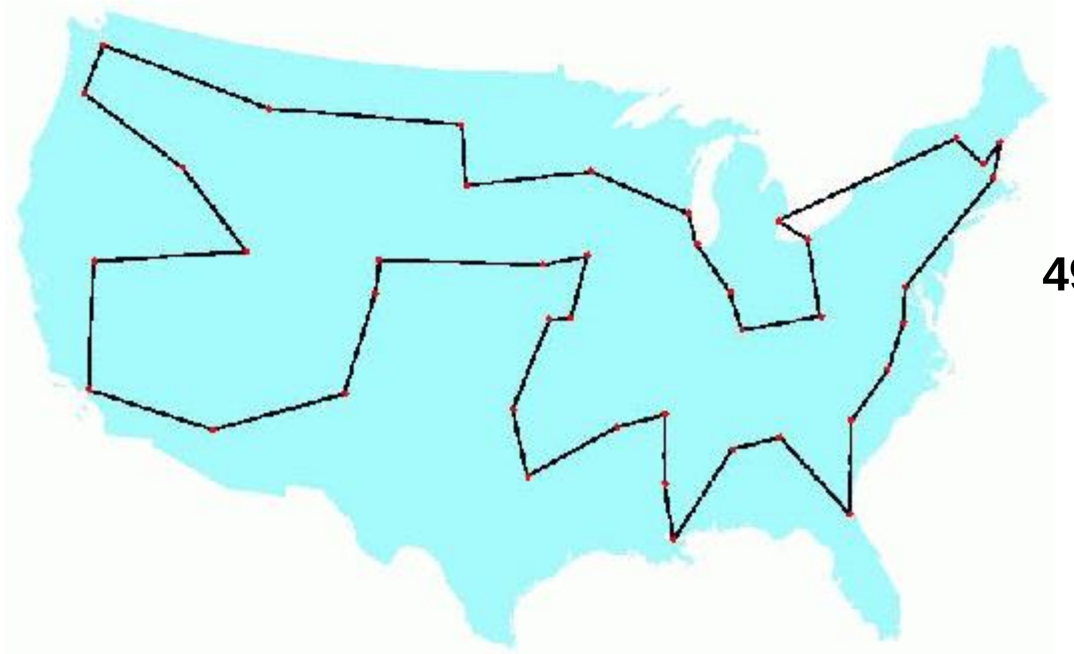
Richard M. Karp. *Reducibility among combinatorial problems*. In R.E. Miller and J.W.Thatcher, editors, Complexity of Computer Computations, Plenum Press, 1972, pp. 85--104.

G. Dantzig, R. Fulkerson, and S. Johnson,
"Solution of a large-scale traveling-salesman problem",
*Operations Research* **2** (1954), 393-410.



**49 cities in the U.S.A.**

G. Dantzig, R. Fulkerson, and S. Johnson,
"Solution of a large-scale traveling-salesman problem",
*Operations Research* **2** (1954), 393-410.



**49 cities in the U.S.A.**

# The Big Bang

# The Dantzig-Fulkerson-Johnson cutting-plane method

# The Dantzig-Fulkerson-Johnson cutting-plane method

applies to any problem minimize $c^T x$ subject to $x \in \mathcal{S}$

such that ***S*** is a finite subset of a Euclidean space

# The Dantzig-Fulkerson-Johnson cutting-plane method

applies to any problem minimize $c^T x$ subject to $x \in \mathcal{S}$

such that ***S*** is a finite subset of a Euclidean space

**choose** a system $Ax \leq b$ of inequalities satisfied by all points of $\mathcal{S}$
**repeat** $x^*$= an extreme point of $\{x : Ax \leq b\}$ that minimizes $c^T x$;
**if** $x^*$ belongs to $\mathcal{S}$
**then** return $x^*$;

**end**
**end**

# The Dantzig-Fulkerson-Johnson cutting-plane method

applies to any problem minimize $c^T x$ subject to $x \in \mathcal{S}$

such that ***S*** is a finite subset of a Euclidean space

**choose** a system $Ax \le b$ of inequalities satisfied by all points of $\mathcal{S}$
**repeat** $x^*$= an extreme point of $\{x : Ax \le b\}$ that minimizes $c^T x$;
    **if** $x^*$ belongs to $\mathcal{S}$
    **then** return $x^*$;
    **else** find a linear inequality satisfied by all points of $\mathcal{S}$
        and violated by $x^*$;
        add this inequality to $Ax \le b$;
    **end**
**end**

# The Dantzig-Fulkerson-Johnson cutting-plane method

applies to any problem minimize $c^T x$ subject to $x \in \mathcal{S}$

such that ***S*** is a finite subset of a Euclidean space

**choose** a system $Ax \le b$ of inequalities satisfied by all points of $\mathcal{S}$
**repeat** $x^*$= an extreme point of $\{x : Ax \le b\}$ that minimizes $c^T x$;
  **if** $x^*$ belongs to $\mathcal{S}$
  **then** return $x^*$;
  **else** find a linear inequality satisfied by all points of $\mathcal{S}$
    and violated by $x^*$;
    add this inequality to $Ax \le b$;
  **end**
**end**

cutting plane

# The Dantzig-Fulkerson-Johnson cutting-plane method

applies to any problem minimize $c^T x$ subject to $x \in \mathcal{S}$
such that ***S*** is a finite subset of a Euclidean space

**choose** a system $Ax \le b$ of inequalities satisfied by all points of $\mathcal{S}$
**repeat** $x^*$= an extreme point of $\{x : Ax \le b\}$ that minimizes $c^T x$;
  **if** $x^*$ belongs to $\mathcal{S}$
  **then** return $x^*$;
  **else** find a linear inequality satisfied by all points of $\mathcal{S}$
    and violated by $x^*$;
    add this inequality to $Ax \le b$;
  **end**
**end**

cutting plane

The Dantzig-Fulkerson-Johnson initial TSP box:

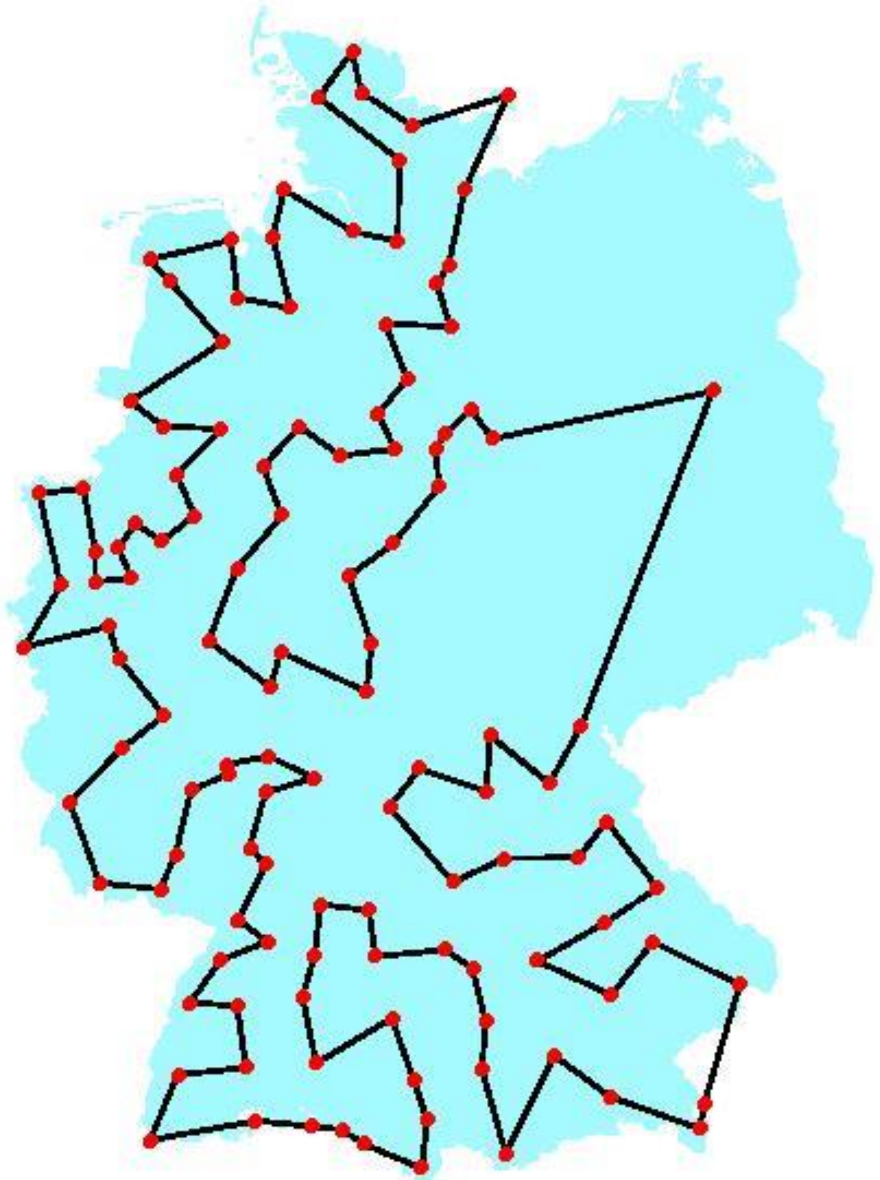
$$0 \le x_{vw} \le 1 \quad \text{for all edges } vw$$
$$\sum_w x_{vw} = 2 \quad \text{for all vertices } v$$

# 1962 Procter & Gamble $10,000 contest: a 33-city instance

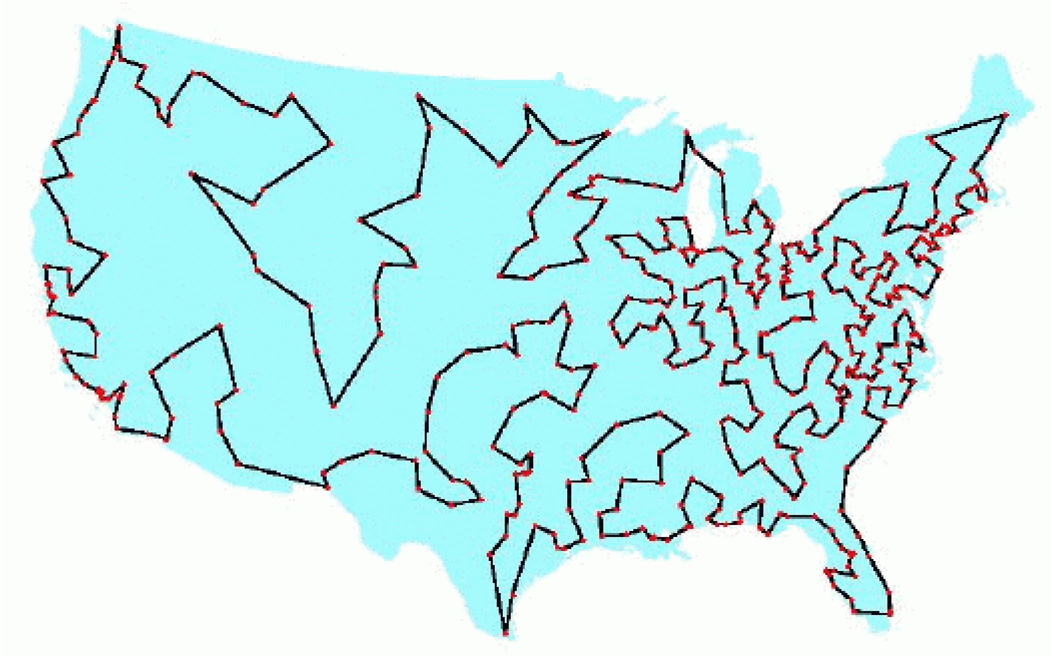
# 1962 Procter & Gamble $10,000 contest: a 33-city instance


Tiebreaker: Write a short essay on one of Procter & Gamble's products

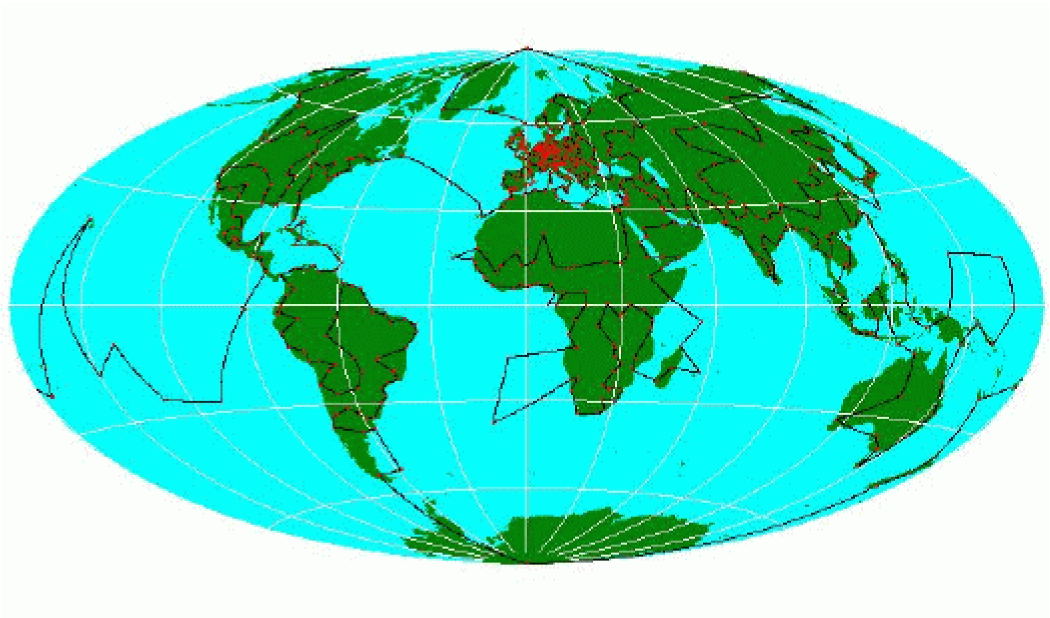
# 1977 Martin Grötschel: a 120-city instance



