Circuit generation.

The accuracy of detection probability lower bound estimation

has been exercised. If this happens, an improved version of the cutting algorithm can be applied to cover the tricky faults [14].

VI. CONCLUSIONS

CACOP, a new testability analyses method, has been proposed in this paper. The examples demonstrate that CACOP is quite suitable to derive the lower bounds of the detection probabilities. Most faults can be covered in the first several propagations. The benefits of using CACOP are twofold: 1) CACOP can be used to alleviate the computing complexity of the cutting algorithm without losing the tightness of DPLBs; 2) CACOP can be used as a one-sided (i.e., never overestimates detection probabilities) probabilistic testability measuring tool with the aid of the improved cutting algorithm if the computing time is tolerable [14].

In 1984, at the International Test Conference, a panel on Will Testability Analysis Replace Fault Simulation concluded with a clear identification of two problems: 1) fault simulation will be too expensive for the million-device chips of the future; and 2) improvements are needed in the testability analyses techniques [13]. Since fault simulation is too expensive, testability analysis is a possible alternative. CACOP is a compromise of $O(n^2)$ and $O(n)$ testability analyses methods from the viewpoint of computing complexity, and the accuracy of detection probability lower bound estimation is potentially better than $O(n^2)$ testability analyses methods.

In future, consideration will be given to select good lines to be broken such that most faults can be covered by only a limited number of tree circuits. The bound distortion is significant if the broken lines feed AND gates, and this should preferably be avoided in the tree circuit generation.

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Adaptive Sliding Mode Coordinated Control of Multiple Robot Arms Attached to a Constrained Object

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Abstract—When a common object, attached to multiple robot arms, is cooperatively manipulated to move along a constrained surface, the control task requires the simultaneous control of the motion trajectory of the attached object on the constrained surface; the constrained force due to the contact with the surface; and the internal force exerted by the arms on the object. To accomplish such a control objective, an adaptive sliding mode control algorithm is presented by developing a new concise dynamic model of the system and exploiting its particular properties. Detailed analysis on the tracking properties of the object's position, the constrained force, and the internal force are given. The stability analysis shows that the proposed algorithm can achieve satisfactory tracking performance.

I. INTRODUCTION

In recent years, the coordinated control of multiple robotic manipulators has been investigated by many researchers [1]-[10]. The potential applications of such a system cover a wide range; for example, material handling and assembly, grasping and manipulation.
by a multi-fingered robot hand, and all other tasks recognized to
be beyond the capability of a single arm. When multiple robots
attach to and manipulate a common object cooperatively, the mul-
tiple manipulators, together with the attached object, form a closed
kinematical chain. In such a situation, the multiple manipulators are
kinematically and dynamically constrained, and the resulting dynamic
equations are extremely nonlinear and coupled. The control of such a
system becomes more complicated since a set of holonomic equality
constraints are imposed and the number of actuators available exceeds
the mobility of the system. Thus, the control objective is not only the
motion of the manipulated object but also the internal force exerted
on the object by the arms, which does not affect its motion.

A number of methods have been proposed for solving this problem
[11]-[10]. These methods can be classified in two categories: i) master/slave method [11]-[3], where one or a group of robot arms
play the role of the master, and the rest of the arms form the slave
group which are moved in conjunction with the master arm(s); ii) hybrid position/force control method [4]-[5], where the position
of the object is controlled in certain directions of the workspace, and the
force is controlled in the other directions. Each robot arm is controlled
using both position and force error.

In addition to the above research efforts, most researches on
the coordinated control of two or more robotic manipulators have
concentrated on the problem in free space, ignoring the presence
of the environment in the workspace. In a variety of tasks, such
as bolt assembly or line draw, however, the motion of the object is
constrained in some directions due to interaction between the attached
object and the environment. In such a case, in addition to the control
of the motion of the object and the internal force it is often necessary
to control the contact force between the environment and the attached
object.

When the environment is modelled as a rigid frictionless constraint
surface, a kinematic constraint is imposed on the attached object.
This has been extensively studied in recent years for a single arm
[11]-[14], and is referred to as nonlinear singular systems [11].
For multiple manipulators the control problem becomes even further
complicated, since we have to control simultaneously the motion of
the attached object on the constrained surface, the contact force
between the attached object and the constrained surface, and the internal force exerted on the object by the arms.

