AXIOMATIC THEORY OF DESIGN MODELING

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In this paper, an axiomatic theory is established for studying design. Using this theory, a formal model of design is derived to represent the syntactic structure of hierarchical evolving design objects and the dynamic design process. The axiomatic theory can be used by design theorists and computer scientists as a formal logical tool to model design whereas the derived formal model can be used by designers to organize their design activities and by software developers to implement various CAD systems. The axiomatic theory includes two groups of axioms: axioms of objects and axioms of the human thought. They address the character of two central entities in the design process: nature and the human thought. Two important notions derived from the axioms are structure and range operations, which naturally represent basic thought processes, such as aggregation and generalization, in the design process. Related work is reviewed to show the history of the axiomatic approach to the modeling of design. Future directions are also given.

1. Introduction

Design is a human activity that has a history as long as human beings themselves. For decades, people have attempted to understand the nature of design for the intellectual merit as well as its instrumental value. The knowledge about design has been applied to improve the quality and efficiency of design activities in many ways, such as the development of computer-aided design systems, design methodologies, and design curriculum. These applications, in turn, are used as experimental tools to test and verify acquired knowledge about design.

There are many ways to acquire the knowledge about design, just like there are many ways to acquire the knowledge about any other phenomena. In engineering sciences, two widely used approaches are bottom-up and top-down strategies. In the context of design studies, they are shown in Fig. 1. Both strategies share one major objective, which is to establish design theories to address the nature and models of the design process, design objects, and design knowledge. A design theory can be verified by applying it to case studies or by comparing it to commonly accepted understanding of design properties. It can also be applied to improve the design practice by managing the identified factors implied in the design process model established in design theories.

The research with the bottom-up strategy attempts to generalize design theories by observing engineer's design activities. Protocol analysis has been one of the popular methods used for this purpose. Speculation and retrospection are also used to develop assumptions, insights, and theories about design. Important results from these explorations include the systematic design methodology (Pahl and Beitz, 1988; Hubka, 1980; Hubka and Eder, 1988), the theory of inventive problem-solving (Altshuller, 1988), axiomatic design (Suh, 1990), the decision-based design theory (Allan and Mistree, 1997), the artificial intelligence-based design (Gero, 2000), etc. Many publications can be found in conferences such as the International Conference on Engineering Design, the ASME Annual Design Engineering Technical Conferences, and Artificial Intelligence in Design, as well as journals such as *Design Studies*, *Research in Engineering Design*, *Journal of Engineering Design*, *Artificial Intelligence in Engineering*, *Journal of Integrated Design and Process Science*, etc. In different contexts, these results are described by natural languages, flow charts, graphic illustration, and/or mathematical symbols.

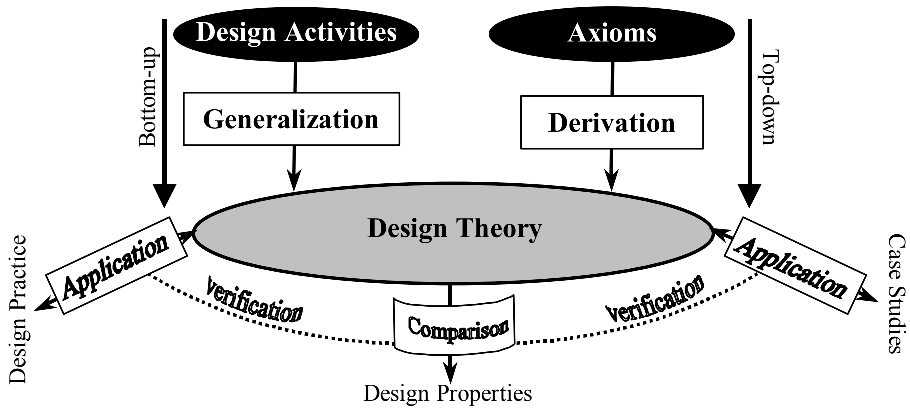

Fig. 1 Two main strategies of design research.

The research with the top-down strategy attempts to derive design theories from the first principles. The axiomatic approach is one of the major tools for this purpose. It usually targets the general design problem and the general model of design so that models for concrete design problems can be logically deduced. At the base of this strategy are axioms, which are self-evident truths that can be taken as the premises for inference. No proof is considered to be required for axioms. Using logic, properties about design activities can be derived from the axioms. These derived design properties are called theorems of design, which can be well-known characteristics of design or new understanding and insights into design. A group of theorems together constitute a design theory. The axioms, theorems, and other objects appearing in the theory are usually described using mathematical symbols for the sake of clarity and consistency. Over the last three decades, different efforts have been made along this research line, such as the general design theory (Yoshikawa, 1981; Tomiyama and Yoshikawa, 1985), the axiomatic theory of design information (Salustri and Venter, 1992), the formal theory of design (Braha and Maimom, 1998), and the axiomatic theory of design modeling (Zeng, 2001).

Apart from the common objective of establishing design theories, the above two strategies differ from each other in that the process in the top-down approach to developing a design theory can be logically identified whereas the theory-building process for the bottom-up strategy is *ad hoc* and mainly attributed to the researcher’s talent and creativity. The extra information about this theory-building process inherited in the top-down strategy makes the theory more accountable. As a result, the top-down approach can be used in two ways. One is as a formal logical and scientific tool to derive formal design theories and computational design models. Another is use the derived design theories and models to organize designers’ activities and to implement various CAD systems. An example is the research conducted in Tomiyama’s group. Based on the General Design Theory (Yoshikawa, 1981; Tomiyama and Yoshikawa, 1985), which is indeed an axiomatic system, Tomiyama and his group have established a knowledge-intensive design theory (Tomiyama, 1994). Application design software systems have also been developed within this framework (Sekiya, 1998). Another example is the formal theory of design (Braha and Maimon, 1998) where the research results in the above two aspects are formally illustrated with set theory. The author's own research and development experience also involves the application in these two levels. Based on various versions of the axiomatic theory of design modeling, a formal model of design was derived to represent different types of design requirements uniformly as well as to manage the design process logically (Zeng, 2001; Zeng and Gu, 2001). This model will be concisely presented in Section 3 of this paper. Furthermore, this formal model has been implemented in a prototype software system for a mechanism design (Zeng, 2001). Finally, this model has been used to guide the author in the design of the system architecture for an on-going industry project "Intelligent Sketching System for Mechanical Design" (Pardasani *et al*, 2002). In addition to the above design-specific application, the attempt made in this paper could be interesting to mathematicians and philosophers from a broader scientific exploration point of view. Coincidently, a part of the mathematics established in this paper happens to be isomorphic to a branch of philosophy called Mereology, which has involved many renowned philosophers, mathematicians, and artificial intelligence researchers (Whitehead, 1929; Leonard and Goodman, 1940; Lesniewski, 1983; Varzi, 1996). In recent years, this is finding robust applications to Qualitative Spatial Reasoning (Cohn and Hazarika, 2001), which is a key technique in the domain such as CAD/CAM, robotics, and artificial intelligence. There is even one application in product modeling (Salustri and Lockledge, 1999).

However, despite its potential advantage, the top-down axiomatic approach has not yet been as popular and thus as fruitful as other approaches in the modeling of design. The main reasons for this are that studies through this approach generally take much longer to mature, and that its usefulness is usually not visible and tangible in the early stages of development. There are a few challenges facing this exploration. First, design looks to be a random, stylish, creative, and complex activity while the axiomatic approach has the nature of consistency, determinacy, and accuracy. These two subjects seem to oppose each other. Whether and how these two seemingly opposing subjects can be united is a question that cannot be avoided. Kryssanov *et al* (2001) addressed this question. Zeng (2001) partly answered this question through the nonlinear dynamic mechanism underlying the design process. Second, due to the complexity of the design activity, any initial endeavor to formalize it could be more complicated than necessary. This fact, plus the abstract nature of the axiomatic method, has made the theory based on this approach relatively difficult to be understood. Third, because of the logical nature underlying the axiomatic method, any flaw in this kind of exploration can be more easily detected. Fourth, since most of the audiences of this research are from engineering community, the usefulness of this research in engineering design has to be justified. Two things need to be verified before it can be claimed useful: the theorems of design must be proven useful in improving design practice; and these theorems are truly logically derivable from the axioms. As is shown in Fig. 1, the implementation of these two processes can be time-consuming. Fifth, because of its formal and metaphysical nature, mathematical parsimoniousness and philosophical insight is another requirement of this research. All of these factors, among others, have largely increased the difficulty of conducting this type of research.

This paper presents the author's recent attempt to understand and model design through the axiomatic approach. It includes two groups of axioms and five theorems. The axioms can be used by design theorists and/or computer scientists to derive models of different design aspects. The theorems can be used by designers to organize their design activities and/or by software developers to develop various CAD systems.

The two groups of axioms are axioms of objects and axioms of the human thought. Axioms of objects state that everything in the universe is an object, and that there are relations between objects. They provide the premise for studying objects in both nature and the human rational system. Axioms of the human thought identify the nature of the reasoning and recognition processes in the human rational system. They state that human beings are bounded in rationality (Simon, 1969, 1982), that human beings do not recognize objects accurately, and that causal relation is the only plausible relation among all relations between causes and effects.