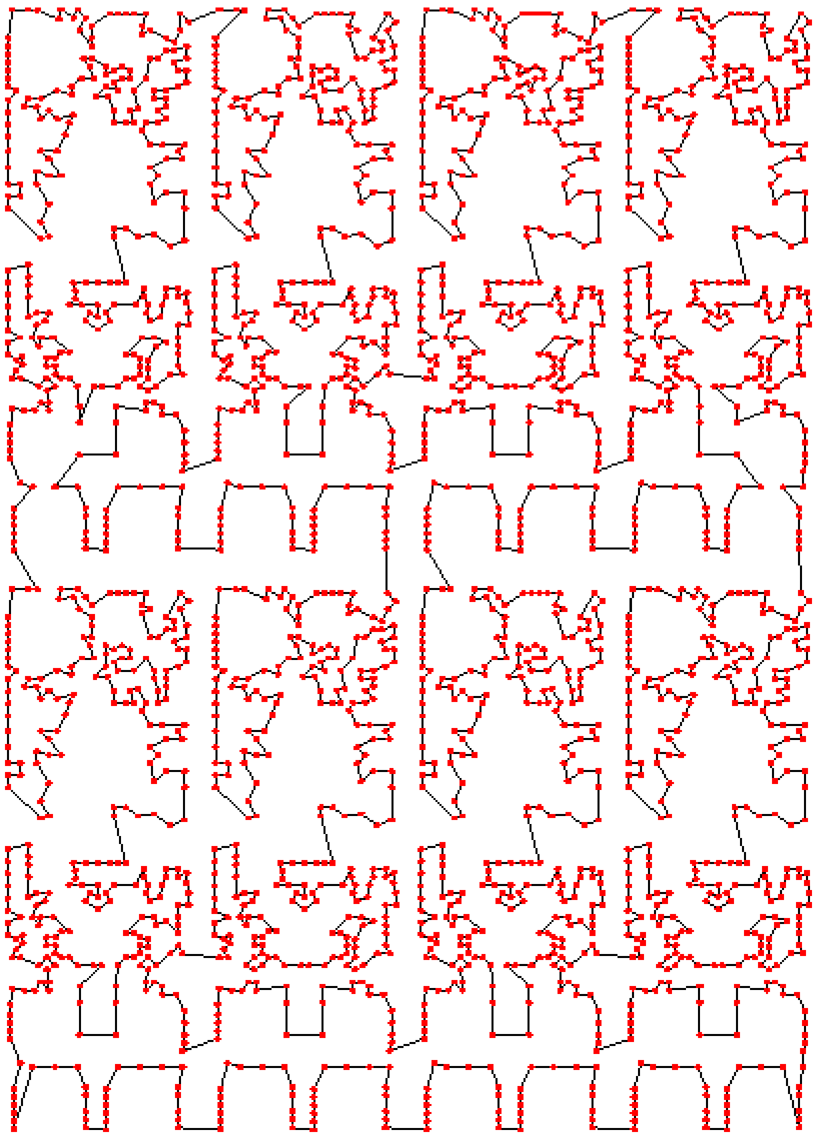
# 1986 Manfred Padberg and Giovanni Rinaldi: a 532-city instance

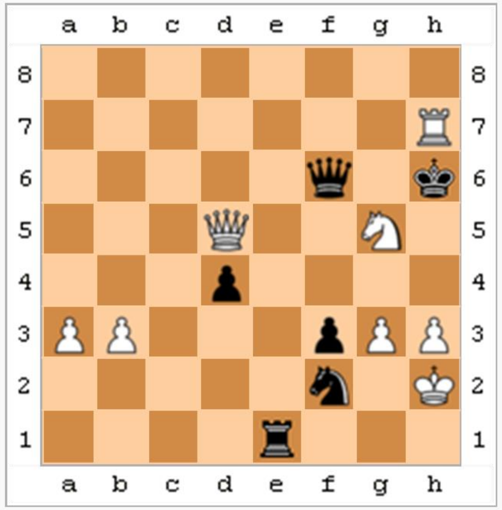


# 1987 Martin Grötschel and Olaf Holland: a 666-city instance

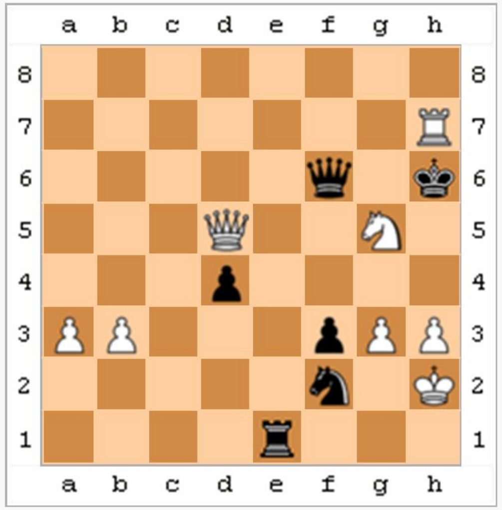


# 1987 Manfred Padberg and Giovanni Rinaldi: a 2,392-city instance



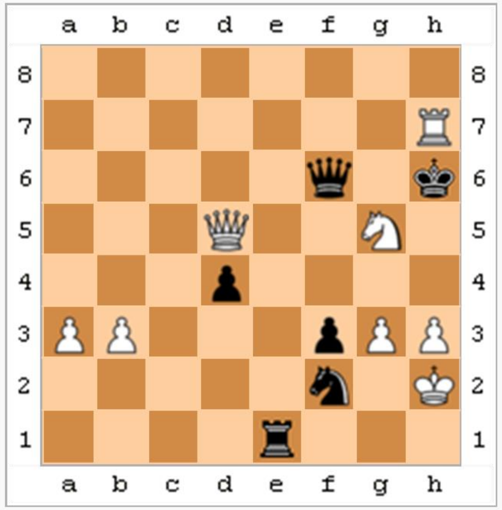



After 37. Rxh7+



After 37. Rxh7+

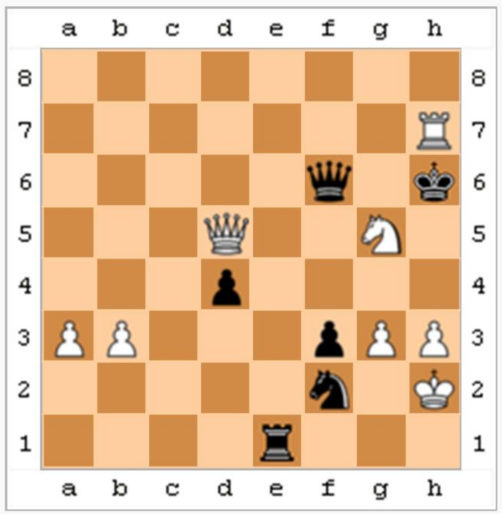
February 10, 1996



After 37. Rxh7+

February 10, 1996

IBM’s “supercomputer” *Deep Blue* beats Garry Kasparov



After 37. Rxh7+

February 10, 1996

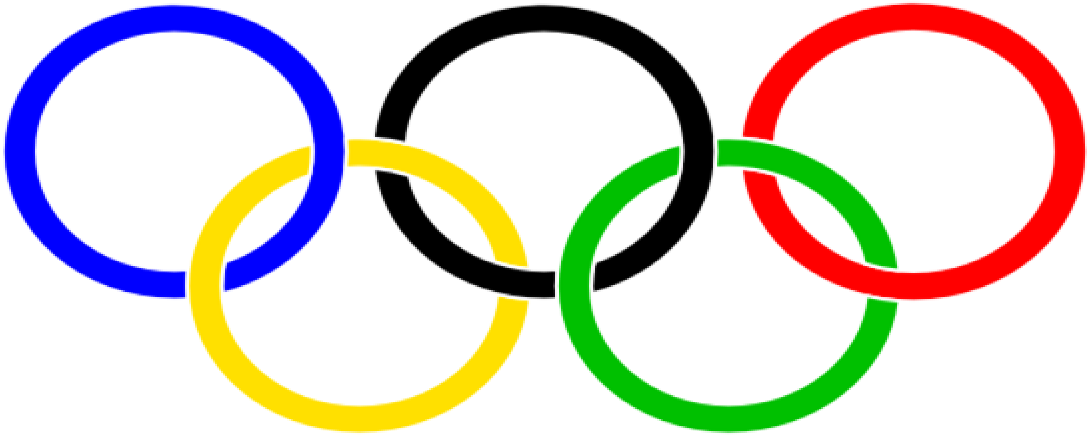
IBM’s “supercomputer” *Deep Blue* beats Garry Kasparov

From an interview with V.C. published in 1996:

*The traveling salesman problem is to mathematical programming what chess is to artificial intelligence: thoroughly useless and fiercely competitive sport that serves as a testing ground of your techniques*.

# The sport of solving the TSP

# The sport of solving the TSP



My daddy can beat up your daddy

# APPLICATION: Genome sequencing

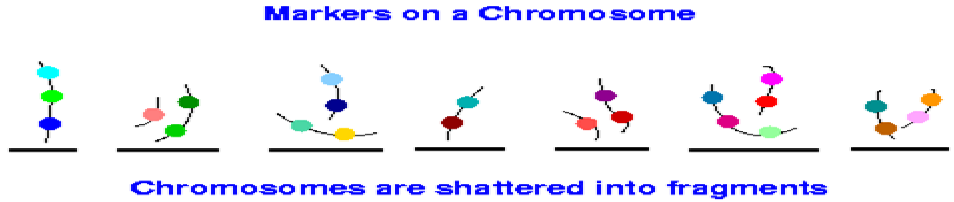
# APPLICATION: Genome sequencing


In which order are markers arranged on a genome?

# APPLICATION: Genome sequencing


In which order are markers arranged on a genome?

**Radiation hybrid technique:** Break the DNA into pieces by X-rays and grow hybrid cells from these pieces

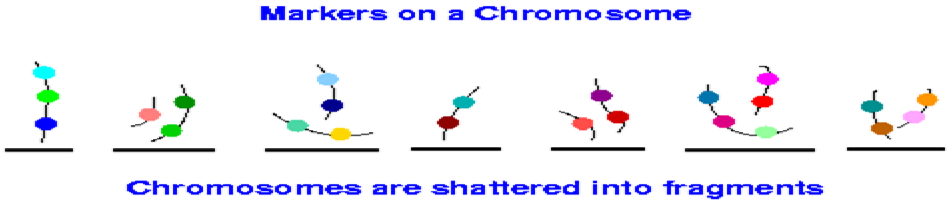



The closer two markers are to each other on the DNA, the more often they appear together in the hybrid cells.

# APPLICATION: Genome sequencing


In which order are markers arranged on a genome?

**Radiation hybrid technique:** Break the DNA into pieces by X-rays and grow hybrid cells from these pieces




The closer two markers are to each other on the DNA, the more often they appear together in the hybrid cells.

**R. Karp, W. Ruzzo, and M. Tompa (1996):**
**To find a sequence that best fits the radiation hybrid data, solve a TSP**

# APPLICATION: NASA's StarLight mission

# APPLICATION: NASA's StarLight mission

*The two StarLight mission spacecraft will orbit the sun, sort of tagging along behind Earth. Both spacecraft will carry telescope mirrors. Both telescope mirrors will be turned to look at the same star. Star light from the two mirrors will be combined to create a very good image, which is then sent back to Earth. Laser beams are used to keep the two spacecraft very precisely aligned.*

# APPLICATION: NASA’s StarLight mission

*The two StarLight mission spacecraft will orbit the sun, sort of tagging along behind Earth. Both spacecraft will carry telescope mirrors. Both telescope mirrors will be turned to look at the same star. Star light from the two mirrors will be combined to create a very good image, which is then sent back to Earth. Laser beams are used to keep the two spacecraft very precisely aligned.*

**slew** *also* **slue** *transitive verb* **1 :** to turn (as a telescope or a ship's spar) about a fixed point that is usually the axis

# APPLICATION: NASA's StarLight mission

*The two StarLight mission spacecraft will orbit the sun, sort of tagging along behind Earth. Both spacecraft will carry telescope mirrors. Both telescope mirrors will be turned to look at the same star. Star light from the two mirrors will be combined to create a very good image, which is then sent back to Earth. Laser beams are used to keep the two spacecraft very precisely aligned.*

**slew** *also* **slue** *transitive verb* **1 :** to turn (as a telescope or a ship's spar) about a fixed point that is usually the axis

Bailey, C., McLain, T., and Beard, R. Fuel Saving Strategies for Dual Spacecraft Interferometry Missions, *Journal of the Astronautical Sciences,* Volume 49, Number 3, pp. 469-488, July-September 2001.

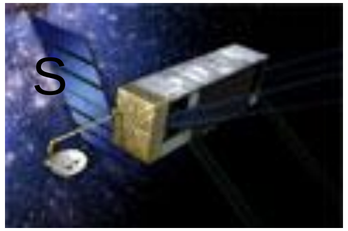
# APPLICATION: NASA's StarLight mission

*The two StarLight mission spacecraft will orbit the sun, sort of tagging along behind Earth. Both spacecraft will carry telescope mirrors. Both telescope mirrors will be turned to look at the same star. Star light from the two mirrors will be combined to create a very good image, which is then sent back to Earth. Laser beams are used to keep the two spacecraft very precisely aligned.*

**slew** *also* **slue** *transitive verb* **1 :** to turn (as a telescope or a ship's spar) about a fixed point that is usually the axis

Bailey, C., McLain, T., and Beard, R. Fuel Saving Strategies for Dual Spacecraft Interferometry Missions, *Journal of the Astronautical Sciences,* Volume 49, Number 3, pp. 469-488, July-September 2001.