The study of control of the constrained object attached to mul-
tiple robots was an open problem until recent efforts described in
[15]-[19]. By using the linearizable method, Yun [15], Yoshikawa
and Zheng [17] respectively developed their schemes. Cole [16]
also proposed a computed torque control method. But it should
be noted that the success of the aforementioned schemes relies on
the full knowledge of the complex dynamics of the multi-
arm system. Care should therefore be taken if there is uncertainty
about the system dynamics, as the controller so designed may give
degraded performance and may incur instability. To deal with the
uncertainties in the dynamics, Hu and Goldenberg [18] proposed an
adaptive control law based on the Popov hyperstability theory.
But their controller needs the measurements of acceleration and force
derivative. Very recently, Yao et al. [19] proposed a sliding mode
control scheme for a two-arm model. However, the derivation is only
based on a simplified model, neglecting the shape of the object.

In this paper, an adaptive sliding mode algorithm for coordinating
multiple robot arms is proposed, such that the multi-arm attaches
to a common object and moves it along a rigid constrained surface
while maintaining a desirable contact force. To do this, firstly, a
concise dynamic model of attached object in the object coordinate
space is proposed. Then, by recognizing that the degrees of freedom
of the generalized coordination decrease while the attached object
is constrained, a reduced dynamic model suitable for motion and
constraint force control is derived. By exploiting the particular
structure of this dynamic model, some fundamental properties are
obtained to facilitate controller design. Based on this reduced dynamic
model, an adaptive sliding mode controller, using only positions,
velocities, and force signals, is proposed. Stability analysis shows
the asymptotical tracking of position, contact force, and internal force
without requiring any knowledge of the dynamic model.

The organization of this article is as follows: In Section II, the robot
dynamics and its structure properties, in the generalized coordinate
space of attached object system, are derived. In Section III, a reduced
model suitable for control purposes is proposed, and based on this
model an adaptive sliding mode control algorithm is proposed. In
Section IV some conclusions are presented.

II. SYSTEM DYNAMIC EQUATION FORMULATION

Although several authors [15]-[19] have proposed a variety of
dynamic models for a multi-arm system, in this section we take a
different point of view, by combining dynamic equations of n non-
redundant arms with that of a rigid attached object, constrained by a
rigid surface, to formulate a complete multi-arm model. This model
will then be used to develop a sliding mode coordinated control law
in Section III.

Consider n robot arms holding a rigid object as shown in Fig. 1,
in which all robot end-effectors hold the same object moving along a
rigid surface in a coordinated fashion. Define the coordinate system
as follows:

- $O_1$: Inertial reference frame.
- $O_c$: Object coordinate frame fixed at the mass center of the object.
- $O_i$: Constraint coordinate frame fixed at the contact point on the
  object surface.
- $O_{ai}$: ith arm coordinate frame fixed at the ith end-effector located
  at the grasping point.

We also use notations defined as follows.

- $r_i \in \mathbb{R}^3$: Position vector of the origin of the frame $O_i$ to the
  centroid of the object $O_c$.
- $R_i \in \mathbb{R}^{3 \times 3}$: Orientation of $O_i$.
- $w_i \in \mathbb{R}^3$: Angular velocity vector of the object.
- $r \in \mathbb{R}^3$: Position vector from the origin of the frame $O_i$ to the
  origin of the ith end-effector frame $O_{ai}$.
- $w \in \mathbb{R}^3$: Angular velocity vector of the arm coordinate system.
- $v_i \in \mathbb{R}^3$: Position vector of the origin of the object frame $O_c$
  to the origin of the frame $O_{ai}$.
- $f_i \in \mathbb{R}^3$: Force applied to the object through the attaching point
  by the ith arm.
- $m_i \in \mathbb{R}^3$: Moment applied to the object through the attaching
  point by the ith arm.
- $M$: Mass of the object.
- $I$: Inertial tensor of the object represented by the frame $O_c$.

To facilitate the dynamic formulation, the following assumptions
are made.

A1: Each manipulator is nonredundant, hence all manipulators have
the same number of joints.
A2: All the end-effectors of the manipulators are rigidly attached
to the common object so that no relative motion occurs between
the object and any end-effector.

It should be noted that if rolling and sliding contacts exist between
the object and any end-effector [25], [26], the problem will become
even more complicated, due to the existence of nonholonomic con-
straints. This problem, however, is beyond the scope of our discussion
in this paper.
For simplicity, it is assumed that each contact point is fixed and has a known location on the object. Each robot applies a force \( f \) and a moment \( \tau \), through the contact point \( C \), to the object. There are in total \( n \) robots acting on the same payload. Firstly, we consider the situation where the environmental constraints on the object are not taken into account. In this case the motion equation of the object manipulated by robot arms is expressed as follows:

\[
\dot{M} \ddot{x} + \dot{\tau} = F + \tau
\]  

(1)

\[
\dot{\tau} = \sum_{i=1}^{n} \left( \mathbf{r}_i \times f_i \right)
\]  