Five theorems of design are logically derived based on the above two groups of axioms. These theorems cover the construction of an engineering system, the formulation of design requirements, and the model of the design process. They are:

1. Theorem 1. An engineering system is made up of the product structure, the environment structure, and the mutual relations between the product and the environment.
2. Theorem 2. In an engineering domain, a limited amount of performance knowledge about a limited number of primitive products exists to represent the causal relations from actions to responses.
3. Theorem 3. Design requirements can be divided into structural requirements and performance requirements. Structural requirements are constraints on the product structure while performance requirements are constraints on the product performance. These requirements can be decomposed in terms of the product environment in which the product is expected to work.
4. Theorem 4. A product's performances can be analyzed through performance knowledge by gradually separating each component from the other components.
5. Theorem 5. Given a collection of design requirements, design solutions can be found by decomposing the product environment implied in the definition of design requirements. Each step of environment decomposition engenders a partial design solution, which redefines the environment and in turn the design requirements. This process halts when all design requirements are satisfied.

The rest of this paper is organized as follows: Section 2 introduces the axioms. Section 3 focuses on the derivation of theorems of design from the axioms. Section 4 reviews related work. Section 5 concludes the paper and indicates the future directions of this research. These sections together provide a syntactic discussion of design modeling. Their semantics depend on more problem-specific context. Furthermore, only the framework of the whole research is presented in this paper. Detail explanation of theorems, case studies, and implementations are not included. Interested readers can refer to Zeng (2001). The major steps in the logic deduction demonstrate how the axioms are used to establish models of design. A diagram showing the relationships between major notions in this paper is given in Appendix A.

2. Axioms

An axiomatic system contains a set of primitive concepts and axioms. Axioms are also called postulates. Primitive concepts are usually informally described and left undefined. Axioms are self-evident truths that can be taken as the basis for inference. They often look simple and trivial. A typical example is Euclidean Geometry, which is given as an example in Appendix B.

Design is an activity to create products that can function in desired manners. It involves both nature and the human thought. Nature is where the products are supposed to function while the human thought is where the design ideas are generated. All objects appearing in the design process are called design objects, which include design requirements, design solutions, and design knowledge. These objects reside in nature and the human thought as well. The design process deals with the relations between these objects. Hence, the modeling of design depends on the assumptions underlying the nature of design objects and the design thought process. The following two groups of axioms address assumptions for this theory.

- Axioms of objects

1. Axiom 1. Everything in the universe is an object.
2. Axiom 2. There are relations between objects in the universe.

- Axioms of the human thought

1. Axiom 3. Human beings are bounded in rationality (Simon, 1969; 1982).

2. Axiom 4. Human beings do not recognize objects accurately.
3. Axiom 5. Causal relation is the only plausible relation in all relations between causes and effects.

To present this axiomatic system in a logically consistent way, the following mathematical symbols are used throughout this paper:

1. Predicate symbols: $\supseteq$(inclusion), = (identity), $\neq$ (inequality)
2. Operation symbols: $\cup$ (union), $\cap$ (intersection), $\otimes$ (relation), $\oplus$ (structure), Θ (range)
3. Logical symbols: $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction), $\forall$ (universal quantification: read as "for every"), $\exists$ (existential quantification: read as "there exists one"), $\exists!$(existential quantification: read as "there exists exactly one"), $\rightarrow$ (logical implication), $\leftrightarrow$ (if and only if), $\xleftrightarrow{\Delta}$ (defined by)
4. Auxiliary symbols: (,), [,], $\lfloor$, $\rfloor$

Logical symbols have the same meaning as they have in any branches of mathematics and logic (van Dalen, Doets, and de Swart, 1978). Auxiliary symbols are self-evident in the context. Predicate and operation symbols will be described or defined based on the axioms of objects.

2.1. Axioms of Objects

Axiom 1. Axiom of the Universal Object

Everything in the universe is an object.

In this axiom, universe and object are two primitive concepts. Informally, universe is the whole body of things and phenomena observed or postulated. An object is any element that can be observed or postulated in the universe. This axiom looks trivial and simplistic. However, it makes this theory different from set theory where concrete and abstract objects are distinguished by set and element. In our theory, the universe is the only abstract concept, which sets the boundary for the discourse of our discussion. Every other object is treated as the same in that it is an object in the universe. This brings convenience into the uniform representation of design objects in the evolutionary design process.

Axiom 2. Axiom of Object Relation

There are relations between objects in the universe. Symbolically,

$$A \otimes B, \forall A, B, \tag{1}$$

where A and B are objects. A$\otimes$B is read as "A relates to B". A relation of one object to itself is called the relation on the object itself.

In this axiom, relation is an aspect or quality that connects two or more objects as being or belonging or working together or as being of the same kind. Relation can also be a property that holds between an ordered pair of objects. This axiom has many implications. Obviously, different types of relations will lead to different concrete axiomatic systems. Corollary 2.1 defines an inclusion relation between two objects:

Corollary 2.1. Every object in the universe includes other objects. Symbolically,

$$A \supseteq B, \forall A \exists B. \tag{2}$$

B is called a subobject of A. The symbol $\supseteq$ is inclusion relation.

<u>Remark</u>: A special case of Corollary 2.1 is: $A \supseteq A, \forall A$.

Based on Corollary 2.1, we have following definitions

Definition 1. Identity (=)

An object A is identical to an object B if and only if every object included in A is also included in B and vice versa. Symbolically,

$$A = B \overset{\Delta}{\longleftrightarrow} \forall x[(x \subseteq A) \leftrightarrow (x \subseteq B)]. \quad (3)$$

Definition 2. Inequality (≠)

An object A does not equal to an object B if and only if at least one object included in A is not included in B or at least one object included in B is not included in A. Symbolically,

$$A \neq B \overset{\Delta}{\longleftrightarrow} \exists x \neg[(x \subseteq A) \leftrightarrow (x \subseteq B)]. \quad (4)$$

Definition 3. Union (∪)

An object A is the union of an object B and another object C if and only if every object included in A is either included in B or included in C and every object included in B or C is included in A. Symbolically,

$$A = B \cup C \overset{\Delta}{\longleftrightarrow} \forall x[(x \subseteq A) \leftrightarrow (x \subseteq B \vee x \subseteq C)]. \quad (5)$$

Equation (5) is read as "Object A is the union of objects B and C". However, "A" should be simply seen as the symbolic substitution of the expression "B∪C".

Definition 4. Intersection (∩)

An object A is the intersection of an object B and an object C if and only if every object included in A is included in both B and C. Symbolically,

$$A = B \cap C \overset{\Delta}{\longleftrightarrow} \forall x[(x \subseteq A) \leftrightarrow (x \subseteq B \wedge x \subseteq C)]. \quad (6)$$

Equation (6) is read as "Object A is the intersection of objects B and C".

Axiom 1, Corollary 2.1, and Definition 1 together constitute the basis of mereology (Varzi, 1996) where parthood is defined to capture the inclusion relation. Starting from these premises, part-whole relations can be established. Hence, all results from mereology can be applied to this research. However, because of its nominastic nature, mereology does not seem to be enough facility to support all important notions appearing in the design process, such as generalization and aggregation. The following corollary establishes the notion of abstraction with the mereological representation of objects.

Corollary 2.2. The structure of an object O, denoted by ⊕O, is the union of the object and the relation on the object itself. That is

$$\oplus O = O \cup (O \otimes O), \ \forall O. \quad (7)$$

This is indeed the logical result of Axioms 1, 2, and Corollary 2.1. According to Axiom 1, the structure of an object is also an object. The structure of object is illustrated in Fig. 2.

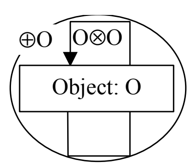

Fig. 2 Object structure.

Remark: According to Axiom 1, the relation between two objects is also an object. According to Axiom 2, relations always exist between subobjects of an object. Therefore, in decomposing an object in terms of its subobjects, the relations among these subobjects should always be attached (i.e. $A \cup B \to A \otimes B$). This implies that the whole is greater than the sum of the parts.

Please note that the structure operation is the notion that originally brought the author's research to the theory presented in this paper. Traditionally, the structure of an object has been represented as a tuple $\langle O, O \times O \rangle$ where $\times$ is the Cartesian product in set theory. To capture the hierarchical structure of object evolution in the design process, relations between subobjects are treated as objects (Zeng and Gu, 1999b, where object and subobject are called product and components, respectively). Zeng (2001) later proposed the structure operator ($\oplus O = O \cup O \times O$) to support this notion. But the study was still in the framework of set theory, where O and O×O are in different levels of abstraction. This treatment implies logical inconsistency, and thus cannot consistently lead through the whole derivation process of modeling design. To deal with this issue in a logically consistent manner, Axioms 1 and 2 were developed. This shift from the set-theoretic representation has made it natural and easy to model evolving hierarchical design objects in the design process. This notion will be used extensively in this paper. It was fascinating to find out that a part of mathematical theory developed based on the axioms of objects is completely isomorphic to mereology, a subject studying the foundation of mathematics (Lesniwski, 1983). The implication of our research in mereology will be treated in detail separately[i].