StarLight cancelled in 2002



# OTHER APPLICATIONS

# OTHER APPLICATIONS

- postal deliveries
- meals on wheels
- inspection tours
- school bus routing

# OTHER APPLICATIONS

- postal deliveries
- meals on wheels
- inspection tours
- school bus routing
- X-ray crystallography

# OTHER APPLICATIONS

- postal deliveries
- meals on wheels
- inspection tours
- school bus routing
- X-ray crystallography
- drilling problems

# OTHER APPLICATIONS

- postal deliveries
- meals on wheels
- inspection tours
- school bus routing
- X-ray crystallography
- drilling problems
- post-manufacture computer chip testing (“scan chains”)

# OTHER APPLICATIONS

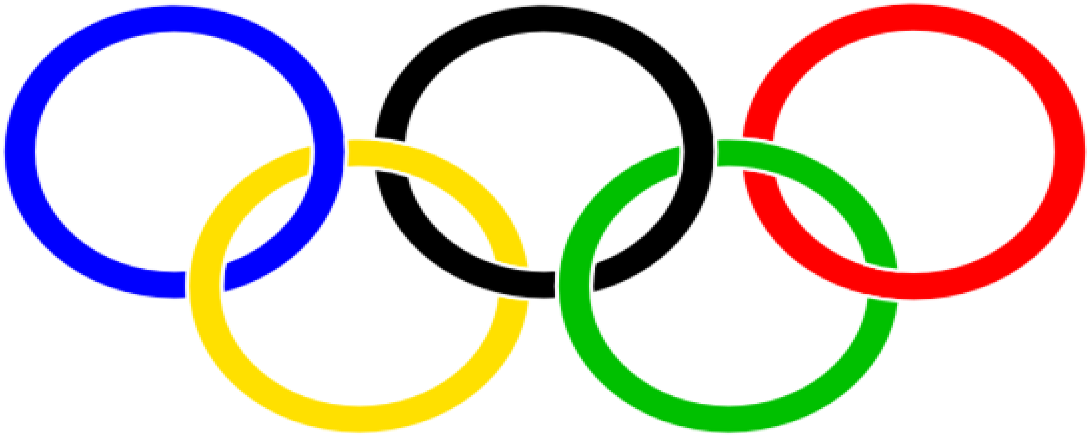
- postal deliveries
- meals on wheels
- inspection tours
- school bus routing
- X-ray crystallography
- drilling problems
- post-manufacture computer chip testing (“scan chains”)
- data clustering

# OTHER APPLICATIONS

- postal deliveries
- meals on wheels
- inspection tours
- school bus routing
- X-ray crystallography
- drilling problems
- post-manufacture computer chip testing (“scan chains”)
- data clustering
- etc. etc. etc.

# The sport of solving the TSP

# The sport of solving the TSP



**TSPLIB:**

a library of 111 instances
collected by Gerhard Reinelt (Heidelberg) in 1991

# TSPLIB

a280
ali535
att48
att532
bayg29
bays29
berlin52
bier127
brazil58
brd14051
brg180
burma14
ch130
ch150
d198
d493
d657
d1291
d1655
d2103
d15112
d18512
dantzig42
dsj1000
eil51
eil76
eil101
fl417
fl1400
fl1577
fl3795
fnl4461
fri26
gil262
gr17
gr21
gr24
gr48
gr96
gr120
gr137
gr202
gr229
gr431
gr666
hk48
kroA100
kroB100
kroC100
kroD100
kroE100
kroA150
kroB150
kroA200
kroB200
lin105
lin318
linhp318
nrw1379
p654
pa561
pcb442
pcb1173
pcb3038
pla7397
pla33810
pla85900
pr76
pr107
pr124
pr136
pr144
pr152
pr226
pr264
pr299
pr439
pr1002
pr2392
rat99
rat195
rat575
rat783
rd100
rd400
rl1304
rl1323
rl1889
rl5915
rl5934
rl11849
si175
si535
si1032
st70
swiss42
ts225
tsp225
u159
u574
u724
u1060
u1432
u1817
u2152
u2319
ulysses16
ulysses22
usa13509
vm1084
vm1748

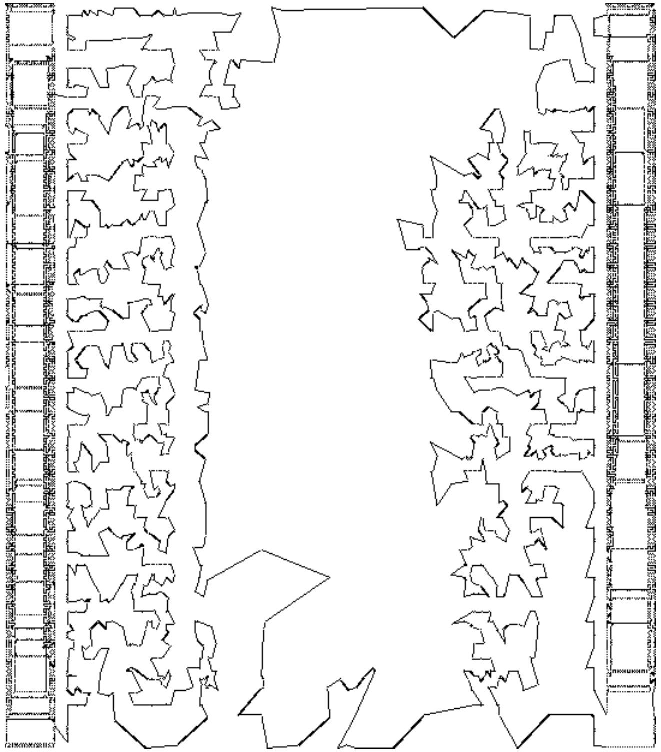
# TSPLIB

| | | | | | |
|---|---|---|---|---|---|
| a280 | d2103 | gr48 | lin318 | pr264 | si1032 |
| ali535 | d15112 | gr96 | linhp318 | pr299 | st70 |
| att48 | d18512 | gr120 | nrw1379 | pr439 | swiss42 |
| att532 | dantzig42 | gr137 | p654 | pr1002 | ts225 |
| bayg29 | dsj1000 | gr202 | pa561 | pr2392 | tsp225 |
| bays29 | eil51 | gr229 | pcb442 | rat99 | u159 |
| berlin52 | eil76 | gr431 | pcb1173 | rat195 | u574 |
| bier127 | eil101 | gr666 | pcb3038 | rat575 | u724 |
| brazil58 | fl417 | hk48 | pla7397 | rat783 | u1060 |
| brd14051 | fl1400 | kroA100 | pla33810 | rd100 | u1432 |
| brg180 | fl1577 | kroB100 | pla85900 | rd400 | u1817 |
| burma14 | fl3795 | kroC100 | pr76 | rl1304 | u2152 |
| ch130 | fnl4461 | kroD100 | pr107 | rl1323 | u2319 |
| ch150 | fri26 | kroE100 | pr124 | rl1889 | ulysses16 |
| d198 | gil262 | kroA150 | pr136 | rl5915 | ulysses22 |
| d493 | gr17 | kroB150 | pr144 | rl5934 | usa13509 |
| d657 | gr21 | kroA200 | pr152 | rl11849 | vm1084 |
| d1291 | gr24 | kroB200 | pr226 | si175 | vm1748 |
| d1655 | | lin105 | | si535 | |