(2)

where \( F \) and \( \tau \) are the resultant force and moment of the external forces and moments, respectively, applied to the object by robot arms.

\[
F_n = \sum_{i=1}^{n} f_i
\]  

(3)

\[
\tau_n = \sum_{i=1}^{n} \left( \mathbf{r}_i \times f_i \right)
\]  

(4)

\[
F = \mathbf{W}_d f_1 + \mathbf{W}_2 f_2 + \cdots + \mathbf{W}_n f_n
\]

(5)

where \( f_i \) and \( \mathbf{r}_i \) are the equivalent forces and moments, respectively, applied to the object by the \( i \)th arm. Equations (3) and (4) can be rewritten as:

\[
\begin{bmatrix}
F_n \\
\tau_n
\end{bmatrix} = \sum_{i=1}^{n} \begin{bmatrix}
\mathbf{W}_d \\
\mathbf{W}_2 \\
\vdots \\
\mathbf{W}_n
\end{bmatrix} \begin{bmatrix}
f_i \\
\mathbf{r}_i \times f_i
\end{bmatrix}
\]

(6)

where \( \mathbf{W}_d \in \mathbb{R}^{6 \times n} \) and \( \mathbf{W}_2 \in \mathbb{R}^{6 \times n} \).

\[
F = \begin{bmatrix}
F_1 \\
F_2 \\
\vdots \\
F_n
\end{bmatrix} \in \mathbb{R}^{6n}
\]

(7)

In general, the position and orientation of the object are of interest. By using the position and orientation of the object, denoted by \( X_o = [r_o, \omega_o]^{T} \), where orientation \( \omega_o \) can be expressed as Euler angles, roll-pitch-yaw-angles, or in any other manner, then the expression of the object coordinates is given by:

\[
X_o = \mathbf{T}_o r_o
\]

(8)

where \( \mathbf{T}_o \) is a transform matrix and expressed as \( \mathbf{T}_o = \begin{bmatrix}
0 & -S_o & 0 \\
S_o & C_o & 0 \\
0 & 0 & 1
\end{bmatrix} \). Consider now a point \( C \) on the object is constrained to follow a physical surface. Supposing the constraints imposed are described by a holonomic smooth manifold, then the algebraic equation for the constraint can be written in the contact point \( X_o = [r_o, \omega_o]^{T} \) as:

\[
\Phi(X_o) = 0
\]

(9)

where the mapping \( \Phi : \mathbb{R}^n \rightarrow \mathbb{R}^m \) is twice continuously differentiable.

Since \( \Phi(X_o) = 0 \) is identically satisfied, it is evident that \( J_o \Phi(X_o) = 0 \) is normal to the constraint surface and we can write:

\[
f_c = J_o \lambda
\]

(10)

where \( \lambda \in \mathbb{R}^m \) is the associated Lagrangian multiplier [11]. The constraint force \( f_c \) at the contact point between the object and constraint surface will produce a resultant force/moment \( F_c \) at the object center of mass, given by:

\[
F_c = S_o T^{-1} f_c
\]

(11)

where \( S_o = \begin{bmatrix}
E_3 \\
0
\end{bmatrix} \) is the associated Lagrangian multiplier [11].

Since the position of frame \( O_o \), relative to the frame \( O_o \), is given by the expression \( \mathbf{r}_c = \mathbf{R}_o \), the relation between \( X_o \) and \( X_c \) can be expressed as:

\[
X_o = \begin{bmatrix}
\mathbf{r}_c + \mathbf{R}_o \mathbf{p}_c \\
\omega_o + \text{constant}
\end{bmatrix}
\]

(12)

which is known.

Then, based on (6) the dynamic equation of the object, taking into account the environmental constraint, can be expressed as:

\[
\begin{bmatrix}
M E_3 \\
0 \\
0
\end{bmatrix} \ddot{X}_o + \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} \omega_o \times I T_o + I T_o \dot{\omega}_o + \begin{bmatrix}
\dot{\omega}_o \times I T_o \\
0 \\
\dot{\omega}_o \times I T_o
\end{bmatrix} \dot{X}_o + \begin{bmatrix}
\dot{M} g \\
0
\end{bmatrix} = W F + F_c
\]

(13)

where \( \Phi(\mathbf{H}(X_o)) = 0 \) is given by [17].

\[
T_o = \begin{bmatrix}
0 & -S_o & 0 \\
S_o & C_o & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(14)

where \( S_o = \sin(\phi), C_o = \cos(\phi), \) etc.
B. Kinematical Constraints on the Attached Object

At each attaching point \( C_j \), the following kinematic constraint relation between the position \( r_i \) and \( r_j \) holds:

\[
v_i = r_i + R_ip_i.
\]