Corollary 2.3. The range of an object O, denoted by ΘO, is the intersection between the object O and the relation on the object O, i.e.

$$\Theta O = O \cap (O \otimes O). \qquad (8)$$

The range of the object can be a type, generalization, and classification of the object.

In this paper, the structure and range operations will only be used for the aggregation and generalization purposes. In this context, the following conditions will be assumed for the relation ⊗:

$$\begin{aligned} A \otimes (B \cup C) &= (A \otimes B) \cup (A \otimes C), \qquad (9)\\ (A \cup B) \otimes C &= (A \otimes C) \cup (B \otimes C), \\ A \otimes (B \cap C) &= (A \otimes B) \cap (A \otimes C), \\ (A \cap B) \otimes C &= (A \otimes C) \cap (B \otimes C). \end{aligned}$$

Remark: Any operation that does not satisfy the conditions specified by Equation (9) is not a valid relation for the aggregation and generalization.

The following are some properties of structure and range operations in representing aggregation and generalization:

1. Aggregation of objects

In the case of aggregation of objects, the relation satisfies the following condition:

$$(A \otimes B) = (C \otimes D) \leftrightarrow (A = C) \wedge (B = D). \qquad (10)$$

Examples of the object structure that applies this relation are Equations (20) and (36).

According to Corollary 2.1, any object includes other objects. Suppose that an object O includes m subobjects O_i (i=1,2,…,m).

$$O = \bigcup_{i=1}^{m} O_i, \qquad (11)$$

where m is a finite natural number.

According to Corollary 2.2, the structure of an object O, ⊕O, is expanded as

$$\oplus O = O \cup (O \otimes O) = \Big(\bigcup_{i=1}^{m} \oplus O_i\Big) \cup \Big(\bigcup_{i=1}^{m} \bigcup_{\substack{j=1 \\ j\neq i}}^{m} (O_i \otimes O_j)\Big). \tag{12}$$

The above equation renders the structure of an object recursive and hierarchical, as is shown in Fig. 3. In this hierarchical structure, $O(k,i_k,j_{k-1})$ represents the node at the i_k^{th} position in the k^{th} layer with a parent node at the $j_{(k-1)}^{th}$ position in the $(k-1)^{th}$ layer.

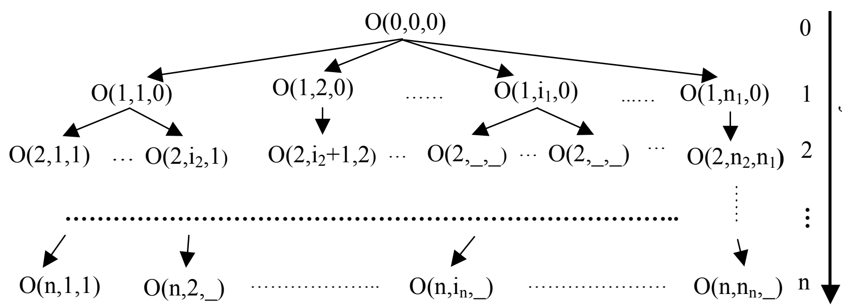

Fig. 3 Hierarchical object representation.

The aggregation of an object is then defined as

$$O[0] = O(0,0,0) = \oplus O, \tag{13}$$

$$O[k] = \oplus\Big(\bigcup_{i_k=1}^{n_k} O(k, i_k, j_{k-1})\Big), \forall k,$$

where k is the layer of object structuring and O[k] is the object structure in the k^{th} layer. Fig. 4 is an example of the object aggregation. The details of this representation are given in Zeng and Gu (1999b) and Zeng (2001).

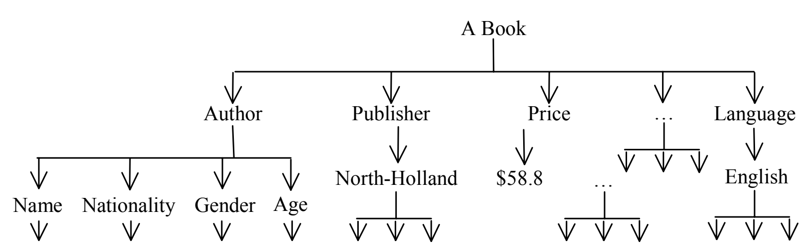

Fig. 4 Aggregation of objects.

2. Generalization of objects

In the case of generalization of objects, the relation satisfies the following condition:

$$(A \otimes B) = (B \otimes A). \tag{14}$$

Generalization comes from the common parts of a collection of objects. Suppose that an object O is the generalization of m objects O_i ($i=1,2,\ldots,m$).

$$O = \bigcap_{i=1}^{m} O_i, \tag{15}$$

where m is a finite natural number.

According to Corollary 2.3, the generalization of an object O, ΘO, is expanded as

$$\Theta O = O \cap (O \otimes O) = (\bigcap_{i=1}^{m} \Theta O_i) \cap (\bigcap_{i=1}^{m} \bigcap_{\substack{j=1 \\ j \neq i}}^{m} (O_i \otimes O_j)). \tag{16}$$

The above equation renders the generalization of an object also recursive and hierarchical, as is shown in Fig. 3.

The generalization of an object is then defined as

$$\begin{aligned} O\lfloor 0 \rfloor &= O(0,0,0) = \Theta O, \\ O\lfloor k \rfloor &= \Theta(\bigcup_{i_k=1}^{n_k} O(k, i_k, j_{k-1})), \forall k, \end{aligned} \tag{17}$$

where k is the layer of object generalization and $O\lfloor k \rfloor$ is the object generalization in the k^{th} layer. Fig. 5 is an example of the object generalization.

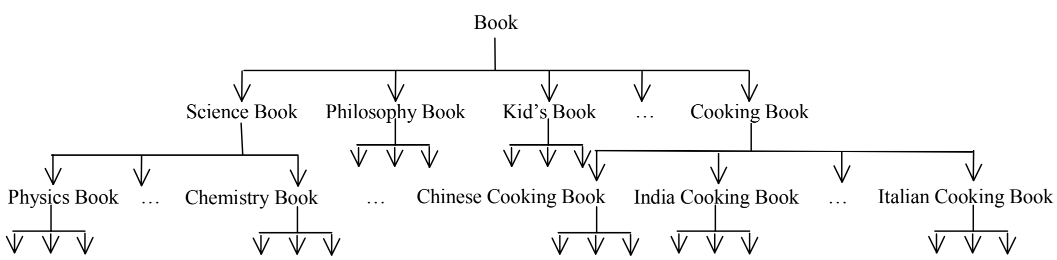

Fig. 5 Generalization of objects.

The type, classification, and specification of objects are embedded in the generalization while the decomposition is implied in the aggregation. These six essential ordering schemes (aggregation, decomposition, generalization, type, classification, and specification) in the design process are represented uniformly. This provides a formal approach to representing and studying the hierarchical system proposed by Simon (1969). The details of these mechanisms are discussed separately[i].

3. Relations between operations:

The following operation rules hold:

$$\begin{aligned} \Theta(A \cup B) &= (\Theta A) \cup (\Theta B), \\ \Theta(A \otimes B) &= (\Theta A) \otimes (\Theta B), \\ \Theta(\oplus A) &= \oplus(\Theta A). \end{aligned} \tag{18}$$

The above equation will only be used in the formulation of design requirements. It can be derived by defining a null object and the difference between union and structure operations. Since it is not closely related to the discussion of this paper, it will be discussed elsewhere.

The above discussion has given all required definitions and mathematical symbols needed for this paper. Other notions, such as null objects, difference and complement, are presented elsewhere[i].

2.2. Axioms of the Human Thought

Different from traditional sciences dealing with understanding nature, design studies attempt to investigate nature and the designer's thought at the same time. By thought we mean the sum of information that human beings can think of and reason with. In this paper, we will not specify what is the carrier of the thought as this has been one of the oldest problems in the philosophy of mind since Descartes. As such, a new object, world, is defined.

Definition 5 World

The world is an object in the universe, which is made up of two objects: nature and the human thought. The world, nature, and the human thought are denoted by W, N and M, respectively, i.e.

$$W = N \cup M. \tag{19}$$

By applying structure operation ⊕ defined in Equation (7) to the object W in Equation (19), the structure of the world, ⊕W, is

$$\begin{aligned} \oplus W &= \oplus(N \cup M) \\ &= (N \cup (N \otimes N)) \cup (M \cup (M \otimes M)) \cup (N \otimes M) \cup (M \otimes N) \\ &= (\oplus N) \cup (\oplus M) \cup (N \otimes M) \cup (M \otimes N). \end{aligned} \tag{20}$$

Alternatively, Equation (20) can be illustrated in Fig. 6a). Fig. 6b) is its representation in terms of Fig. 2, which illustrates the recursive nature of object structure more symbolically.