The red instances were unsolved in 1991

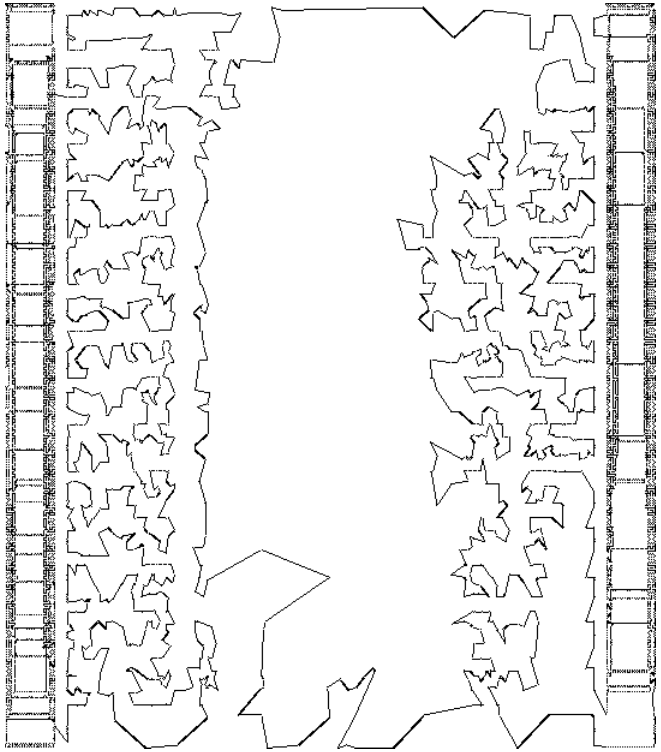
# MISCONCEPTION #8:  SIZE IS IMPORTANT

# MISCONCEPTION #8:  SIZE IS IMPORTANT

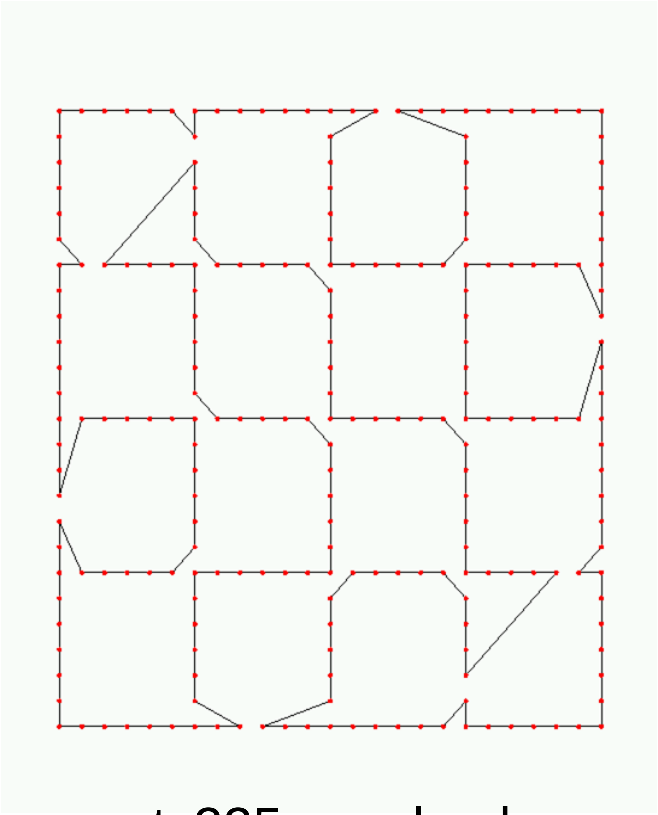


pla7397 solved
in October 1994

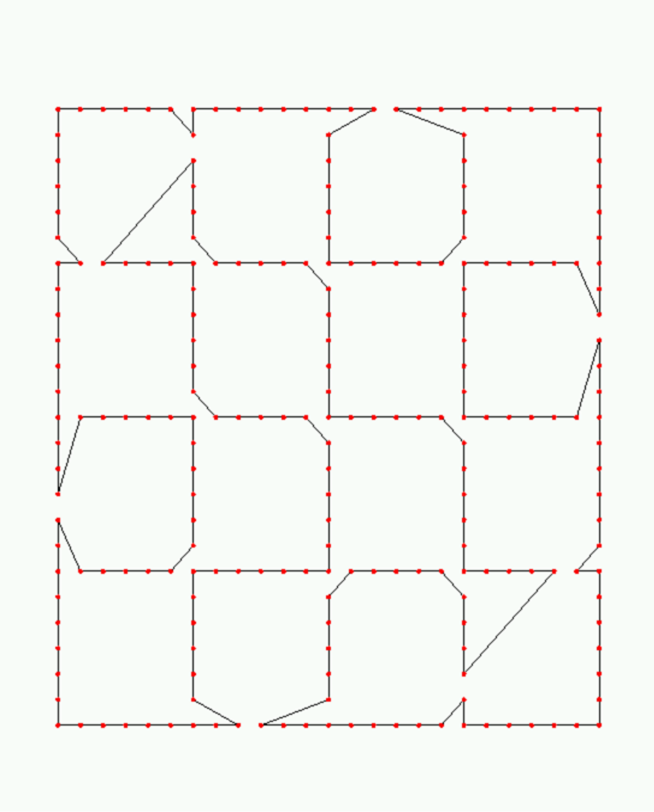
# MISCONCEPTION #8: SIZE IS IMPORTANT



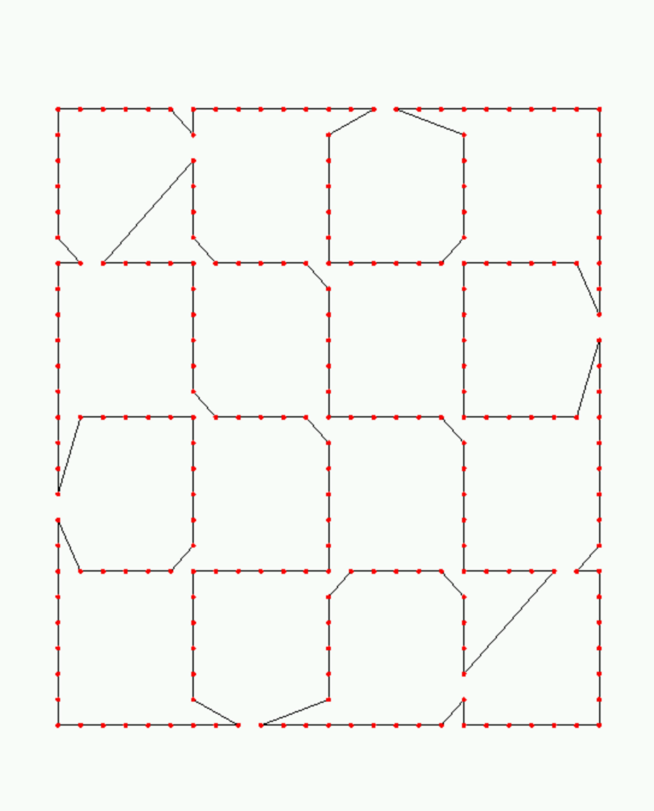
pla7397 solved
in October 1994



ts225 unsolved
in October 1994

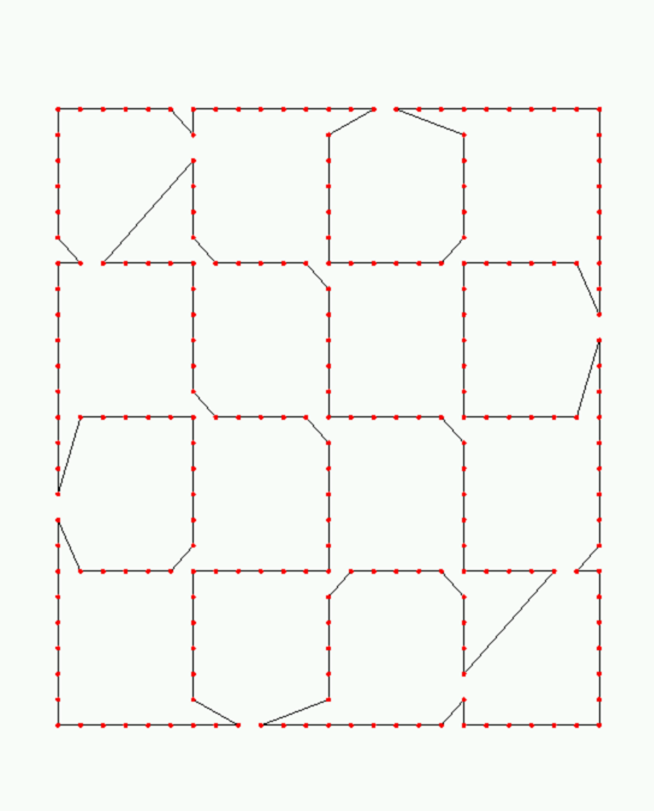


ts225 constructed by Stefan Tschöke



ts225 constructed by Stefan Tschöke

with malice aforethought



ts225 constructed by Stefan Tschöke

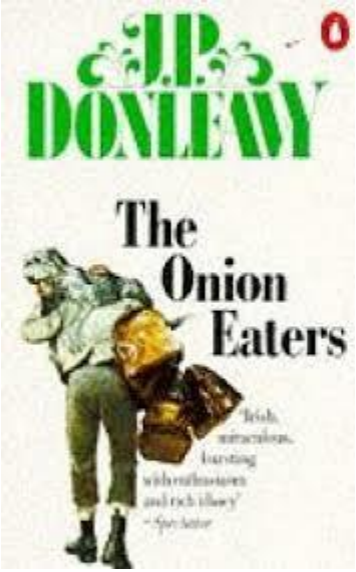
with malice aforethought

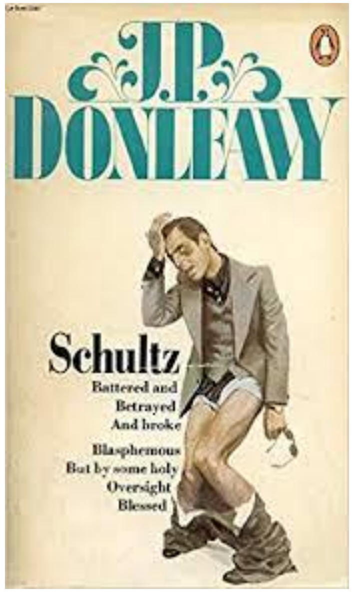
LESSON #1:
MISCHIEF IS A POWERFUL ENGINE OF DISCOVERY

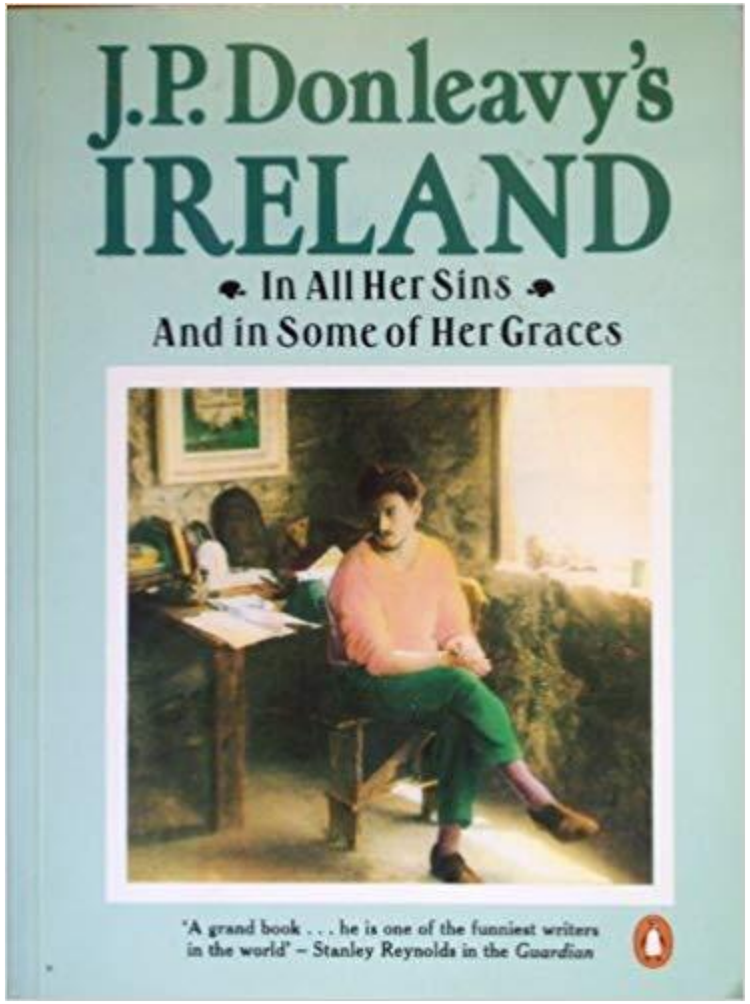
# The sport of solving the TSP

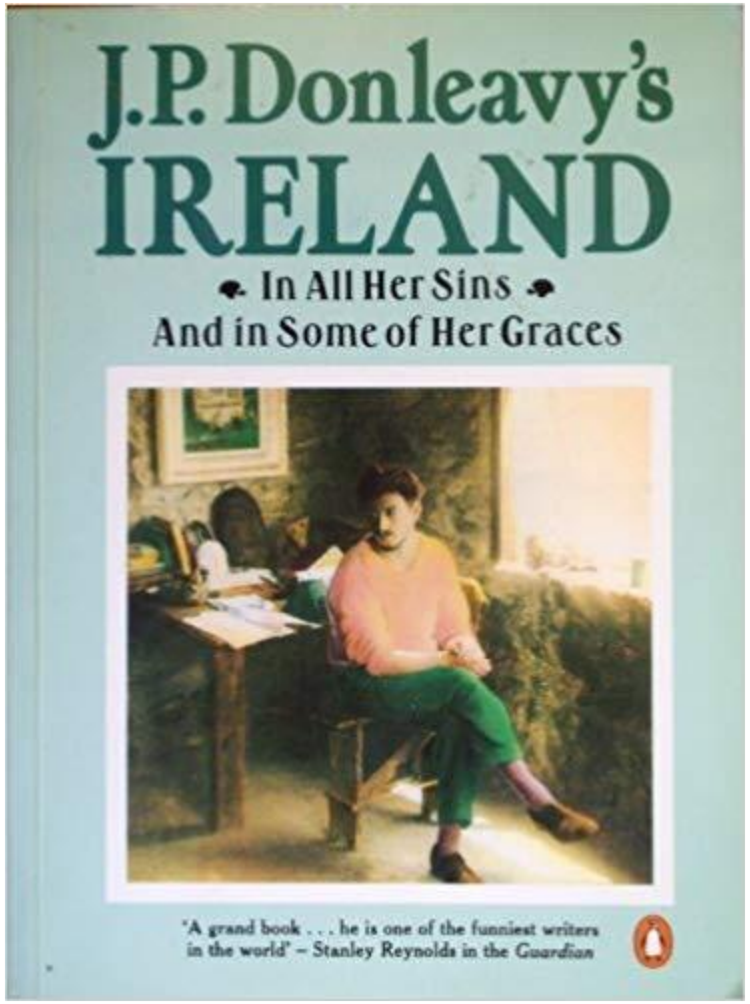
My daddy can beat up your daddy

# Bill Cook

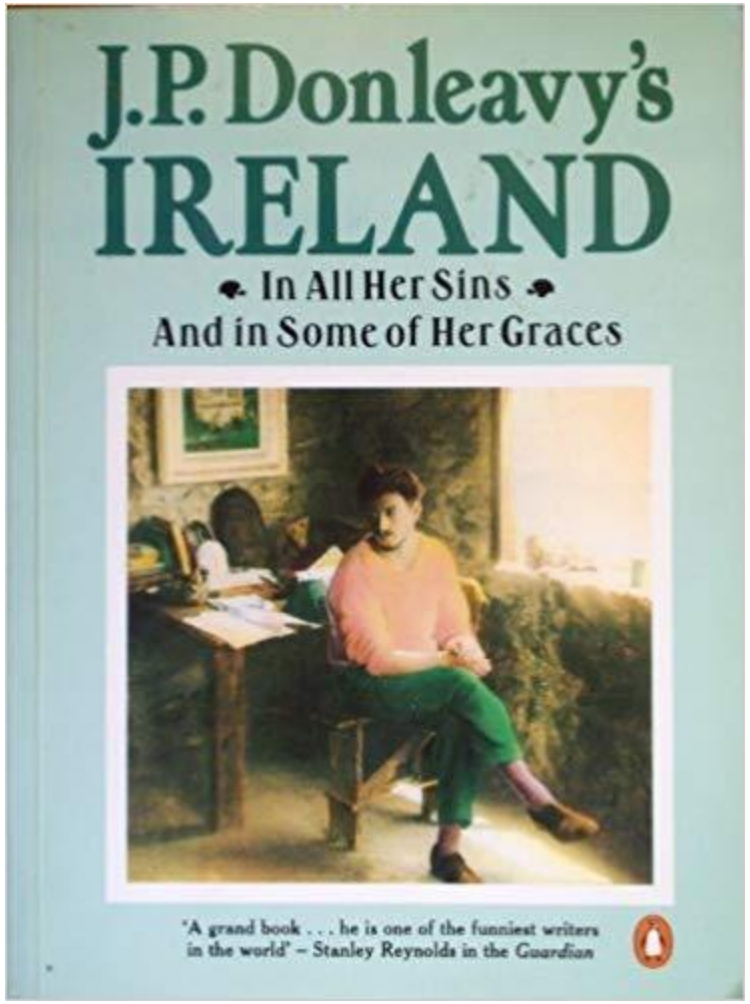

J.P.
DONLEAVY
The Onion Eaters


J.P.
DONLEAVY
Schultz
Battered and
Betrayed
And broke
Blasphemous
But by some holy
Oversight
Blessed


J.P. Donleavy's
IRELAND
In All Her Sins
And in Some of Her Graces
'A grand book . . . he is one of the funniest writers in the world' – Stanley Reynolds in the Guardian

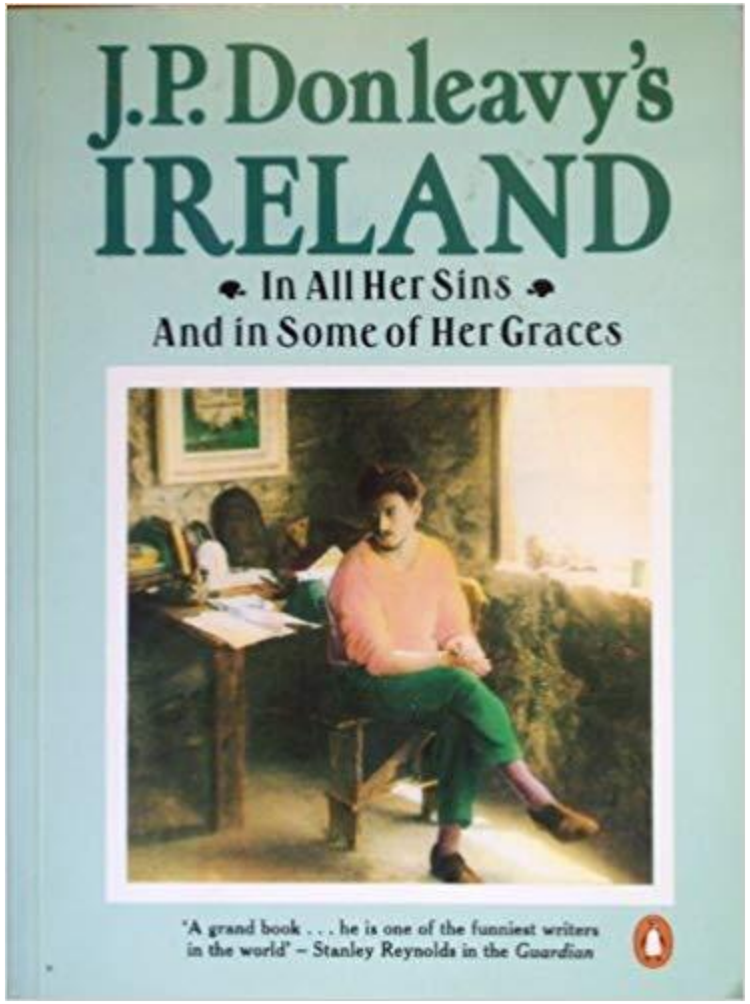



There is also the oft-repeated recollection in Donleavy's autobiographical writing that his Irish parents grew up "without a pot to piss in".