Differentiating the above with respect to time yields:

\[
\dot{v}_i = \dot{r}_i + R_ip_i.
\]

where \( w_i \times R_i = 0 \), hence, at the attaching point \( C_j \), the following relation can be established:

\[
\begin{bmatrix}
\dot{v}_i \\
w_{ai}
\end{bmatrix} =
\begin{bmatrix}
E_3 & -(R_ip_i) \\
0 & E_3
\end{bmatrix}
\begin{bmatrix}
\dot{r}_i \\
w_i
\end{bmatrix}
\equiv
s_n
\begin{bmatrix}
\dot{r}_i \\
w_{ai}
\end{bmatrix}.
\]

Let the position and the orientation of the end-effector of the \( i \)th arm at attaching point \( C_j \) be denoted by \( p_i \equiv [\alpha_i \beta_i \gamma_i] \), the velocity including orientational elements by \( v_i = [v_i^r \ v_i^w] \in \mathbb{R}^7 \), and the joint variable vector by \( q_i \in \mathbb{R}^n \). Then:

\[
v_i = J_i q_i
\]

where \( J_i \) is the generalized Jacobian matrix. Using the (15), it can be shown that

\[
s_n
\begin{bmatrix}
\dot{r}_i \\
w_{ai}
\end{bmatrix} = J_i q_i
\]

It is assumed in the following, that each robot works in a nonsingular region. Thus the inverse of the matrix \( J_i \) exists.

Considering all the robots acting on the object at the same time, the following kinematical constraints are obtained:

\[
J q = S \begin{bmatrix}
\dot{r}_1 \\
\vdots \\
\dot{r}_i \\
\vdots \\
\dot{r}_m
\end{bmatrix} = ST \dot{X}_o
\]

where \( J = \text{block diag}(J_1, \ldots, J_m) \in \mathbb{R}^{n \times n}, q = [q_1 \cdots q_m] \in \mathbb{R}^n, S = [s_1 \cdots s_m]^T \in \mathbb{R}^{n \times m}, T = \text{block diag}(E_3, T_o) \in \mathbb{R}^{m \times 3} \).

Concerning the matrices \( W \) in (5) and \( S \) in (18), the following useful properties hold:

Property 2.1:

1) \( S \) and \( W \) are full rank, i.e., \( \text{rank}(W) = \text{rank}(S) = 6 \).
2) \( ST^T = W \).

C. The Combined Robots/Object Dynamics

We now derive the dynamic equation for the whole system in terms of the object variable \( X_o \). If the object moves along the constraint surface in response to manipulation by the multiple arms, it has been shown that the dynamics of the 7th robot reflecting object effects is given by:

\[
D_i(q)\ddot{q} + B_i(q, \dot{q})\dot{q} + G_i(q) = u_i - J_i^T F,
\]

Equation (19) can be expressed more concisely as

\[
D(q)\ddot{q} + B(q, \dot{q})\dot{q} + G(q) = u - J^T F
\]

where \( q \in \mathbb{R}^n \) is defined in (18), \( F \in \mathbb{R}^{n \times m} \) is defined in (5).

The robot model (20) is characterized by the following structural properties, which are of importance to our controller design.

Property 2.2:

1) \( D(q) \) is symmetric positive definite.
2) A suitable definition of \( B(q, \dot{q}) \) makes matrix \( (\dot{D} - 2B) \) skew-symmetric [21]. In such a case, \((\dot{D} - 2B) \) is also skew-symmetric, i.e.,

\[
\dot{x}^T(\dot{D} - 2B)x = 0.
\]

3) The matrices \( D, B, \) and \( G \) are linear in the dynamic parameters of robot \( i \). More specifically, the following decomposition will hold [21]:

\[
D_i(q, \dot{q})\ddot{q} + B_i(q, \dot{q})\dot{q} + G_i(q) = Y_i(q_i, \dot{q}_i, \ddot{q}_i, \dot{r}_i, \phi_i)
\]

where \( i \) is a regressor matrix of known function of \( q_i, \dot{q}_i \) and \( \alpha_i \in \mathbb{R}^{n \times m} \) is a vector of equivalent parameters.

It is worth noting that there are in total \((6n + 1) \) position variables in the (11), (12), and (20), but in fact, only 6 of the variables are independent. This can be easily seen since once the trajectory of the object is given, the joint trajectory of each robot is uniquely determined due to the assumption A1 and A2. Owing to the dependence among position variables, the (11), (12), and (20) would not be suitable for one to analyze the dynamic behavior of this system.

With this observation in mind, we will treat elements of the object coordinate \( X_o \) as independent position variable and reformulate the dynamic (20) in terms of these variables so as to describe the whole system behavior.