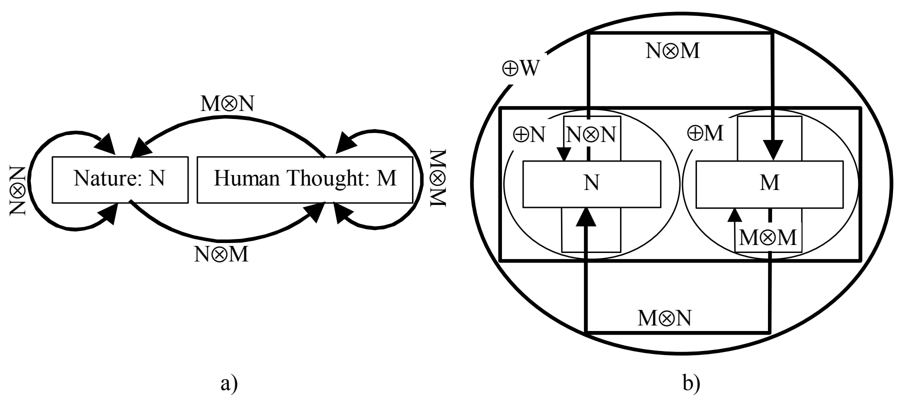

Fig. 6 Structure of the world.

Corresponding to Equation (20) and Fig. 6, we can define four objects: the natural system, the human rational system, recognition, and action.

Definition 6 Natural system

The natural system is the structure of nature. Symbolically,

$$\oplus N = N \cup (N \otimes N). \tag{21}$$

Natural law L is a subobject of the relation N⊗N, and

$$L \subseteq N \otimes N. \tag{22}$$

Definition 7 Human rational system

The human rational system is the structure of the human thought. Symbolically,

$$\oplus M = M \cup (M \otimes M). \tag{23}$$

Knowledge K is a subobject of the relation M⊗M, and

$$K \subseteq M \otimes M. \tag{24}$$

Definition 8 Recognition

Recognition is a relation f^R from nature N to the human thought M.

$$f^R \subseteq N \otimes M. \tag{25}$$

Definition 9 Action

Action is a relation f^A from the human thought M to nature N, which influences nature N by implementing a plan or a design produced by the human thought M. Symbolically,

$$f^A \subseteq M \otimes N. \tag{26}$$

Design is an activity happening in the human rational system. The results have to be sent back to the natural system through action. Once a design is materialized in nature, it has to follow natural laws. Whether and how a design can survive in nature depends on the answers to the following questions:

1. What is the character of nature?
2. What is the character of recognition?
3. What is the character of the human rational system?

The answer to the first question can be philosophical and theological, for it depends on how human beings understand the world. Any answer to this question would only be relatively true with respect to the answers to the last two questions, since it is itself a result from human recognition and reasoning (Knight, 1999). In accordance to human commonsense, three axioms are developed to address the last two questions.

First, according to Corollary 2.2, the structure of an object depends on what objects are included in the object. As we can see in Fig. 6 and Definition 8, the goal of the recognition process is to define the structure of an object in nature N with the object(s) in the human thought M. If the human thought have a one-to-one correspondence to nature, then they would have perfect knowledge. This is the axiom of correspondence in Yoshikawa's General Design Theory (Yoshikawa, 1981). But it is against the human commonsense. The axioms of bounded rationality and recognition address this character.

Axiom 3. Axiom of Bounded Rationality

Human beings are bounded in rationality.

This axiom is adopted from Simon (1969, 1982). In this axiom, rationality is the quality or state of being having reason or understanding. The main manifestation of this axiom is the limitation of resources (e.g., time and memory) in the rational system. A direct result is the limited number of objects in the human thought M. That is

$$M = \bigcup_{i=1}^{n} O_i^a, \tag{27}$$

where i and n are finite natural numbers. Each object O_i^a is called a primitive object.

Substituting Equation (24) into Equation (27),

$$K \subseteq (\bigcup_{i=1}^{n} O_i^a) \otimes (\bigcup_{i=1}^{n} O_i^a). \tag{28}$$

Since n is a finite natural number, knowledge K is limited. This means that the amount of human knowledge is limited.

This axiom is represented in Fig. 7. The choosing of primitive objects is artificial. It can be observable and/or measurable properties, or even objects with complex structure. One of the tasks of scientific research is to look for the minimum number of primitive objects to describe various natural phenomena.

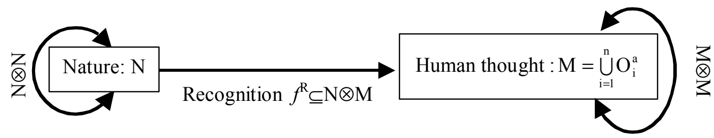

Fig. 7 Human recognition process.

Another issue in the human recognition process is the nature of recognition. This is addressed by the following axiom:

Axiom 4. Axiom of Recognition

Human beings do not recognize objects accurately. That is,

$$(O' = f^R(O)) \wedge (O' \neq O), \forall O \subseteq N \; \exists O' \subseteq M. \tag{29}$$

Though this axiom will not be used in the rest of this paper, its application is addressed in Zeng and Gu (2002a)[ii].

Second, let us consider a special case where an object O in the human rational system consists of three subobjects: cause O^c, object O^a, and effect O^e, which is shown in Fig. 8.

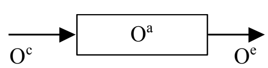

Fig. 8 Object consisting of three subobjects.

This object is symbolically represented as

$$O = O^c \cup O^a \cup O^e, \text{for } O, O^c, O^a, O^e \subseteq M. \tag{30}$$

Since the formal definitions of cause and effect are rather philosophical (Menzies, 2002), we will not identify their semantic differences in this paper. This treatment does not affect the present discussion. In terms of Equation (28), knowledge K for this object can be represented as

$$\begin{aligned} K \subseteq O \otimes O &= (O^c \cup O^a \cup O^e) \otimes (O^c \cup O^a \cup O^e) \\ &= (O^c \otimes O^c) \cup (O^a \otimes O^a) \cup (O^e \otimes O^e) \cup \\ &\quad (O^c \otimes O^a) \cup (O^a \otimes O^e) \cup (O^c \otimes O^e) \cup \\ &\quad (O^e \otimes O^a) \cup (O^a \otimes O^c) \cup (O^e \otimes O^c). \end{aligned} \tag{31}$$

The relations in Equation (31) can be divided into three groups:

1. $O^c \otimes O^c, O^a \otimes O^a, O^e \otimes O^e$.
2. $(O^c \otimes O^a) \cup (O^a \otimes O^e) \cup (O^c \otimes O^e)$.

3. $(\mathrm{O}^e \otimes \mathrm{O}^a) \cup (\mathrm{O}^a \otimes \mathrm{O}^c) \cup (\mathrm{O}^e \otimes \mathrm{O}^c)$.

The three relations in the first group include knowledge about each object itself. The relations in the second and third groups lead to the inductive knowledge regarding the object O^a: $\mathrm{O}^c \otimes \mathrm{O}^e$ or $\mathrm{O}^e \otimes \mathrm{O}^c$. As will be shown in Equations (38) and (42), O^c and O^e can be defined with respect to O^a.

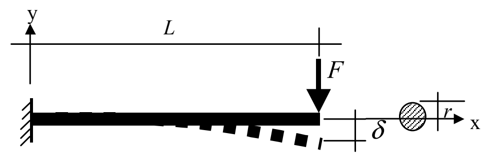

Fig. 9 Knowledge about the cantilever beam.

Taking the cantilever beam in Fig. 9 as an example, we have the next three objects: $\mathrm{O}^c = F$, $\mathrm{O}^a = beam$, and $\mathrm{O}^e = \delta$. Correspondingly, examples of possible knowledge include:

1. the knowledge about the section area A ($\mathrm{O}^c \otimes \mathrm{O}^c, \mathrm{O}^a \otimes \mathrm{O}^a, \mathrm{O}^e \otimes \mathrm{O}^e$):

$$A = \pi r^2. \tag{32}$$

2. knowledge about the displacement under the force ($\mathrm{O}^c \otimes \mathrm{O}^e$):

$$EI\frac{d^2 y}{dx^2} = M(x) = Fx, \quad \delta = y|_{x=L}. \tag{33}$$

According to Equation (33), the displacement in any point of the beam is solely determined if a force F is given.

3. knowledge about the forces causing the displacement ($\mathrm{O}^e \otimes \mathrm{O}^c$):

In this case, the displacement is not enough information to determine the type, position, direction, and quantity of the forces that triggered the displacement. A force such as that in Fig. 10 may also produce the same amount of displacement δ. That is, we cannot determine causes from effect.

To address the difference between $\mathrm{O}^c \otimes \mathrm{O}^e$ and $\mathrm{O}^e \otimes \mathrm{O}^c$, the causal relation is defined in the following manner:

Definition 10 Causal relation

The causal relation is the relation from cause to effect in the human rational system.

Axiom 5. Axiom of Causality

The causal relation is the only plausible relation in all relations between causes and effects.

$$\mathrm{e_e} = \mathrm{C^r}(\mathrm{e_c}), \forall \mathrm{e_c} \subseteq \mathrm{O^c} \subseteq \mathrm{M}, \ \exists! \mathrm{e_e} \subseteq \mathrm{O^e} \subseteq \mathrm{M}, \exists \mathrm{C^r} \subseteq \mathrm{O^c} \otimes \mathrm{O^e}, \tag{34}$$

where $\mathrm{C^r}$ is a causal relation between causes O^c and effects O^e.