There is also the oft-repeated recollection in Donleavy's autobiographical writing that his Irish parents grew up "without a pot to piss in".

J. P. writes in his memoir: "I had come to this peasant land with my nice big American pot to piss in."




There is also the oft-repeated recollection in Donleavy's autobiographical writing that his Irish parents grew up "without a pot to piss in".

J. P. writes in his memoir: "I had come to this peasant land with my nice big American pot to piss in."

… the author's rise to artistic glory, when he finally found in his home at Levington Park, with eleven toilets, a pot to piss in.

# McGill 1987: Chính Hoàng's Ph.D. defense

# McGill 1987: Chính Hoàng’s Ph.D. defense

# McGill 1987: Chính Hoàng's Ph.D. defense

Professor Cook first from the left (in the doorway)

Discrete Mathematics 86 (1990) 47–57
North-Holland 

# THE BONDAGE NUMBER OF A GRAPH

John Frederick FINK, Michael S. JACOBSON, Lael F. KINCH and
John ROBERTS
*Department of Mathematics, University of Louisville, Louisville, KY 40292, USA*

Received 2 December 1988

A set $D$ of vertices in a graph $G$ is a *dominating set* if each vertex of $G$ that is not in $D$ is adjacent to at least one vertex of $D$. The minimum cardinality among all dominating sets in $G$ is called the *domination number* of $G$ and denoted $\sigma(G)$. We define the *bondage number* $b(G)$ of a graph $G$ to be the cardinality of a smallest set $E$ of edges for which $\sigma(G - E) > \sigma(G)$. Sharp bounds are obtained for $b(G)$, and the exact values are determined for several classes of graphs.

Discrete Mathematics 86 (1990) 191–198
North-Holland


# THE DISCIPLINE NUMBER OF A GRAPH

V. CHVÁTAL
*Department of Computer Science, Rutgers University, New Brunswick, NJ 08903, USA*

W. COOK
*Graduate School of Business, Columbia University, New York, NY 10027, USA*

Received 2 December 1988

## 1. Introduction

The *domination number* $\gamma(G)$ of a graph $G$ is the size of a smallest set $D$ of vertices such that every vertex outside $D$ has at least one neighbour in $D$; Fink, Jacobson, Kinch, and Roberts [4] defined the *bondage number* $b(G)$ of a graph $G$ as the least number of edges whose deletion makes $\gamma(G)$ increase. As we are about to point out, computing $b(G)$ amounts to solving an integer linear program.

Define a *whip* in a graph $G$ as any spanning subgraph $F$ of $G$ such that each component of $F$ is a star and $F$ has precisely $\gamma(G)$ components; let $E(G)$ denote the set of edges of $G$ and let $W(G)$ denote the set of all whips in $G$. Obviously, $b(G)$ is the optimal value of the problem

$$\begin{aligned} &\text{minimize} && \textstyle\sum \{x_e : e \in E(G)\} && \\ &\text{subject to} && \textstyle\sum \{x_e : e \in E(F)\} \geq 1 && \text{for all } F \text{ in } W(G), \\ & && x_e \geq 0 && \text{for all } e \text{ in } E(G). \\ & && x_e = \text{integer} && \text{for all } e \text{ in } E(G). \end{aligned} \tag{1}$$

By the *fractional bondage number* $b^*(G)$ we shall mean the optimal value of the 'linear programming relaxation' of (1),

$$\begin{aligned} &\text{minimize} && \textstyle\sum \{x_e : e \in E(G)\} && \\ &\text{subject to} && \textstyle\sum \{x_e : e \in E(F)\} \geq 1 && \text{for all } F \text{ in } W(G), \\ & && x_e \geq 0 && \text{for all } e \text{ in } E(G). \end{aligned} \tag{2}$$

By the duality theorem of linear programming, $b^*(G)$ equals the optimal value of the dual of (2),

$$\begin{aligned} &\text{maximize } \sum \{y_F : F \in W(G)\} \\ &\text{subject to } \sum \{y_F : e \in E(F)\} \leqslant 1 \quad \text{for all } e \text{ in } E(G), \\ &\qquad y_F \geqslant 0 \qquad\qquad\qquad \text{for all } F \text{ in } W(G). \end{aligned} \tag{3}$$

Since (3) can be seen as the linear programming relaxation of

$$\begin{aligned} &\text{maximize } \sum \{y_F : F \in W(G)\} \\ &\text{subject to } \sum \{y_F : e \in E(F)\} \leqslant 1 \quad \text{for all } e \text{ in } E(G), \\ &\qquad y_F \geqslant 0 \qquad\qquad\qquad \text{for all } F \text{ in } W(G), \\ &\qquad y_F = \text{integer} \qquad\qquad \text{for all } F \text{ in } W(G), \end{aligned} \tag{4}$$

problems (1) and (4) are in a sense dual. Therefore we refer to the optimal value of (4) as the *discipline number* dis$(G)$ of $G$.

We have

$$1 \leqslant \text{dis}(G) \leqslant b^*(G) \leqslant b(G) \tag{5}$$

for all graphs $G$. Apart from establishing upper bounds on $b(G)$, Fink et al. computed the bondage number of cycles, paths, and complete multipartite graphs and studied the bondage number of trees (several of these results can also be found in Bauer, Harary, Nieminen, and Suffel [1]). The purpose of this paper is to provide ties with analogous results for the fractional bondage number and for the discipline number.

## 2. The fractional bondage number

The principle restraining device of this section goes as follows.

**Theorem 1.** *If $G$ has $n$ vertices and $m$ edges then $b^*(G) \leqslant m/(n-\gamma(G))$.*

**Proof.** Observe that the constraints of (2) are satisfied by $x_e = 1/(n-\gamma(G))$ for all $e$. □

As usual, let $\Delta(G)$ denote the largest degree of a vertex in $G$. Fink et al. conjectured that $b(G) \leqslant \Delta(G)+1$.

**Theorem 2.** $b^*(G) \leqslant \Delta(G)$.

**Claim 5.** *If $k \geq 3$, then $|Q| \geq 4$.*

**Proof of Claim 5.** Assume the contrary: $k \geq 3$ but $|Q| \leq 3$. Since $G$ has at least four vertices, some vertex $w$ lies outside $Q$; since $F_1, F_2, \ldots, F_k$ are edge-disjoint, $w$ is adjacent to at least $k$ distinct vertices in $Q$. Hence $|Q| = k = 3$. Now no $S_j$ can include a vertex from $Q$ and a vertex $w$ outside $Q$ ($w$ has to be adjacent to at least three distinct vertices in $Q$); since $|S_j| \geq 2$ for all $j$, it follows that $Q = S_j$ for some $j$. Finally, this $S_j$ includes some vertex $w$ distinct from $u_1$ and $v_1$, a contradiction: $w$ must be adjacent to at least one of $u_1$ and $v_1$. □

**Claim 6.** *If $k \geq 4$, then $k \leq a + \lfloor b/2 \rfloor$.*

**Proof of Claim 6.** By virtue of Claim 4, we only need show that $|Q| = 2k$. For this purpose, assume the contrary: without loss of generality $u_1 = u_2$. Write

$$Q_0 = \{u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4\}$$

and consider the graph $H_0$ whose set of vertices is $Q_0$, two vertices being adjacent in $H_0$ if and only if they are adjacent in some $F_i$ with $1 \leq i \leq 4$. Since each $F_i$ with $1 \leq i \leq 4$ contributes $|Q_0| - 2$ edges to $H_0$, we have

$$4(|Q_0| - 2) \leq \binom{|Q_0|}{2};$$

observing that $|Q_0| \leq 7$ (since $u_1 = u_2$) and $|Q_0| \geq 4$ (by Claim 5), we conclude that $|Q_0| = 7$. Now $H_0$ has twenty edges, which is a contradiction: $\binom{7}{2} = 21$ and no $u_i$ with $2 \leq i \leq 4$ is adjacent to $v_i$ in $H_0$. □

**Claim 7.** *If $k = 3$, then $k \leq a + \lfloor b/2 \rfloor$ or $a + b \geq 4$.*

**Proof of Claim 7.** Claim 4 allows us to assume that $|Q| \leq 5$; Claim 5 guarantees that $|Q| \geq 4$. Defining $H$ as in the proof of Claim 4, observe that $H$ has $3(|Q| - 2)$ edges. It follows that $H$ (and hence also $G$) contains four pairwise adjacent vertices. □

**Claim 8.** *If $a = 0$ and $b = 2$, then $k \neq 2$.*

**Proof of Claim 8.** Assume the contrary: $k = 2$ but $a = 0$ and $b = 2$. Claim 4 implies that $|Q| \leq 3$ and so, without loss of generality, $u_1 = u_2 \in S_1$. Since $S_1$ includes a vertex distinct from both $u_1$, $v_1$ but adjacent to at least one of them, we must have $v_1 \in S_2$; a symmetric argument shows that $v_2 \in S_2$. But then $S_2$ includes a vertex outside $Q$ and adjacent to only one vertex in $Q$, a contradiction. □

This ties down the proof of Theorem 9. □

The reader interested in additional results in a similar vein is directed to [2, Chapter 5].

## Acknowledgement

We thank the two referees for their thoughtful comments which helped to improve the presentation of our results.

## References

[1] D. Bauer, F. Harary, J. Nieminen and C.L. Suffel, Domination alteration sets in graphs, Discrete Math. 47 (1983) 153–161.
[2] K.J. Devlin, Constructibility (Springer, Berlin, 1984).
[3] J.F. Fink, M.S. Jacobson, L.F. Kinch and J. Roberts, On graphs having domination numbers half their order, Per. Math. Hungar. 16 (1985) 63–69.
[4] J.F. Fink, M.S. Jacobson, L.F. Kinch and J. Roberts, The bondage number of a graph, preprint.
[5] O. Ore, Theory of Graphs, Amer. Math. Soc. Colloq. Publ. 38 (Amer. Mathematical Soc., Providence, RI, 1962).
[6] M. Padberg and G. Rinaldi, Optimization of a 532-city symmetric traveling salesman problem by branch and cut, Oper. Res. Lett. 6 (1987) 1–8.
[7] D.A.F. de Sade, Oeuvres Completes, 2–3 (J.-J. Pauvert, Paris, 1955).

# December 1987

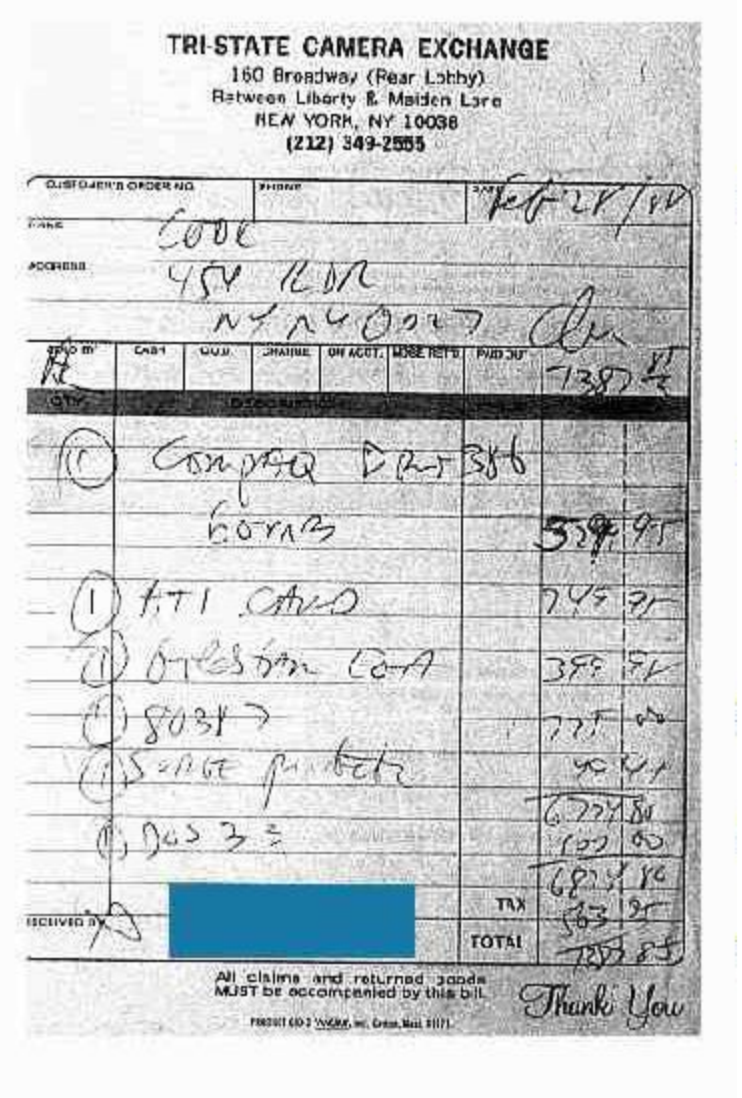
LESSON #1:
MISCHIEF IS A POWERFUL ENGINE OF DISCOVERY

**January 1988**

# David Applegate

*Don’t leave home without it!*

*Don't leave home without it!*



TRI-STATE CAMERA EXCHANGE
160 Broadway (Rear Lobby)
Between Liberty & Maiden Lane
NEW YORK, NY 10038
(212) 349-2555