In review of (18) and noting that \( w_o = T_o c_o \), it is readily obtained that

\[
\dot{q} = J_i^{-1} ST \dot{X}_o
\]

\[
\ddot{q} = J_i^{-1} ST \ddot{X}_o + \frac{d}{dt}(J_i^{-1} ST) \dot{X}_o
\]

With these relations, the dynamic model of the multi-robot system (20) can be reformulated, in terms of the object coordinate \( X_o \) as

\[
D_{1}J_i^{-1} ST \ddot{X}_o + D_{2} \frac{d}{dt}(J_i^{-1} ST) \dot{X}_o + B_{1}J_i^{-1} ST \dot{X}_o + G(X_o) = u - J^T F
\]

Premultiplying both sides of (23) by \( T^T \) and using Property 2.1, then, the dynamics model of the multi-robot system, coupled with the object dynamics (11), is

\[
D_i(X_o) \dot{X}_o + B_i(X_o, \dot{X}_o) \dot{X}_o + G_i(X_o) = u_i + T^T S^T \ddot{X}_o + T^T J_i^T \lambda
\]

\[
\Phi(X_o) = 0
\]
By the implicit function theorem, the constraint (25) can always be expressed explicitly as [11]

\[ X_2 = \sigma(X_1^i). \]  

It is assumed that the elements of \( X_1^i \) are chosen to be the first \( 6-r \) components of \( X_1 \). If this is not the case, (24) can always be reordered so that the first \( 6-r \) equation correspond to \( X_2^i \) and the last \( r \) equation to \( X_2^r \). Defining

\[ L(X_1^i) = \left[ \frac{\partial \sigma(X_1^i)}{\partial X_1^r} \right], \]  

Then, from (29)

\[ \dot{X}_e = L(X_1^i) \hat{X}_e \]  

\[ \dot{X}_e = L(X_1^i) \hat{X}_e + B_1(X_1^i, \hat{X}_e) \hat{X}_e + G_1(X_1^i) \]  

Therefore, the dynamic model (24), which is restricting to the constraint surface, can be expressed in a reduced form as

\[ D_e(X_1^i) \dot{X}_e + B_1(X_1^i, \hat{X}_e) \hat{X}_e + G_1(X_1^i) = \mathbf{u}_e + T' S_\iota T^{-1} J_\iota \lambda \]  

where \( B_1 \) is defined as

\[ B_1(X_1^i, \hat{X}_e) = D_e(X_1^i) \hat{X}_e + B_1(X_1^i, \hat{X}_e) \hat{X}_e + G_1(X_1^i) \]

Remark: Equation (32) is suitable for control purposes which forms the basis for the subsequent development. This is because the quality constraint equation are embedded into the dynamic equation, resulting in an affine nonlinear system without the constraints.

By exploiting the structure of the (32), three properties can be obtained.

Property 3.1: Define the matrix

\[ A(X_1^i) = L'(X_1^i) D_e(X_1^i) L(X_1^i), \]

then

\[ \dot{A}(X_1^i) = 2L'(X_1^i) B_1(X_1^i, \hat{X}_e) \]

is skew-symmetric.

Proof:

\[ \dot{A} - 2L'(X_1^i) B_1 = L(D_e - 2B_1) L = L T'[T^{-1} J_\iota \lambda], \]

From Theorem 2.1 that \( \dot{D}_e - 2B_1 \) is skew-symmetric, it is easy to know that \( \dot{A} - 2L'(X_1^i) B_1 \) is also skew-symmetric.

Property 3.2:

\[ J_\iota T^{-1} S_\iota T L = L' S_\iota S_\iota T^{-1} J_\iota = 0. \]

Proof: Based on the (7), we have

\[ \frac{d}{dt} H(X_e) = 0. \]  

By using (10), \( dH(X_e)/dt \) can be expressed as

\[ \frac{dH(X_e)}{dt} = T^{-1} S_\iota \dot{X}_e. \]

From (30), (33) becomes

\[ J_\iota T^{-1} S_\iota T L(X_1^i) \hat{X}_e = 0 \]

Since \( X_1^i \) is linearly independent, therefore, we obtain

\[ J_\iota T^{-1} S_\iota T L = L' S_\iota S_\iota T^{-1} J_\iota = 0. \]

Property 3.3: Motion (32) is still linear in terms of a suitably selected set of parameters, i.e.,

\[ D_e(X_1^i) L(X_1^i) \hat{X}_e + B_1(X_1^i, \hat{X}_e) \hat{X}_e + G_1(X_1^i) = Y_1(X_1^i, \hat{X}_e) \]

This property can be easily proved from Property 2.3.