<u>Remark</u>: This axiom determines that human reasoning is not symmetric. Only deductive reasoning is deterministic and all other reasoning modes are implausible.

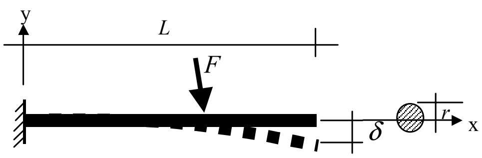

Fig. 10 Displacement alone does not determine force for a cantilever beam.

Basically, the above three axioms describe the nature of the human rational system, which determines the characters of intelligent activities. Up to this point, two major notions in Simon's distinguished work (1969): bounded rationality and hierarchical systems, have been logically embodied in one formal framework. The next section will apply these axioms to derive theorems about engineering design. This derivation process can be used to verify the axioms. The axiomatic approach to studying design will be shown to be feasible and useful.

3. Theorems of Design

In this section, we are going to derive theorems about engineering design from the axioms given in Section 2. These theorems indeed constitute a formal model of design, including the engineering system, design requirements, and the design process. They cover the core of an engineering design problem, which is shown in Fig. 11.

Fig. 11 Design problem.

3.1. Engineering System

3.1.1. Engineering System

Definition 11 Product

A product is an object in nature created by human beings.

Definition 12 Environment

The environment of a product is the collection of all objects in nature excluding the product.

Definition 13 Engineering system

Nature can be seen as the union of a product (S) and its environment (E). Under this context, the structure of nature is called the engineering system, denoted by $\oplus\Omega$. That is,

$$\oplus\,\Omega = \oplus(E \cup S). \tag{35}$$

Theorem 1. An engineering system is made up of the product structure, the environment structure, and the mutual relations between the product and the environment.

[Proof] Applying Equation (12), the engineering system in Equation (35) can be represented as

$$\oplus\,\Omega = (\oplus E) \cup (\oplus S) \cup (E \otimes S) \cup (S \otimes E). \tag{36}$$

Equation (36) includes four components: environment structure ⊕E, product structure ⊕S, the relation from the environment to the product E⊗S, and the relation from the product to the environment S⊗E. This proves the theorem.

The engineering system is shown in Fig. 12. It is fascinating to see that this diagram is isomorphic to Fig. 6, which is the structure of the world. At this point, the author cannot help praise the order, beauty, and uniformity of nature.

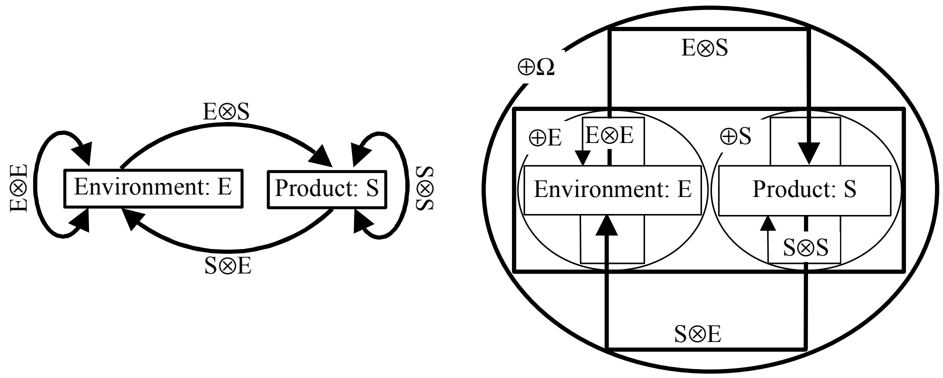

Fig. 12 Engineering system.

Revisiting the cantilever example from Fig. 9, the cantilever-environment system is illustrated in Fig. 13.

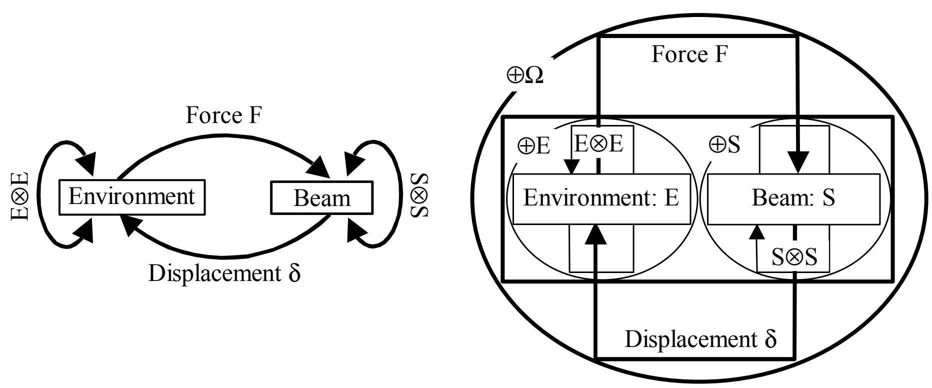

Fig. 13 Cantilever-environment system.

Since product, environment as well as engineering system are all objects in nature, they should assume the same form. Equations (1) to (18) apply to all three objects. According to Equation (13),

$$\Omega[n] = E[n] \cup S[n] \cup E \otimes S \cup S \otimes E. \quad (37)$$

Next subsection will study the relations between the product and the environment in Equation (36).

3.1.2. Product Performance

Fig. 12 can be simplified as in Fig. 14, which defines product performance.

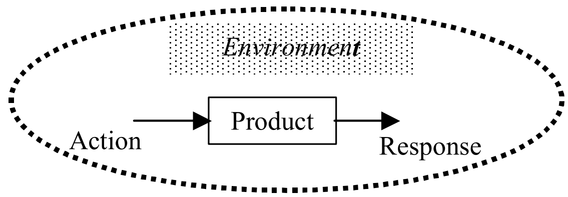

Fig. 14 Product performance in environment.

Definition 14 Product performance

Product performances, denoted by P, are relations from actions (A) to responses (R). Actions are relations from the environment (E) to the product (S). Responses are relations from the product (S) to the environment (E).

$$P \subseteq A \otimes R, \quad (38)$$
$$A \subseteq E \otimes S,$$
$$R \subseteq S \otimes E.$$

According to Corollary 2.1, product and environment may consist of multiple subobjects. Suppose that they have n and m subobjects, respectively. We may have n×m potential actions and responses:

$$A \subseteq (\bigcup_{k=1}^{n} E_l) \otimes (\bigcup_{l=1}^{m} S_k), \quad (39)$$
$$R \subseteq (\bigcup_{k=1}^{n} S_k) \otimes (\bigcup_{l=1}^{m} E_l).$$

Totally, there are $(n\times m)^2$ possible product performances.

Remark: In Equation (39), the environment and the product are decomposed into subobjects. This can be done more formally based on algebra.

Using again the example of the cantilever beam in Fig. 9, action A may include force, ambient temperature, vibration of the left supporting end and so on, while response R may embody the displacement, stress, and strain at any point in the beam. Hence, we may have performances such as:

- Strain under ambient temperature
- Strain under force
- Strain under end vibration
- Stress under ambient temperature
- Stress under force
- Stress under end vibration
- Displacement under ambient temperature
- Displacement under force
- Displacement under end vibration

Therefore, product performance can be investigated from two perspectives:

1. The focus is on every single object included in each performance relation, i.e.,

$$p_i = a \otimes r \subseteq P, \forall a_i \subseteq A, \exists r \subseteq R. \quad (40)$$

2. The focus is on the form of the relation from action to performance, i.e.,

$$\forall S_i \subseteq S, \exists A, \exists R\, [\forall A_i \subseteq A, \exists R_i \subseteq R, \exists L_i \subseteq L, L_i \subseteq A_i \otimes R_i], \quad (41)$$

where L is the natural law defined in Equation (22). Through the human recognition process, L becomes a set of performance knowledge. Obviously, product performance is an object defined by Equation (30). Action, product, and response correspond to cause, object, and effect, respectively. According to Equation (34), performance knowledge can be represented as

$$K \subseteq A \otimes R. \quad (42)$$

Here, R, K, A correspond to O^e, C^r, and O^c, respectively.

By comparing Equations (38) and (42), it can be noted that product (S) is implied in the representation of performance knowledge in Equation (42). The following equation makes this item explicit,

$$K \subseteq A \otimes R, \forall S, \exists K. \quad (43)$$

From Axiom 4, it is logical to state that there exists a set of n_a primitive products (S^a) in a design domain

$$S^a = \bigcup_{i=1}^{n_a} S_i^a. \quad (44)$$

Remark: the number of primitive products is not fixed, though limited. New primitive product can always be added when the need arises.