CUSTOMER'S ORDER NO. | PHONE | DATE Feb 28/88

NAME Cook
ADDRESS 454 12 DR
NY NY 00027

SOLD BY | CASH | C.O.D. | CHARGE | ON ACCT. | MDSE RETD. | PAID OUT

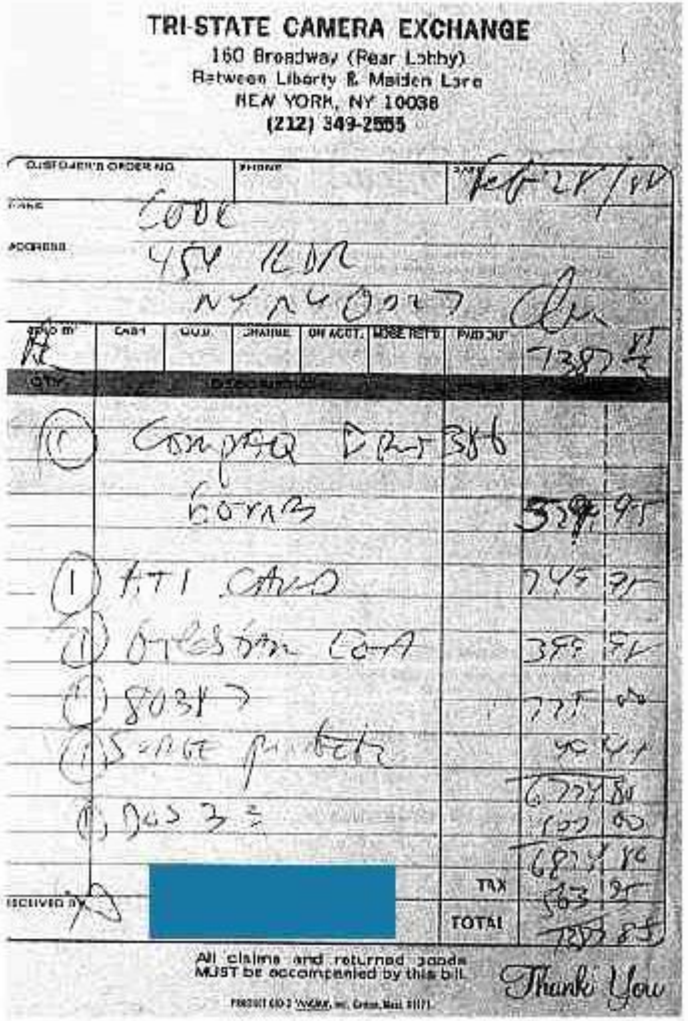
| QTY | DESCRIPTION | | |
|---|---|---|---|
| 1 | Compaq DP-386 | | |
| | 60 MB | 3599 | 95 |
| 1 | ATI Card | 749 | 95 |
| 1 | Quest/Card | 389 | 95 |
| 1 | 80387 | 775 | 00 |
| 1 | Serial parallel | 99 | 95 |
| | | 6222 | 80 |
| 1 | Dos 3.3 | 100 | 00 |
| | | 6822 | 80 |
| | TAX | 563 | 25 |
| | TOTAL | 7386 | 05 |

RECEIVED BY [redacted]

All claims and returned goods MUST be accompanied by this bill.

Thank You

*Don't leave home without it!*


TRI-STATE CAMERA EXCHANGE
160 Broadway (Rear Lobby)
Between Liberty & Maiden Lane
NEW YORK, NY 10038
(212) 349-2555

CUSTOMER'S ORDER NO. | PHONE | DATE Feb 28/88
NAME Cook
ADDRESS 45 Bdr
NY NY 10027

SOLD BY | CASH | C.O.D. | CHARGE | ON ACCT. | MDSE RET'D | PAID OUT
7387.85

QTY | DESCRIPTION

(1) Compaq DPro 386 60MB | 5399.95
(1) ATI card | 745.95
(1) Everex ... | 395.95
(1) 80387 | 775.00
(1) Surge protector | 90.45
(1) DOS 3.3 | 
6773.80
100.00
6823.80
TAX 563.95
TOTAL 7387.85

RECEIVED BY [redacted]

All claims and returned goods MUST be accompanied by this bill.

Thank You

February 28, 1988

Compaq DeskPro 386

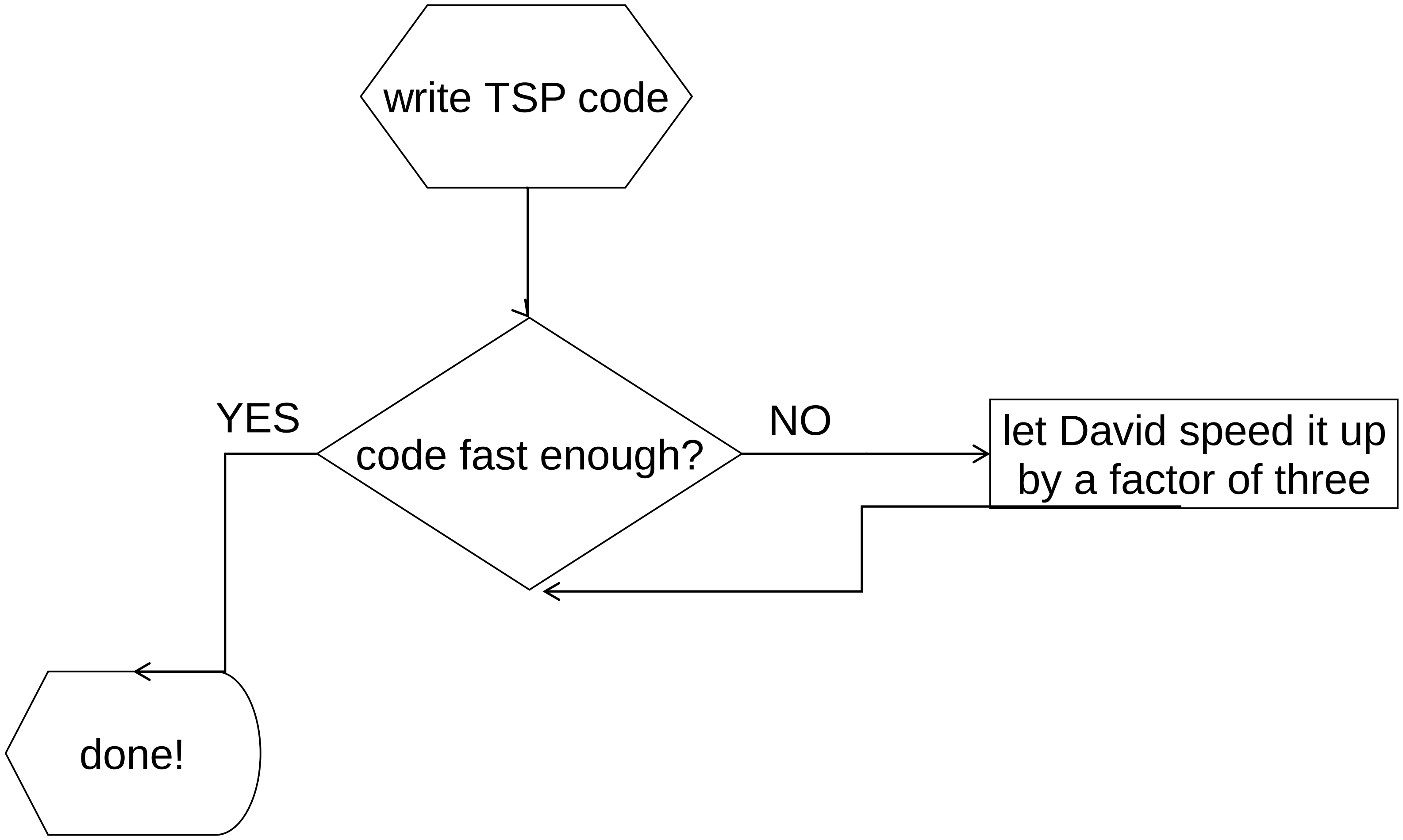
80387

DOS 3.3

$7387.85

# Bill Cook’s original TSP algorithm

# Bill Cook’s original TSP algorithm



# 1988 – 1989: The Age of Innocence

# 1988 – 1989: The Age of Innocence

Battle cry:

If everyone does it in a certain way, we will do it differently!!

OUT WITH THE OLD, IN WITH THE NEW

# 1988 – 1989: The Age of Innocence

Battle cry:

If everyone does it in a certain way, we will do it differently!!

OUT WITH THE OLD, IN WITH THE NEW

## LESSON #3:

SOMETIMES THE OLD IS USED FOR A GOOD REASON

# 1988 – 1989: The Age of Innocence

Battle cry:

If everyone does it in a certain way, we will do it differently!!

OUT WITH THE OLD, IN WITH THE NEW

## LESSON #3:

SOMETIMES THE OLD IS USED FOR A GOOD REASON

## Case in point:

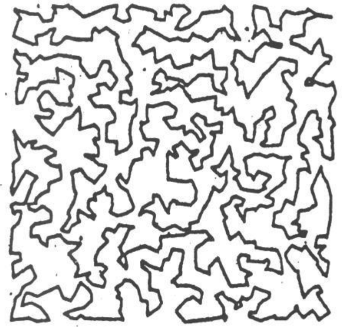
The simplex method of linear programming

*The New York Times*, March 12, 1991

# Math Problem, Long Baffling, Slowly Yields

## The traveling salesman problem still isn't solved, but computers can now get most answers.

By GINA KOLATA



A.T.&T.

Approximation of best route for 1,000 cities.

A CENTURY-OLD math problem of notorious difficulty has started to crumble. Even though an exact solution still defies mathematicians, researchers can now obtain answers that are good enough for almost all practical applications.

The traveling salesman problem, as it is known, crops up in many practical applications, from the design of computer chips to the designation of work orders in factories.

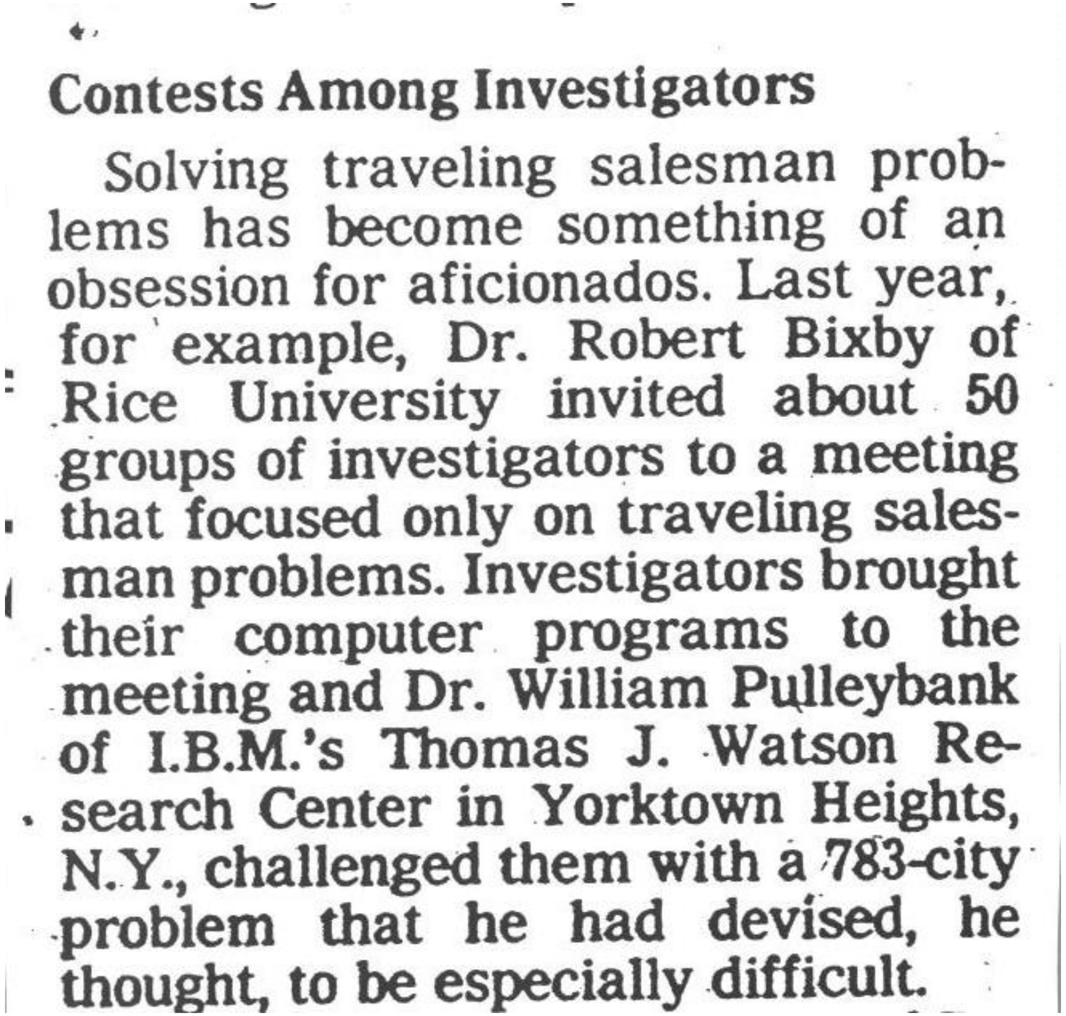
Brute number-crunching by computers can now produce answers to most such problems, even though not an immaculate solution. "Everybody likes to point to the

*The New York Times*, March 12, 1991

## Contests Among Investigators

Solving traveling salesman problems has become something of an obsession for aficionados. Last year, for example, Dr. Robert Bixby of Rice University invited about 50 groups of investigators to a meeting that focused only on traveling salesman problems. Investigators brought their computer programs to the meeting and Dr. William Pulleybank of I.B.M.'s Thomas J. Watson Research Center in Yorktown Heights, N.Y., challenged them with a 783-city problem that he had devised, he thought, to be especially difficult.