The above properties are fundamental for designing the adaptive sliding mode laws.
B. Adaptive Sliding Mode Controller

The controller design problem is as follows: given the desired object trajectory $X_d^i$ and desired constraint force $f_d^i$, or identically desired multiplier $\lambda_i$, which satisfy the imposed constraints, i.e., $\Phi(H(X_d^i)) = 0$ and $f_d^i = f^i \lambda_i$, to determine a sliding control law such that for all $(X, X_d^i) \in \Omega$, that $X_d^i \rightarrow X_d^i$ and $f_d^i \rightarrow f_d^i$ as $t \rightarrow \infty$. It should be noted that, since $X_d^i = \sigma(X_d^i)$, it is only required to find a sliding control law to satisfy $X_d^i - X_d^i$ as $t \rightarrow \infty$.

Now, we are ready to introduce the sliding mode coordinated control algorithm by borrowing some conceptual development of the sliding mode control scheme proposed in [14]. Defining

$$c_m = X_d^i - X_d^i, \quad \dot{X}_m^i = X_d^i - X_m^i$$

where $c_m$ is the tracking error, $X_m^i$ is the auxiliary trajectory; $\lambda$ is tunable positive definite matrix whose eigenvalues are strictly in the right-half complex plane.

Defining $a_i$, as a constant $m$-dimensional vector, containing the unknown elements in the suitably selected set of equivalent dynamic parameters, then the linear parametrizability of the dynamics (Property 3.3) leads to

$$D(X_d^i)I(X_d^i)X_d^i + B(X_d^i)X_d^i + G(X_d^i) = Y_i(X_d^i, X_d^i, X_d^i, X_d^i, X_d^i),$$

where $Y_i(X_d^i, X_d^i, X_d^i, X_d^i, X_d^i)$ is a $6 \times m$ regressor matrix of known function of $X_d^i, X_d^i, X_d^i, X_d^i, X_d^i$.

The sliding surface is chosen as

$$s_1 = \dot{X}_m^i - \dot{X}_m^i = \dot{c}_m + \lambda \dot{c}_m.$$  \hspace{1cm} (38)

The adaptive sliding mode control law is defined as

$$u_i = Y_i(X_d^i, X_d^i, X_d^i, X_d^i, X_d^i) \dot{c}_m + K_2 s_1 + T^T S^i T^{-1} J^i \lambda_i$$

where $K_2$ is a positive definite design matrix, $\eta_i > 0$ are arbitrary constants; $\lambda$ is a force control defined by

$$\lambda_i = \lambda_i - K_i \dot{c}_m.$$  \hspace{1cm} (42)

where $K_i$ is an $m \times m$ constant matrix of force control feedback gains, $c_m = \lambda - \dot{\lambda}_i \dot{c}_m$.  \hspace{1cm} (43)

The following theorem is proposed.

**Theorem:** Consider an object attached by $n$ robots, each robot having six degrees of freedom. Let a point $P_i = [x_i, y_i, \theta_i]^T$ on the object be constrained to move along a rigid frictionless constraint surface. Then, for the position/orientation $X_n$, a constraint $\Phi(H(X_n)) = 0$ is imposed. Suppose no robot goes through a singularity and the grasp maintains force closure over the trajectory. For such a system which is modelled in the reduced form (32), using the control law (39)-(41), with the sliding surface $s = 0$ described by (38), the closed-loop system is then globally asymptotically stable in the sense that

1. $X_d^i \rightarrow X_d^i$ as $t \rightarrow \infty$
2. steady-state force $F_d^i = F_d^i$ is bounded and inversely proportional to the norm of the matrix $(K_i + I)$.

Proof: The following sliding mode equation can be easily obtained by using (39) and (37)

$$D_i \dot{L}_i = D_i J^i \dot{X}_m^i - D_i \dot{X}_m^i$$

where

$$\dot{X}_m^i = u_i + T^T S^i T^{-1} J^i \lambda_i = D_i J^i \dot{X}_m^i - B_i \dot{X}_m^i - G_i \dot{X}_m^i$$

with

$$Y_i \dot{c}_m = Y_i \dot{X}_m^i - Y_i X_d^i \dot{s}_1$$

which leads to

$$Y_i \dot{c}_m = Y_i \dot{X}_m^i - Y_i X_d^i \dot{s}_1$$

and

$$Y_i \dot{c}_m = Y_i \dot{X}_m^i - Y_i X_d^i \dot{s}_1$$

which implies

$$\dot{c}_m = \frac{1}{\lambda_i} \frac{\partial}{\partial c_m} \left( Y_i \dot{c}_m - Y_i X_d^i \dot{s}_1 \right)$$

Differentiating (46) with respect to time along the solution of (45) gives

$$\dot{V} = \frac{1}{2} \dot{s}_1^2 + \sum_{i=1}^{m} \beta_i |i - \beta_i|^\gamma / \eta_i$$

where $\beta_i = |u_i|, \alpha_i$ is defined in (37), $\gamma$ is its estimate.