Equations (43) and (44) together imply the following theorem:

Theorem 2. In an engineering domain, a limited amount of performance knowledge about a limited number of primitive products exists to represent the causal relations from actions to responses.

$$\forall S_i^a \subseteq S^a [\forall A_j \subseteq A\, \exists R_j \subseteq R, \exists K_{ij} \subseteq A_j \otimes R_j]. \quad (45)$$

3.2. Design Requirements

Definition 15 Constraint

A constraint, denoted by c, defines a relation between an object, denoted by x, and a range of this object, Θx, within which the object can change. Symbolically,

$$c = \lambda(x, \Theta x), \quad (46)$$

where λ is a relation from x to Θx. Range Θx is defined in Equation (8).

Definition 16 Design requirements

Design requirements, denoted by R^d, are constraints on an engineering system to be designed. Symbolically,

$$R^d \subseteq \lambda(\oplus\Omega, \Theta(\oplus\Omega)). \quad (47)$$

Since an engineering system is the aggregation of a product and its environment, Equation (18) applies. According to Equation (36), we have

$$\oplus\Omega = (\oplus E)\cup(\oplus S)\cup A\cup R = (\oplus E)\cup(\oplus S)\cup B, \tag{48}$$
$$\Theta(\oplus\Omega) = (\oplus(\Theta E))\cup(\oplus(\Theta S))\cup\Theta(A\cup R) = (\oplus(\Theta E))\cup(\oplus(\Theta S))\cup\Theta B.$$

where B=A∪R.

Substituting Equation (48) into Equation (47) and using the definition of constraint (only the relations from one object to its own range should be kept in the definition of design requirements), Equation (47) can be written as

$$R^d \subseteq \lambda(\oplus E,\oplus(\Theta E))\cup\lambda(\oplus S,\oplus(\Theta S))\cup\lambda(B,\Theta B). \tag{49}$$

Since the environment is predefined in a design problem, i.e., $E=\Theta E$, the first item in Equation (49) can be eliminated.

$$R^d \subseteq \lambda(\oplus S,\oplus(\Theta S))\cup\lambda(B,\Theta B). \tag{50}$$

From Equation (50), design requirements can be divided into structural and performance requirements corresponding to the objects to be constrained (⊕S and B). For the design of the cantilever beam in Fig. 9, an example of a structural requirement is $2m \le L \le 3m$ while that of a performance requirement is $\delta \le 5cm$.

Moreover, substituting Equations (39) and (48) into Equation (50), then applying the definition of constraint, we have

$$R^d \subseteq \lambda(\oplus S,\oplus(\Theta S))\cup(\bigcup_{i=1}^{n}\lambda(B_i,\Theta B_i)). \tag{51}$$

where $B_i=(S\otimes E_i)\cup(E_i\otimes S)$. This means that design requirements can be decomposed and classified in terms of the product environment.

The above derivation leads to the following theorem

Theorem 3. Design requirements can be divided into structural requirements and performance requirements. Structural requirements are constraints on the product structure while performance requirements are constraints on the product performance. These requirements can be decomposed in terms of the product environment in which the product is expected to work.

The details of this theorem are given in another paper[iii]. The theorem provides a logical way for decomposing a complex design problem.

3.3. Design Process

3.3.1. Design Evaluation

Definition 17 Design evaluation

Design evaluation is a process used to evaluate a product and its performances against prescribed design requirements.

It is straightforward to evaluate structural requirements. However, since human beings only have a limited amount of performance knowledge about a limited number of primitive products, a product's performance may not be able to be directly obtained for evaluation if it is not one of the primitive products. The following derives a model establishing the performances of any product based on primitive products.

Without the loss of generality, the product S can be divided into two parts: S_i and S_i^a where S_i^a is a primitive product defined in Equation (44). Hence,

$$S = S_i \cup S_i^a, \exists S_i^a \subseteq S^a. \tag{52}$$

Remark: if there is no such S_i^a in S^a, then a new primitive product needs to be added. This is one of the tasks of scientific investigation.

Substituting the above equation into Equation (36), we get

$$\begin{aligned} \oplus\,\Omega &= \oplus(E \cup S) = \oplus\,(E \cup S_i \cup S_i^a) \\ &= (\oplus E_i) \cup (\oplus S_i) \cup A_i \cup R_i, \end{aligned} \tag{53}$$

where $E_i = E \cup S_i^a, A_i = (E_i \otimes S_i), R_i = (S_i \otimes E_i)$. By applying the performance knowledge in Equation (45), the product performance, which was the hidden interactions between the extracted component and the left product structure, can then be specified using A_i and R_i. This process is repeated until S_i becomes empty. The union of all intermediate A_i and R_i embodies all performances within a product in a given environment. Therefore, by moving each component of a product into its environment, the performances of a product can be studied. This approach has been widely used in engineering analysis. The free body diagram is one example; finite element analysis is another. The above process leads to the following theorem:

Theorem 4. A product's performances can be analyzed through performance knowledge by gradually separating each component from the other components.

3.3.2. Design Synthesis

From the definition of design requirement, we can define the design synthesis process as below.

Definition 18 Design synthesis

Design synthesis, denoted by d_s, is a process that generates products according to prescribed design requirements. The process can be represented as

$$d_s \subseteq \Theta(\oplus\Omega) \otimes (\oplus\Omega). \tag{54}$$

The initial state of this process is given by $\oplus(\Theta\,\Omega)$, which is represented as

$$\Theta(\oplus\Omega) = \oplus(E \cup \Theta\, S). \tag{55}$$

As can be seen from Equation (48), Equation (55) includes all ranges defining design requirements. In this equation, only the environment E is well-defined before the design problem is solved. To complete the transition from $\Theta(\oplus\Omega)$ to $\oplus\,\Omega$, the initial environment can be divided into two parts: $E'(1)$ and $e(1)$, both of which are still environment. Any intermediate environment $E(i)$ can be divided into: $E'(i+1)$ and $e(i+1)$. That is

$$\begin{aligned} E(0) &= E = E'(1) \cup e(1), \\ E(i) &= E'(i+1) \cup e(i+1). \end{aligned} \tag{56}$$

Substituting Equation (56) into Equation (55), we have

$$\begin{aligned} \Theta(\oplus\Omega) &= \oplus(E'(i) \cup e(i) \cup (\Theta\, S)) \\ &= \oplus(E'(i) \cup (\Theta\, S)) \cup (\oplus(e(i) \cup (\Theta\, S))) \cup (E'(i) \otimes e(i)) \cup (e(i) \otimes E'(i)). \end{aligned} \tag{57}$$

This decomposition of the environment has indeed divided the range of original design requirements into two parts: $\oplus(E'(i) \cup \Theta\, S)$ and $\oplus(e(i) \cup \Theta\, S)$. Suppose that a partial product s(i) can be obtained from $\oplus(e(i) \cup \Theta S)$, based on performance knowledge in Equation (45). Then $\Theta(\oplus\Omega)$ can be updated as

$$\Theta(\oplus\Omega) = \oplus(E'(i) \cup s(i) \cup \Theta S). \tag{58}$$

Therefore,

$$\Theta(\oplus\Omega) = \oplus(E(i) \cup \Theta S), E(i) = E'(i) \cup s(i). \tag{59}$$

The above process continues until $E(i) = E'(i+1)$. This means that all design requirements are satisfied. E(i) embodies the design solutions.

Remark: The transition from Equation (57) to Equation (58) is indeed constrained by Axiom 5. Multiple s(i) can be generated in this process. The logic is not deductive any more (Zeng and Cheng, 1991). This process is described in Zeng and Gu (1999a), Zeng and Gu (2001), and Zeng (2001). Further formulation is in progress[iv].

The above derivation leads to the following theorem:

Theorem 5. Given a collection of design requirements, design solutions can be found by decomposing the environment implied in the design requirements. Each step of environment decomposition engenders a partial design solution, which redefines the environment and in turn the design requirements. This process halts when all design requirements are satisfied.

3.4. Summary

The five theorems derived in this section constitute a formal model of design, which embodies the major parts of a design problem: the design requirements, the product structure, the product performance, the evaluation process, and the synthesis process. In deriving these five theorems, the definition of each problem is firstly formulated using the notions and symbols introduced in Section 2. The whole derivation processes are only based on the axioms or proven theorems. This shows that the axiomatic approach can be used to develop models of design.

The details of the five theorems in this section are discussed in separate papers with refined derivation steps, case studies and/or the computer implementation. A diagram showing the interrelationships between main notions in this paper is given in Appendix A.

4. Related Work

This section attempts to identify the evolution process of our theory and its relation to similar endeavors. To the author's knowledge, there are three groups of work that are directly comparable to the research presented in this paper. They are the general design theory (Yoshikawa, 1981; Tomiyama and Yoshikawa, 1985), the formal design theory (Braha and Maimon, 1998), and the axiomatic theory of engineering design information (Salustri and Venter, 1992). Suh's well-known axiomatic design theory (1990) is on a different level than the one introduced in this paper.