*The New York Times*, March 12, 1991



Contests Among Investigators

Solving traveling salesman problems has become something of an obsession for aficionados. Last year, for example, Dr. Robert Bixby of Rice University invited about 50 groups of investigators to a meeting that focused only on traveling salesman problems. Investigators brought their computer programs to the meeting and Dr. William Pulleybank of I.B.M.'s Thomas J. Watson Research Center in Yorktown Heights, N.Y., challenged them with a 783-city problem that he had devised, he thought, to be especially difficult.

The Texas Shootout: April 22-24, 1990

[Facsimile of a transparency used by V.C. in his talk in Texas]

[Facsimile of a transparency used by V.C. in his talk in Texas]

# SURVEY OF PROGRESS IN TSP SOLVING

[Facsimile of a transparency used by V.C. in his talk in Texas]

# SURVEY OF PROGRESS IN TSP SOLVING

| | | |
|---|---|---|
| 1954 | Dantzig, Fulkerson, and Johnson | 49 cities |

[Facsimile of a transparency used by V.C. in his talk in Texas]

## SURVEY OF PROGRESS IN TSP SOLVING

| | | |
|---|---|---|
| 1954 | Dantzig, Fulkerson, and Johnson | 49 cities |
| 1970 | Held and Karp | 60 cities |

[Facsimile of a transparency used by V.C. in his talk in Texas]

# SURVEY OF PROGRESS IN TSP SOLVING

| | | |
|---|---|---|
| 1954 | Dantzig, Fulkerson, and Johnson | 49 cities |
| 1970 | Held and Karp | 60 cities |
| 1974 | Helbig Hansen and Krarup | 80 cities |

[Facsimile of a transparency used by V.C. in his talk in Texas]

# SURVEY OF PROGRESS IN TSP SOLVING

| | | |
|---|---|---|
| 1954 | Dantzig, Fulkerson, and Johnson | 49 cities |
| 1970 | Held and Karp | 60 cities |
| 1974 | Helbig Hansen and Krarup | 80 cities |
| 1975 | Camerini, Fratta, and Maffioli | 100 cities |

[Facsimile of a transparency used by V.C. in his talk in Texas]

# SURVEY OF PROGRESS IN TSP SOLVING

| | | |
|---|---|---|
| 1954 | Dantzig, Fulkerson, and Johnson | 49 cities |
| 1970 | Held and Karp | 60 cities |
| 1974 | Helbig Hansen and Krarup | 80 cities |
| 1975 | Camerini, Fratta, and Maffioli | 100 cities |
| 1977 | Grötschel | 120 cities |

[Facsimile of a transparency used by V.C. in his talk in Texas]

## SURVEY OF PROGRESS IN TSP SOLVING

| | | |
|---|---|---|
| 1954 | Dantzig, Fulkerson, and Johnson | 49 cities |
| 1970 | Held and Karp | 60 cities |
| 1974 | Helbig Hansen and Krarup | 80 cities |
| 1975 | Camerini, Fratta, and Maffioli | 100 cities |
| 1977 | Grötschel | 120 cities |
| 1980 | Crowder and Padberg | 318 cities |

[Facsimile of a transparency used by V.C. in his talk in Texas]

## SURVEY OF PROGRESS IN TSP SOLVING

| | | |
|---|---|---|
| 1954 | Dantzig, Fulkerson, and Johnson | 49 cities |
| 1970 | Held and Karp | 60 cities |
| 1974 | Helbig Hansen and Krarup | 80 cities |
| 1975 | Camerini, Fratta, and Maffioli | 100 cities |
| 1977 | Grötschel | 120 cities |
| 1980 | Crowder and Padberg | 318 cities |
| 1986 | Padberg and Rinaldi | 532 cities |

[Facsimile of a transparency used by V.C. in his talk in Texas]

# SURVEY OF PROGRESS IN TSP SOLVING

| | | |
|---|---|---|
| 1954 | Dantzig, Fulkerson, and Johnson | 49 cities |
| 1970 | Held and Karp | 60 cities |
| 1974 | Helbig Hansen and Krarup | 80 cities |
| 1975 | Camerini, Fratta, and Maffioli | 100 cities |
| 1977 | Grötschel | 120 cities |
| 1980 | Crowder and Padberg | 318 cities |
| 1986 | Padberg and Rinaldi | 532 cities |
| 1987 | Grötschel and Holland | 666 cities |

[Facsimile of a transparency used by V.C. in his talk in Texas]

# SURVEY OF PROGRESS IN TSP SOLVING

| | | |
|---|---|---|
| 1954 | Dantzig, Fulkerson, and Johnson | 49 cities |
| 1970 | Held and Karp | 60 cities |
| 1974 | Helbig Hansen and Krarup | 80 cities |
| 1975 | Camerini, Fratta, and Maffioli | 100 cities |
| 1977 | Grötschel | 120 cities |
| 1980 | Crowder and Padberg | 318 cities |
| 1986 | Padberg and Rinaldi | 532 cities |
| 1987 | Grötschel and Holland | 666 cities |
| 1987 | Padberg and Rinaldi | 2,392 cities |

[Facsimile of a transparency used by V.C. in his talk in Texas]

# SURVEY OF PROGRESS IN TSP SOLVING

| | | |
|---|---|---|
| 1954 | Dantzig, Fulkerson, and Johnson | 49 cities |
| 1970 | Held and Karp | 60 cities |
| 1974 | Helbig Hansen and Krarup | 80 cities |
| 1975 | Camerini, Fratta, and Maffioli | 100 cities |
| 1977 | Grötschel | 120 cities |
| 1980 | Crowder and Padberg | 318 cities |
| 1986 | Padberg and Rinaldi | 532 cities |
| 1987 | Grötschel and Holland | 666 cities |
| 1987 | Padberg and Rinaldi | 2,392 cities |
| 1990 | Applegate, Chvátal, and Cook | 17 cities |

## Contests Among Investigators

Solving traveling salesman problems has become something of an obsession for aficionados. Last year, for example, Dr. Robert Bixby of Rice University invited about 50 groups of investigators to a meeting that focused only on traveling salesman problems. Investigators brought their computer programs to the meeting and Dr. William Pulleybank of I.B.M.'s Thomas J. Watson Research Center in Yorktown Heights, N.Y., challenged them with a 783-city problem that he had devised, he thought, to be especially difficult.

The winning team, made up of Dr. William Cook of Bell Communications Research, and Dr. David Applegate and Dr. Vasek Chvatal of Rutgers University, solved the problem in half an hour. "It turned out to be easy," Dr. Cook said.

# Bob Bixby

# Bob Bixby

CPLEX

# The sport of solving the TSP

My daddy can beat up your daddy

# The sport of solving the TSP

# The goal: pcb3038

# The goal: pcb3038

# Essential constituents of research in computational mathematics:

Essential constituents of research in computational mathematics:

1. Design an algorithm to solve your problem
2. Choose data structures to implement your algorithm

Essential constituents of research in computational mathematics:

1. Design an algorithm to solve your problem
2. Choose data structures to implement your algorithm
3. Find a cute acronym to name the result

Essential constituents of research in computational mathematics:

1. Design an algorithm to solve your problem
2. Choose data structures to implement your algorithm
3. Find a cute acronym to name the result

**C**ombinatorial
**O**ptimization and
**N**etworked
**C**ombinatorial
**O**ptimization
**R**esearch and
**D**evelopment
**E**nvironment

Essential constituents of research in computational mathematics:

1. Design an algorithm to solve your problem
2. Choose data structures to implement your algorithm
3. Find a cute acronym to name the result

**C**ombinatorial
**O**ptimization and
**N**etworked
**C**ombinatorial
**O**ptimization
**R**esearch and
**D**evelopment
**E**nvironment

# January 1992: The Great Dismay

## January 1992: The Great Dismay

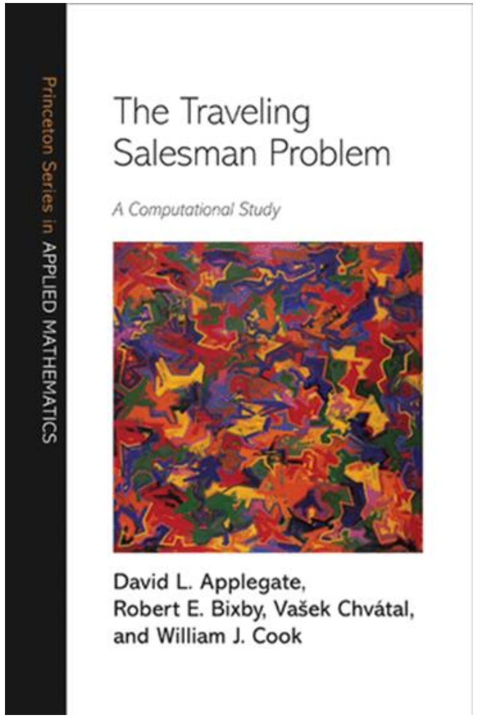
Desperate last-ditch efforts:

- combs from consecutive ones (Chapter 8)
- cut tightening (Section 10.1)
- cut gluing (Section 10.4)
- strong branching (Section 14.3)

January 1992: The Great Dismay

Desperate last-ditch efforts:

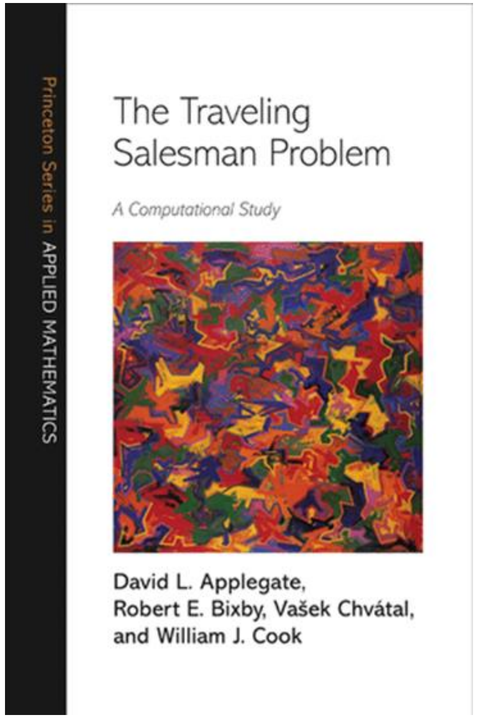
- combs from consecutive ones (Chapter 8)
- cut tightening (Section 10.1)
- cut gluing (Section 10.4)
- strong branching (Section 14.3)



January 1992: The Great Dismay

Desperate last-ditch efforts:

- combs from consecutive ones (Chapter 8)
- cut tightening (Section 10.1)
- cut gluing (Section 10.4)
- strong branching (Section 14.3)



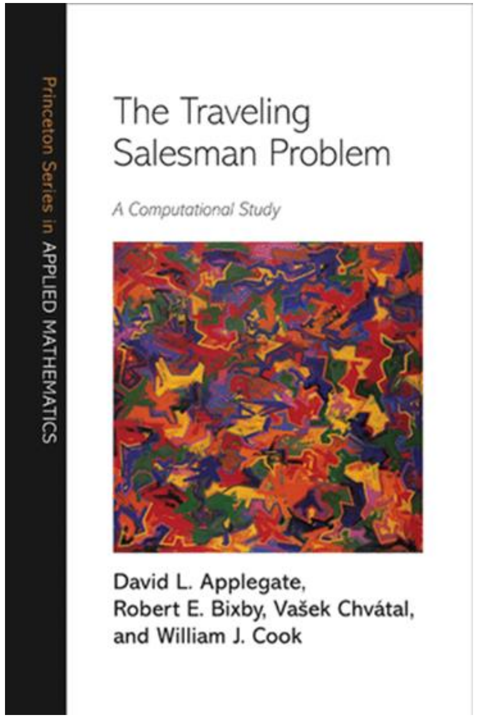

April 1992: pcb3038 falls

January 1992: The Great Dismay

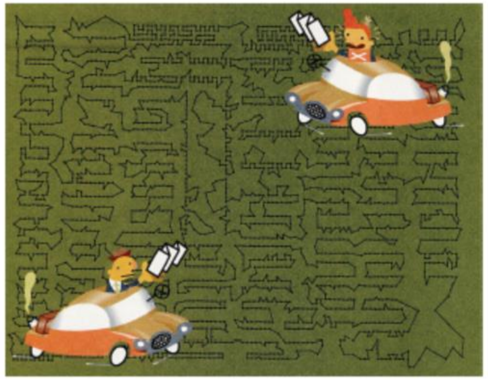
Desperate last-ditch efforts:

- combs from consecutive ones (Chapter 8)
- cut tightening (Section 10.1)
- cut gluing (Section 10.4)
- strong branching (Section 14.3)

April 1992: pcb3038 falls




*The Discover Magazine*, January 1993
"A new TSP record, 3,038 cities"