Using the Property 3.3, the above equation becomes

$$\dot{V} = \frac{1}{2} \dot{s}_1^2 + \sum_{i=1}^{m} \beta_i |i - \beta_i|^\gamma / \eta_i$$

Using the Property 3.1, (47) becomes

$$\dot{V} = \frac{1}{2} \dot{s}_1^2 + \sum_{i=1}^{m} \beta_i |i - \beta_i|^\gamma / \eta_i$$

which implies

$$\dot{V} = \frac{1}{2} \dot{s}_1^2 + \sum_{i=1}^{m} \beta_i |i - \beta_i|^\gamma / \eta_i$$

and

$$\dot{V} = \frac{1}{2} \dot{s}_1^2 + \sum_{i=1}^{m} \beta_i |i - \beta_i|^\gamma / \eta_i$$

where $\dot{c}_m$ is bounded. Substituting the control (39) into the reduced order dynamic model (32) and using Property 3.3, we have

$$Y_i(X_d^i, X_d^i, X_d^i, X_d^i, X_d^i) \dot{c}_m = K_i L(X_d^i) s_1 - T^T S^i T^{-1} J^i \lambda_i$$

where $\lambda_i = \frac{\partial}{\partial x} \left( Y_i(X_d^i, X_d^i, X_d^i, X_d^i, X_d^i) \dot{c}_m \right) - K_i L(X_d^i) \dot{s}_1$.

which can be written as

$$\lambda_i = \lambda_i - K_i L(X_d^i) \dot{s}_1$$

where $\lambda_i = \lambda_i - K_i L(X_d^i) \dot{s}_1$.

So all signals on the right side of (49) are bounded and we can write (51) as

$$\lambda_i = \lambda_i - K_i L(X_d^i) \dot{s}_1$$

where $\lambda_i$ is a bounded function. Using (35), (36), and (41) again allows (51) to be written as

$$\lambda_i = \lambda_i - K_i L(X_d^i) \dot{s}_1$$

and

$$\lambda_i = \lambda_i - K_i L(X_d^i) \dot{s}_1$$

(52)
where $\xi$ is a bounded function. Substituting the force control given in (43) into (52) yields

$$\tau_f = (K_f + I)^{-1} \xi$$

(53)

where $I$ is the $m \times m$ identity matrix. Thus, $\tau_f$ and therefore the force tracking error $F_r - F_j^*$ is bounded and can be adjusted by changing the feedback gain matrix $K_f$.

**Remark:**
1. The control laws (39)–(41) do not require any knowledge of the detailed description of the model, and no additional assumption is imposed on the system. The controller is very general and structurally simple as well as computationally fast.
2. The results for force control is similar to the results presented in [13] for a single arm.
3. The method for computing the regressor matrix $Y_1$ given in (39) was given by several references, such as [23].
4. While assuring the desired behavior, the control law (39) is discontinuous across the sliding surface $s^*$, which leads to control chattering. Chattering, in general, is highly undesirable in practice, since it involves extremely high control activity, and further, may excite high-frequency dynamics neglected during modelling [20]. This can be remedied by smoothing out the control discontinuities in a boundary layer neighboring the sliding surface. To do this, we replace $u_{in}$ by $u_{in} + \varepsilon$, where $\varepsilon$ is boundary layer thickness. It can be proved that this will guarantee the ultimate boundedness of the system to within any neighborhood of the boundary layer [20], [22].

**C. Torque Distribution**

For tracking purposes a control algorithm in terms of $u$ is required. Fortunately, since $W$ is full rank (Property 2.2), there exists a matrix $W^+ = W^T (W W^T)^{-1}$, such that (note that $u_{in} = T^T W^T u$)

$$u = J^T u_{in}$$

(54)

where

$$u_{in} = W^T T^{-T} u_{in} + F_j$$

(55)

is the force causing the motion of the object, in which $u_{in}$ is computed by (39), and $F_j$ represents an internal force vector. Since the only constraint for $F_j$ is that it should lie within the null space of $W$, it is not unique, and depends on the load distribution among the robot arms. Due to different applications requirements different choices for $F_j$, therefore, a many torque distribution methods have been proposed [9], [10].

For this paper, the torque distribution method given in [9] can be applied. The reader may refer to [9].