In his pioneering study of axiomatic design modeling, Yoshikawa (1981) established the General Design Theory (abbreviated as GDT). The basis of the GDT is a human recognition model. In this model, the GDT makes a distinction between an entity and an entity concept. An entity is a concrete existing object, and an entity concept is its abstract, mental impression conceived by a human being. In the form of Fig. 6, entity is the object in nature while entity concept is the object in the human thought. This is illustrated in Fig. 15. Designers create another entity concept by using entity concepts previously shaped through the recognition process. Based on this recognition model, Yoshikawa established three axioms for the general design theory:

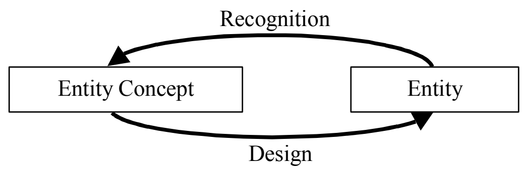

Fig. 15 Yoshikawa's model of the world.

Axiom 1. (Axiom of recognition) Any entity can be recognized or described by the attributes.

Axiom 2. (Axiom of correspondence) The entity set and the entity-concept set have one-to-one correspondence.

Axiom 3. (Axiom of operation) The abstract-concept set is a topology of the entity-concept set.

According to Tomiyama (1994), Axiom 1 and Axiom 2 basically say that any object in nature can be perfectly recognized by the human thought. Axiom 3 signifies that it is possible to logically operate abstract concepts as if they were just ordinary mathematical sets. Therefore, set theory is their basic mathematical tool. Set theory, together with the set of attributes introduced in Axiom 1, provides the language for representing any entity and entity concept in their theory.

Based on Axiom 2, Tomiyama and Yoshikawa (1985) introduced the ideal knowledge that knows all the elements of the entity set perfectly and that can describe each element with abstract concepts without ambiguity. Because of this, the attributes defining the design solution are immediately obtained after the specifications are described using attributes. In recognizing that human recognition is imperfect and therefore the case assumed by Axiom 2 does not happen in general, they developed Extended General Design Theory. In the extended general design theory, they indicated that the situation in ideal knowledge does not apply to real design. They considered design as the process in which the designer builds the goal and tries to satisfy specifications without violating physical constraints. To formalize this process, the model of real knowledge was established. They first defined a physical law as a description of the relationships among physical quantities of entities and the field. This corresponds to the relation $N \otimes N$ in Equation (22) in this paper. The concept of physical laws is one of the abstract concepts formed when one looks at a physical phenomenon as a manifestation of physical laws. This corresponds to the relation $M \otimes M$ in Equation (24) in this paper. Based on the hypothesis that finite subcoverings exist for any coverings of the set of feasible entity concepts made of sets chosen from the set of physical law concepts, they claimed that they can prove finiteness or boundedness of our knowledge. This corresponds to the Axiom 4 in our theory. With these considerations, they formalized design process as a metamodel evolution process. This model indicates that in real design, design is a stepwise transformation process from the function space to the attribute space via the metamodel space. Here, metamodel space is physical phenomenon space, corresponding to the performance knowledge we defined in Equation (42).

Along the similar research line, Braha and Maimon (1998) established a formal design theory (abbreviated as FDT) based on the science of the artificial proposed by Simon(1969). There are five postulates in their theory. The first two is about the representation of product while the last three address the nature of the designer's knowledge.

Postulate 1 (Entity-Relational Knowledge Representation): An artifact representation is built upon the multiplicity of modules (attributes) and relationships among them.

Postulate 2 (Nested Hierarchical Representation): The design of any complex system can be considered at various levels. The general direction of design is from more abstract to less. A design at any level of abstraction is a description of an organized collection of constraints (such as various structural, cause-effect, functional, and performance features) that appear in the physically implemented design.

These two premises lead directly to the formulation of the attribute space as an algebraic structure. The product is represented by the pair <*M*, *C*> where *M* stands for the set of modules that the product is comprised of while *C* denotes the set of relations that represent the relationships among the modules. In order to capture the essence of design, a hierarchical construction of systems from subsystems is also developed. Consequently, the general set of modules is classified into basic and complex modules. Basic modules represent entities that can not be defined in terms of others. This corresponds to primitive objects in our theory. Complex modules are defined hierarchically in terms of other modules, where the effects of their interaction are expressed. This is essentially the structure of the general system established by Yang (1989). Compared to the GDT, these two postulates play the role of Axiom 3, but with a more sophisticated definition of object structure. This object structure has made them able to investigate design in a deeper and more comprehensive way.

Postulate 3 (Incompleteness): Any knowledge representation (as represented by the designer) is incomplete.

Postulate 4 (Bounded rationality): The designer can consider only a subset of knowledge representations at any instant of decision making.

Postulate 5 (Non-Determinism): Several feasible designs can be generated to the level specified by the designer.

By applying these three postulates informally, idealized and real design process models are developed in the FDT. The algebraic structure they established based on the first two postulates is used to support the object representation in the design process.

The author's endeavor in this research started after the logic of design was proposed to classify the reasoning processes in the human thought (Zeng and Cheng, 1991). A formal design process model was developed through formalizing the objects appearing in the logic of design (Zeng, *et al*, 1996). In addition to the logic of design, another base of this formal design process model is a design object model (Zeng, *et al*, 1996). This design object model represents a product as a tuple ($<S^v, S^c>$) of its components (S^v) and the relation (S^c) between the components. This is equivalent of the algebraic structure <M, C> in the FDT. To support the evolving design process, the authors had to introduce two operators based on empirical understanding of the design process. They are abstraction and decomposition operators, which aim to address the layers of object representation in Fig. 3. In recognizing the difficulty of naturally embodying the evolutionary nature of design objects in the design process, Zeng and Gu (1999b) established a new representation scheme by defining the structure of a product as the union of S^v and S^c, which is $S^v \cup S^c$. An implied assumption of this change is that the relation S^c is also a component. This seemingly minor change from the existing structure ($<S^v, S^c>$ or <M, C>) has led to establish the foundation of the work developed in this paper. Motivated by the axiomatic approach underlying the research in the GDT and FDT, Zeng (2001) developed an axiomatic design modeling theory based on the formal model of design (Zeng and Gu, 1999ab). The structure operation $\oplus$ and range operation Θ are proposed to capture the hierarchical structure of product representation. This is a major contribution of this work. It is the foundation of all derived design theorems presented in this paper. To support the argument that the relation S^c is also a component so that structure operation can be logically reliable, two axioms of objects are developed to replace the set-theory based object representation. These axioms have provided a uniform, consistent, and simple way to capture the aggregation, decomposition, generalization, classification, type, and other natures of object structure, which are widely seen in the design process. Based on these two axioms, the structure of the world is logically derived in terms of nature and the human thought. This is another unique character of this research. It provides a context to define axioms of the human thought, which address the nature of the human thought and the relations from nature to the thought as well as on the thought itself. These two groups of axioms have been used to study different aspects of design. As such, two central notions in the Science of the Artificial (Simon, 1969): bounded rationality and hierarchical systems, are formulated in a uniform logical formal framework.

It is interesting to see that axioms in the above three theories can all be divided into two categories: axioms about the representation of objects and axioms about the human thought. Table 1 illustrates the evolving process from the GDT to the FDT to our theory. This is indeed a process of looking for axioms to describe design activities more perspicuously, consistently, and robustly.

Table 1 Evolution of the axiomatic theory of design modelling.

	GDT	FDT	Our Theory
Axiom of Object Representation	**Axiom 3.** The abstract-concept set is a topology of the entity-concept set.	**Postulate 1.** An artifact representation is built upon the multiplicity of modules (attributes) and relationships among them. **Postulate 2.** The design of any complex system can be considered at various levels. The general direction of design is from more abstract to less. A design at any level of abstraction is a description of an organized collection of constraints (such as various structural, cause-effect, functional, and performance features) that appear in the physically implemented design.	**Axiom 1.** Everything in the universe is an object. **Axiom 2.** There are relations between objects in the universe.
Axiom of the Human Thought	**Axiom 1.** Any entity can be recognized or described by the attributes. **Axiom 2.** The entity set and the entity-concept set have one-to-one correspondence.	**Postulate 3.** Any knowledge representation (as represented by the designer) is incomplete. **Postulate 4.** The designer can consider only a subset of knowledge representations at any instant of decision making. **Postulate 5.** Several feasible designs can be generated to the level specified by the designer.	**Axiom 3** Human beings are bounded in rationality. **Axiom 4.** Human beings do not recognize objects accurately. **Axiom 5.** The causal relation is the only plausible relation in all relations between causes and effects.

Instead of investigating design process and design objects as a whole, Salustri and Venter (1992) attempted to develop a design information theory based on axiomatic set theory. Their work is a subset of axiomatic set theory by adding constraints from engineering design. This work established a well formulated theory to represent design information. They developed six main concepts: attribute, object, set of all attributes, set of all objects, domains and ranges of attributes, and view of object from the axiomatic set theory point of view. Not all of these concepts are the sets at the same logic level. Examples include attribute and set of all attributes. The latter is one level higher in the axiomatic set theory. Therefore, some set operations, such as union and intersection, do not always apply. In addition to these six concepts, five ordering schemes are also defined: type, aggregation, classification, specialization, and generalization of objects. One of the major tasks in this theory is to capture all objects and their relations appearing in the evolving design process. To the author's knowledge, this was the first published work that has had such a focused and deep investigation into the formal structure of design objects, though design requirements and process were not formally investigated. There is no doubt that the authors have achieved certain success in completing this mission. To further apply this theory to design modeling raises the following questions: 1) is property an object? 2) is type an object? Since in the evolving design process, the distinctions between property, object, and type are relative, the specification of an object in an early design stage can be a type or a generalization of the object in a later stage. This makes it hard to keep the definition consistent or the axiomatic set theory intact in that operations have to be performed on the sets from different levels of generalization/specification.