# June 1993: fnl4461

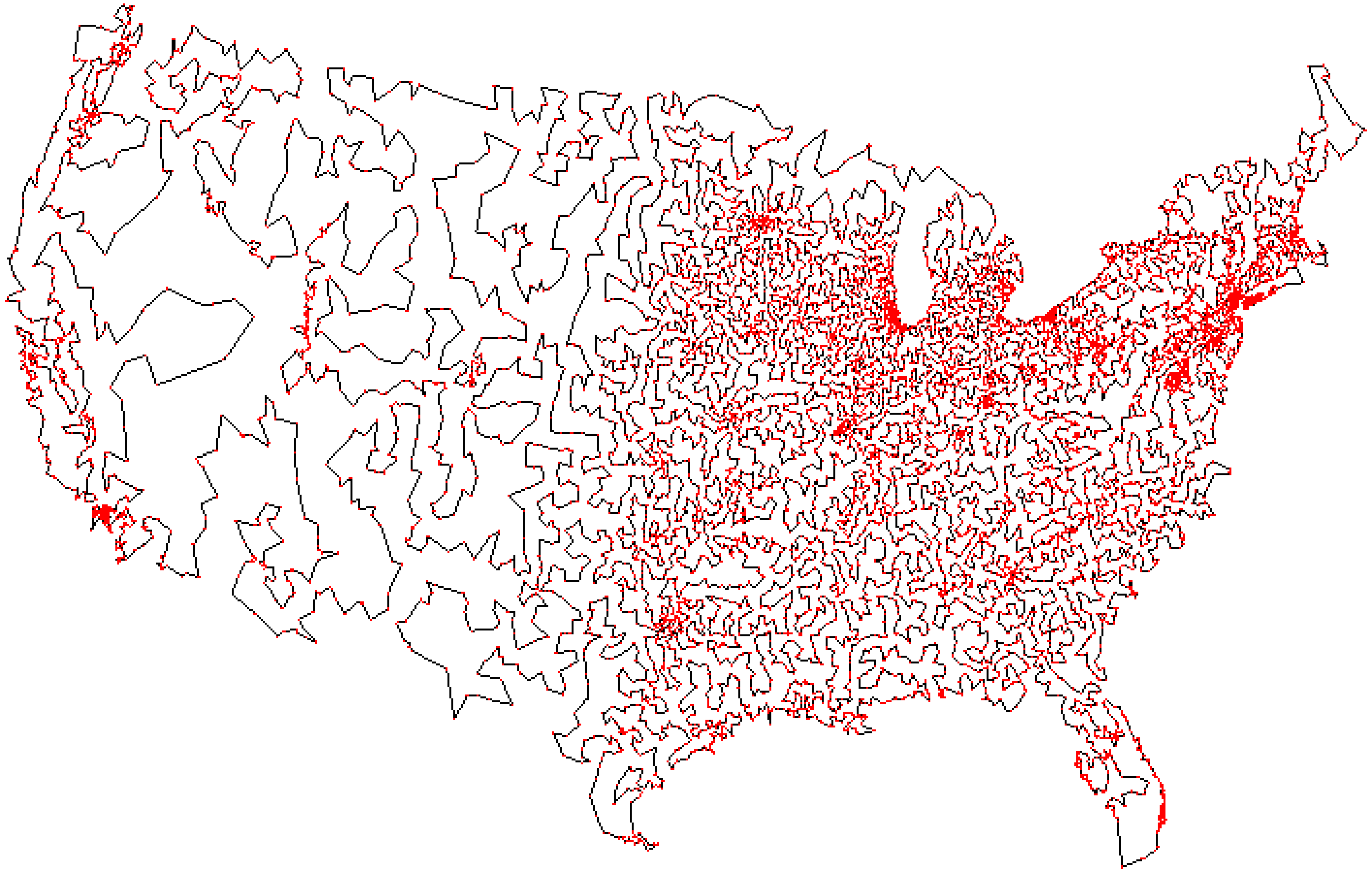
## January 1996

Another breakthrough: local cuts (Chapter 11)

## January 1996

# Another breakthrough: local cuts (Chapter 11)

## 1998: usa13509



# TSPLIB

a280
ali535
att48
att532
bayg29
bays29
berlin52
bier127
brazil58
brd14051
brg180
burma14
ch130
ch150
d198
d493
d657
d1291
d1655

d2103
d15112
d18512
dantzig42
dsj1000
eil51
eil76
eil101
fl417
fl1400
fl1577
fl3795
fnl4461
fri26
gil262
gr17
gr21
gr24

gr48
gr96
gr120
gr137
gr202
gr229
gr431
gr666
hk48
kroA100
kroB100
kroC100
kroD100
kroE100
kroA150
kroB150
kroA200
kroB200
lin105

lin318
linhp318
nrw1379
p654
pa561
pcb442
pcb1173
pcb3038
pla7397
pla33810
pla85900
pr76
pr107
pr124
pr136
pr144
pr152
pr226

pr264
pr299
pr439
pr1002
pr2392
rat99
rat195
rat575
rat783
rd100
rd400
rl1304
rl1323
rl1889
rl5915
rl5934
rl11849
si175
si535

si1032
st70
swiss42
ts225
tsp225
u159
u574
u724
u1060
u1432
u1817
u2152
u2319
ulysses16
ulysses22
usa13509
vm1084
vm1748

The red instances were unsolved in 1991

# TSPLIB

a280
ali535
att48
att532
bayg29
bays29
berlin52
bier127
brazil58
brd14051
brg180
burma14
ch130
ch150
d198
d493
d657
d1291
d1655
d2103
d15112
d18512
dantzig42
dsj1000
eil51
eil76
eil101
fl417
fl1400
fl1577
fl3795
fnl4461
fri26
gil262
gr17
gr21
gr24
gr48
gr96
gr120
gr137
gr202
gr229
gr431
gr666
hk48
kroA100
kroB100
kroC100
kroD100
kroE100
kroA150
kroB150
kroA200
kroB200
lin105
lin318
linhp318
nrw1379
p654
pa561
pcb442
pcb1173
pcb3038
pla7397
pla33810
pla85900
pr76
pr107
pr124
pr136
pr144
pr152
pr226
pr264
pr299
pr439
pr1002
pr2392
rat99
rat195
rat575
rat783
rd100
rd400
rl1304
rl1323
rl1889
rl5915
rl5934
rl11849
si175
si535
si1032
st70
swiss42
ts225
tsp225
u159
u574
u724
u1060
u1432
u1817
u2152
u2319
ulysses16
ulysses22
usa13509
vm1084
vm1748

The red instances were unsolved in 2006

Large TSP instances solved in the new millennium

| | | |
|---|---|---|
| April 2001 | d15112 | 34 CPU years |
| December 2002 | it16862 | 11 CPU years |
| March 2004 | brd14051 | 5 CPU years |
| May 2004 | sw24978 | 85 CPU years |
| October 2004 | pla33810 | 16 CPU years |
| March 2005 | d18512 | 57 CPU years |
| April 2006 | pla85900 | 136 CPU years |

年の9月に日本オペレーションズリサーチ学会の会員に中央大学の伊理正夫を通して
Chvátal からの協力依頼の電子メイルが流れたんだ．一部割愛して引用しよう．

> Date: Tue, 28 Sep 93 18:00:00
>
> Subject: ohisashiburi desu ne
>
> Dear Professor Iri,
>
> you may be surprised to get this message out of the blue. I wonder if you would do me a favor. David Applegate, Bob Bixby, Bill Cook, and I have written a computer code to solve travelling salesman problems. Last year we have solved one with 3038 cities (from a database called TSPLIB), which was the world record at that time; a couple of months ago we have broken the record by solving a problem with 4461 cities, and now we would like to try 7397 cities. We solve those problems in parallel on different workstations; currently we are using about 65 of them. The way it works is that people open accounts for us on their machines and we run parts of our program on them whenever no one else is using them (we have a clever program that kills our computations as soon as somebody touches the keyboard or logs into the machine from home). The 65 workstations are sitting around here in New Jersey, but with our sights on the 7397 cities we would like to recruit as many new machines as we can. David will try to get us some at McGill, we are asking a few other friends in North America to do the same, and thought it would be nice if we had a few helpers in Japan as well. If you know anybody who would be willing to lend his or her machine to contribute to breaking the record again, would you, please, pass this message on to him or her? (This is not terribly important, so please, don't go to too much trouble to help us.)
>
> Thank you very much!
>
> Ogenki de,
>
> Vasek Chvatal

何台ものワークステーションをつなげて世界記録を目指しているが，日本でも協力し

# CONCORDE’S YOUNGER HELPERS

• An implementation of Adam Letchford's *domino parity constraints* developed by Bill Cook, Daniel Espinoza, and Marcos Goycoolea.
Used in the solution of d18512, pla33810, and pla85900.

• Keld Helsgaun's refinement of the Lin-Kernighan heuristic.
Used in the solution of sw24978 and pla85900.

In February 2009, Robert Bosch created a 100,000-city problem.

In February 2009, Robert Bosch created a 100,000-city problem.

In March 2009, Yuichi Nagata found there the following tour of length 5,757,191:

In February 2009, Robert Bosch created a 100,000-city problem.

In March 2009, Yuichi Nagata found there the following tour of length 5,757,191:

In February 2009, Robert Bosch created a 100,000-city problem.

In March 2009, Yuichi Nagata found there the following tour of length 5,757,191:

Techniques for developing such point sets have evolved through work of Bosch and Craig Kaplan.

In February 2009, Robert Bosch created a 100,000-city problem.

In March 2009, Yuichi Nagata found there the following tour of length 5,757,191:

Techniques for developing such point sets have evolved through work of Bosch and Craig Kaplan.

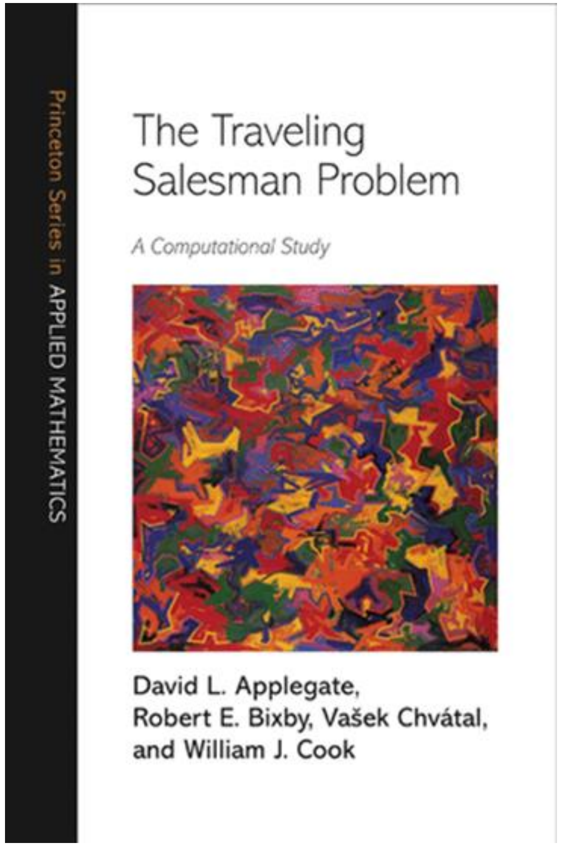
Bill Cook offers $1000 for a shorter tour.

Bill Cook’s enchanting website:

http://www.math.uwaterloo.ca/tsp/

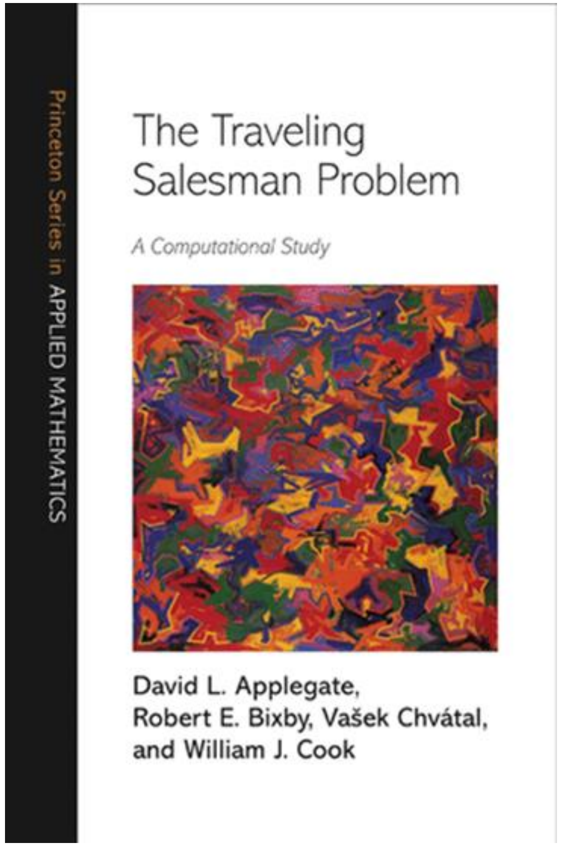
# It is never too early to think of Christmas gifts

# It is never too early to think of Christmas gifts

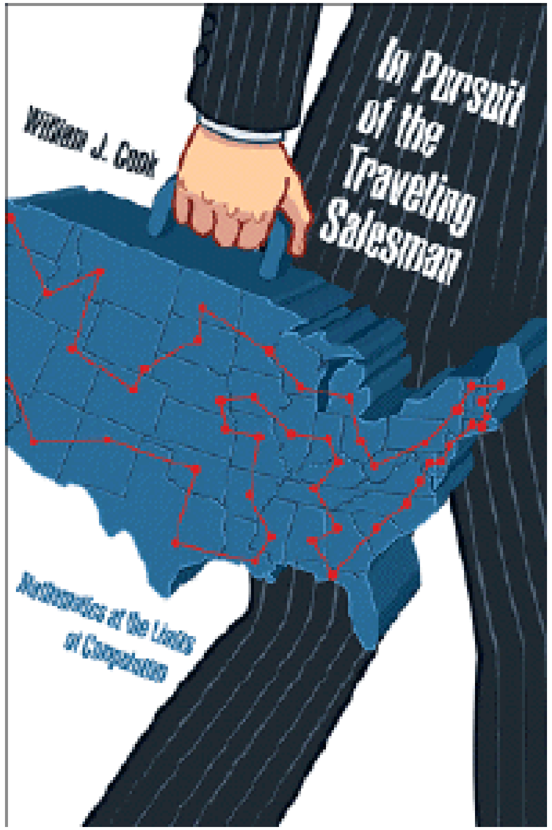



Hardcover $80.81
at amazon.com

# It is never too early to think of Christmas gifts




Hardcover $80.81
at amazon.com




Paperback $13.78
at amazon.com