**IV. CONCLUSION**

A robust coordination scheme, for cooperatively manipulating a common object to follow a constrained surface with multiple arms, was presented in this paper. The kinematic and dynamic constraints together with the redundancy in actuation make the control of such a system a formidable problem. The main contributions of this paper lie in the establishment of a new dynamic model to describe the constrained object motion, which makes it possible to seek a sliding mode control law. The proposed algorithm guarantees the simultaneous control of the motion of the object on the constrained surface; the constrained force due to the contact with the surface; and internal force exerted by the arms on the object, using only the measurements of joint position, velocity, and constraint force. The object can be of any shape as long as its mass-center is known.

The computational efficiency for realizing the proposed scheme can be improved by off-line computation of the nonlinear regressor matrix using the desired positions and velocities instead of the actual measurements. The method for implementation of such a strategy is discussed in detail in [24] for a single robot. Therefore, the way to extend the proposed scheme is a further research topic. In developing the control strategy, it is assumed that each arm firmly attaches to the object through the attaching point. For some advanced applications, rolling and sliding contacts between the object and some end-effectors may be required. Hence, extension of the results to the rolling or sliding case is also an interesting further research topic.

**REFERENCES**

Abstract—This paper deals with the evaluation of human biofeedback response in virtual reality and in direct view. The experiments have been performed with a new paradigm for the evaluation of human biofeedback during the telemanipulation performance of a touch task. The controlled motion of one finger is monitored with the surface EMG, while a mechanical robotized hand finger follows the motion imposed by the human finger. The biofeedback is detected in a direct way, by the operator’s eyes to the display and the blade are respectively 2 meters and 1.30 meters.

The process of visual search in virtual environments has been investigated by Stark et al., as well as the role of visual depth cues and effects of stereo and occlusion on simulated manipulation [5].

Experimental studies were conducted by Massimino and Sheridan to determine the effects of visual and force feedback on human performance in telemanipulation, with varying frame rates and subtended visual angles, with and without force feedback [6]. Kazerooni has proposed a framework for the design of a telerobot controller in which the dynamic behaviors of master and slave systems are mutually dependent [7]. In his book [8], Sheridan provides a wide survey on the efforts that have been made to model the man-in-the-loop and the operator’s role in supervisory control.

Our research provides an experimental evaluation of the different control strategies adopted by the human neuromuscular system when the same teleoperation task is performed with the aid of different man/machine interfaces [9], [10], [11].

The EMG recording during a teleoperation experiment, performed both in conditions of direct visual contact with the remote environment and utilizing different interfaces, allows an investigation of the neuromuscular activity of a human subject. A better understanding of how human control is performed can then be achieved.

The sensory signals processed by the cerebral cortex and the cerebellum represent the feedback aspect in the human control loop. To adjust neuromuscular activity to the desired behavior in anticipation of the sensory signals is performed by a feedforward control as the human motion plan does not contain, in itself, a complete description of the task [12].

In this experiment, the operator wears an exoskeleton system that drives the mechanical finger motion. During the operator’s finger motion, the sensed signals from the exoskeleton change and these changes provide signals to actuate the mechanical finger.

The sensory biofeedback in the tests is obtained by the eyes, which are observing the performance of the telemanipulation action and the contact force of the robotic finger, depicted on a monitor or expressed by the bending of a loaded blade. The line-of-sight distances from the operator’s eyes to the display and the blade are respectively 2 meters and 1.30 meters.

The process monitoring continues throughout the duration of the test. The following signals are sampled and memorized for quantitative analysis: 1) operator’s finger motions, 2) EMG signals, 3) forces exerted by the mechanical finger on the blade.

II. TEST EQUIPMENT

The test equipment makes use of appropriately integrated mechanical, electronic and display components. The integration itself allowed the development of a system which is able to provide different types of feedback to the operator and to carry out a quantitative analysis of the test execution modes.

The main features of the experimental station are (Fig. 1): 1) Telerobotic hand, a mechanical gripping device with three independent fingers with phalanxes articulation, actuated by three motors which stretch and release a metal tendon, developed in the Robotics Laboratory of the Department of Mechanics, Politecnico di Milano. In this first stage of the experiments, was decided that only one of the fingers should be used, to simplify the execution of the test. Therefore, only one of the mechanical fingers was programmed to accept direct control by the operator. The elements of the experiment include the following:

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The controlled motion of one finger is monitored with the surface EMG, as part of the telemanipulation control loop. The biofeedback in the tests is obtained by the eyes, which are observing the performance of the telemanipulation action and the contact force of the robotic finger, depicted on a monitor or expressed by the bending of a loaded blade. The line-of-sight distances from the operator’s eyes to the display and the blade are respectively 2 meters and 1.30 meters.

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