Despite the differences between our theory and all other endeavors, it must be admitted that the dream, vision and insight behind all the related research have always inspired the author in the exploration of the regularity underlying the design activity over the last decade.

5. Concluding Remarks

This paper established an axiomatic approach to studying design. With this approach, design activities were investigated by deriving theorems from axioms. These theorems constitute a formal model of design. The axioms developed in this paper can be used by design theorists and computer scientists to model different aspects of design problem. The formal model of design can be used by designers to organize their activities or by software developers to implement various CAD systems.

The established axiomatic theory includes two groups of axioms: axioms of objects and axioms of the human thought. Axioms of objects state that everything in the universe is an object, and that there are relations between objects. They provide the premise for studying objects in both nature and the human thought. Two important notions developed from these two axioms are the structure and range operations, which naturally represents basic operations, such as aggregation and generalization, in the design process. Axioms of the human thought are introduced in the context of the structure of the world, which is established using the structure operation. These axioms identify the nature of the reasoning and recognition processes in the human thought. They state that human beings are bounded in rationality, that human beings do not recognize objects accurately, and that causal relation is the only plausible relation among all relations between causes and effects. Five basic theorems about engineering design are logically derived from the axioms, mainly with the application of the structure and range operations. These theorems cover the engineering system, the formulation of design requirements, and the model of the design process. This work has formally realized two main notions in Simon's Science of the Artificial (Simon, 1969): bounded rationality and hierarchical systems.

The significance of this work lies in that it provides a logical method for approaching design studies. With this approach, design studies become a scientific exploration through the derivation of mathematical equations plus the explanation of the factual meaning of these equations. This exploration could lead to results that are already known from other approaches. Meanwhile, it also opens the possibility that some new results may be found. For example, Theorems 1, 2, and 4 in this paper have been indeed widely discussed in the literature. They are general properties of design. In terms of the research framework in Fig. 1, this verifies the axiomatic theory in this paper. On the other hand, Theorems 3 and 5 are new approaches to modeling design requirements and the design synthesis process (Zeng, 2001; Zeng and Gu, 2001). This new model addresses the uniform representation of design requirements in terms of product environment and a natural decomposition method of design problem.

The future work of this research includes deriving theorems which can be comparable to existing design theory and methodology, such as the Suh's axiomatic design theory (Suh, 1990), the TRIZ

(Altshuller, 1988), the systematic design methodology (Pahl and Beitz, 1988; Hubka, 1980; Hubka and Eder, 1988), etc. Apart from an initial endeavor to address the relation between the design creativity and axiomatic approach (Zeng, 2001), further investigation needs to be conducted to identify room for flexibility, style, vagueness, randomness, creativity, and complexity for design in the present theory (Wilhelm *et al*, 2001; Salustri, 2000). Since the two groups of axioms are a general description of objects and the human thought, they can also be used to study other intelligent activities. In addition, some on-going collaborations are in development to apply this theory to general system theory and software engineering. Another issue that needs serious investigation is the philosophical foundation of this work. The axioms of objects share some foundation with mereology while the axioms of the human thought basically addresses the problem of universals. These two philosophical branches are closely related. This paper indeed attempted to establish a science of design by joining philosophy, logic, mathematics, and engineering. It is worthwhile to explore the relation of this research to the agenda of process science (Gatchel and Tanik, 2001; Ertas, *et al*, 2000). The author would like to conclude this paper by saying that the science of design is an attempt to engineer philosophy and to philosophy the engineering.

6. Acknowledgements

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7. References

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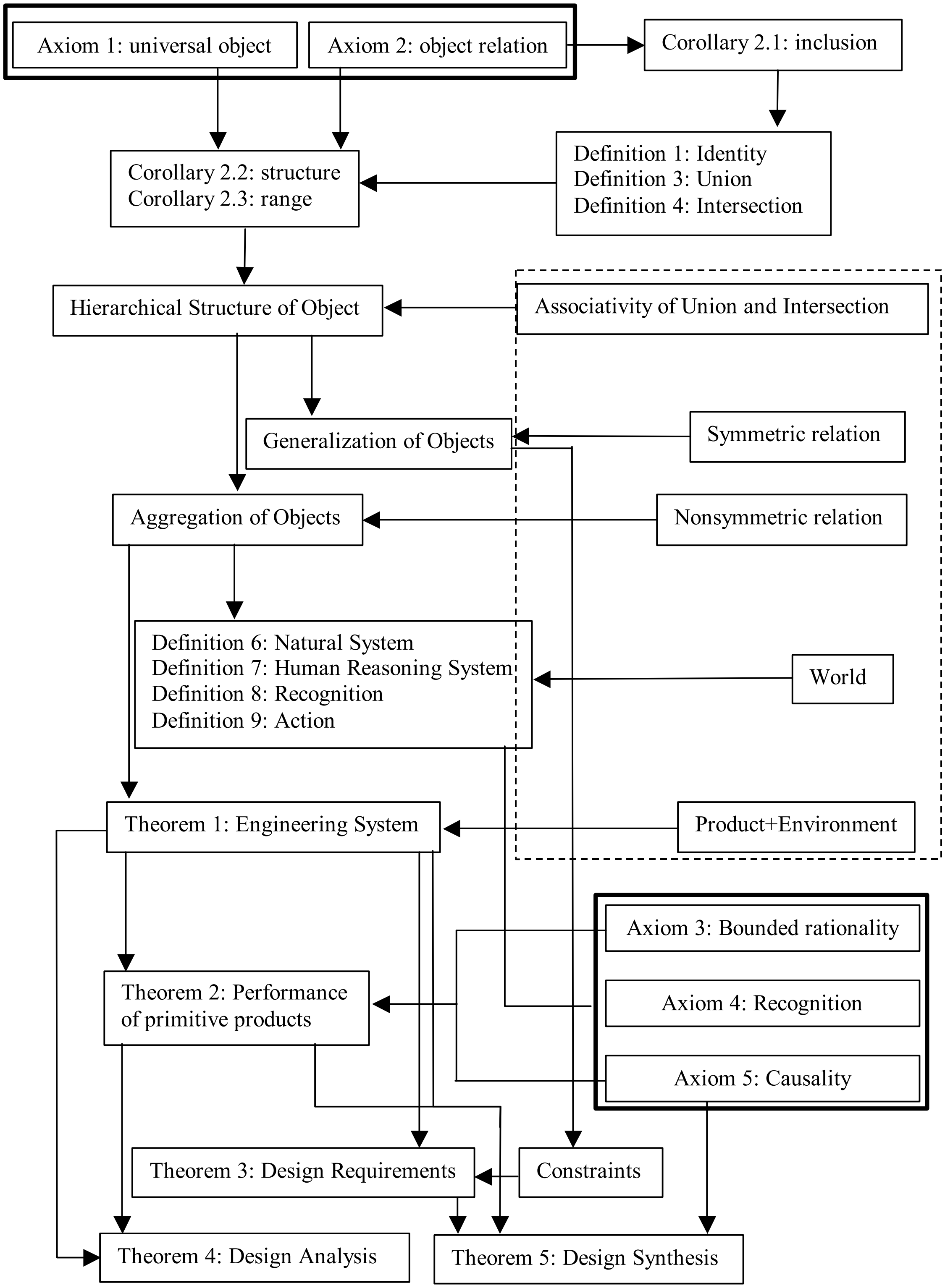
8. Appendix A. Interrelationships Between the Notions in This Paper



In the above illustration, the notions included in the dashed box are introduced for the formulation of concrete problems so that the axiomatic theory can be applied for the modeling purpose.

9. Appendix B. Axioms and Primitive Notions in Euclidean Geometry

Postulates:

1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To describe a circle with any centre and distance.
4. That all right angles are equal to one another.
5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

Primitive notions:

1. A *point* is that which has no part.
2. A *line* is of breadthless length.
3. The extremities of a line are points.
4. A *straight line* is a line which lies evenly with the points on itself.
5. A *plane angle* is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.
6. When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is *right* ....

Logic rules:

1. Things which are equal to the same thing are also equal to one another.
2. If equals are added to equals, the wholes are equal.
3. The whole is greater than the part.

Notes

[i] Zeng, Y. and Wu. F., 2002, Axiomatic object theory, in preparation.

[ii] Zeng, Y. and Gu, P., 2002a, Dynamic mechanism underlying the design evolution process. This paper is in the process of being adapted to the current axiomatic theory of design modeling. Its early version is published in NSF Design and Manufacturing Grantees Conference**,** Jan. 3-6 2000, Vancouver, Canada.

[iii] Zeng, Y. and Gu, P., 2002b, Formulation of design requirements based on environment. This paper is in the final stage of preparation.

[iv] Zeng, Y., 2002, Design evolution based on environment decomposition. This paper is in the final stage of preparation.