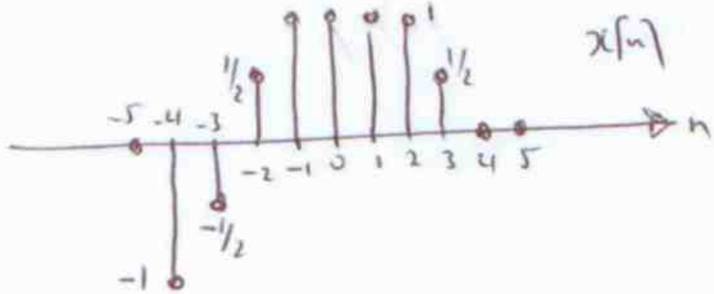
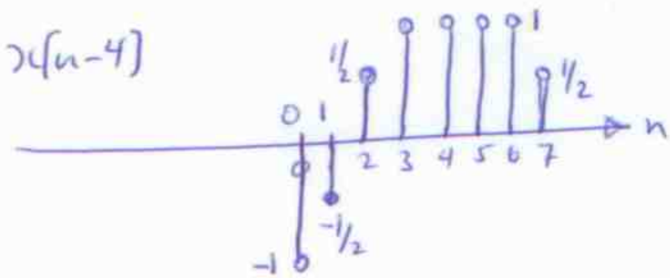
Solutions for Tutorial week1:

1.22



a) $x[n-4]$

The signal should be shifted 4 units to the right



b) $x[3-n]$

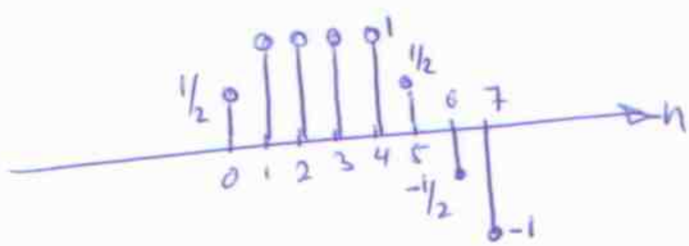
First, signal should be flipped then shifted 3 units to the right.

Or

we can say, the value of $x[n]$ at $n=n_0$ is equal the value of $x[3-n]$, in other words

$$x[n=n_0] = x[3-n] \longrightarrow n_0 = 3-n \longrightarrow n = 3-n_0$$

Therefore,



d) $x[3n+1]$

First, signal should be shifted to the left then compressed by factor $1/3$.

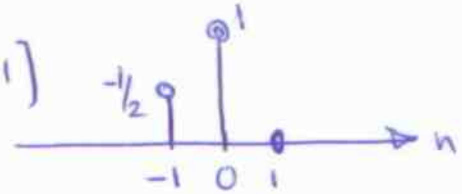
Or we can say

$$x[n_0] = x[3n+1] \rightarrow n_0 = 3n+1 \rightarrow n = \frac{n_0 - 1}{3}$$

* Always we first shift then compress / stretch !!

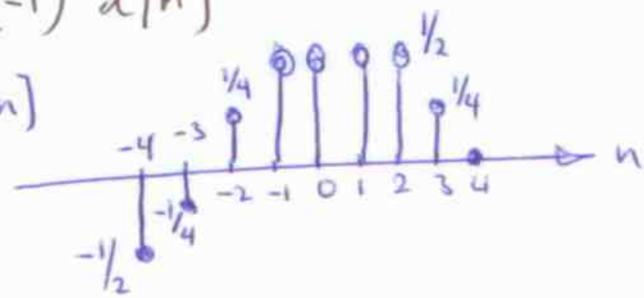
* n is an integer therefore, non-integer values are not acceptable !!

$x[3n+1]$



g) $\frac{1}{2}x[n] + \frac{1}{2}(-1)^n x[n]$

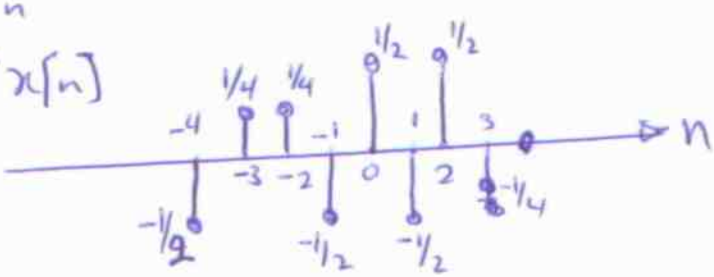
$\frac{1}{2} \times x[n]$



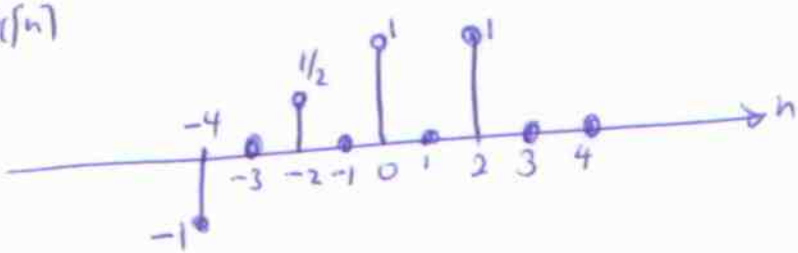
+

$(-1)^n \begin{cases} \rightarrow 1 \text{ for even } n \\ \rightarrow -1 \text{ for odd } n \end{cases}$

$\frac{1}{2}(-1)^n x[n]$

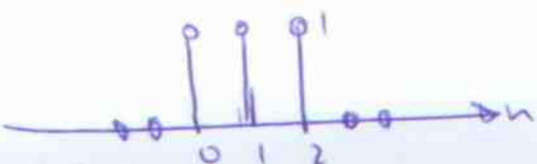


$\frac{1}{2}x[n] + \frac{1}{2}(-1)^n x[n]$



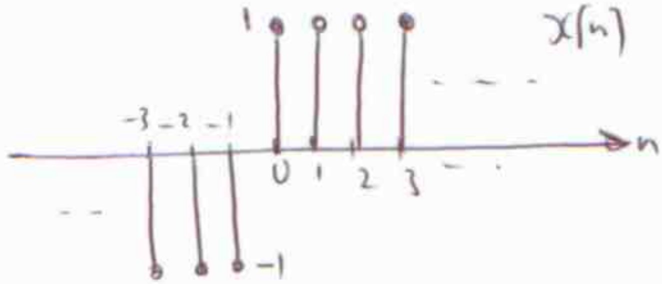
h) $x[(n-1)^2]$

$$x[n = n_0] = x[(n-1)^2] \rightarrow n = \pm\sqrt{n_0} + 1$$



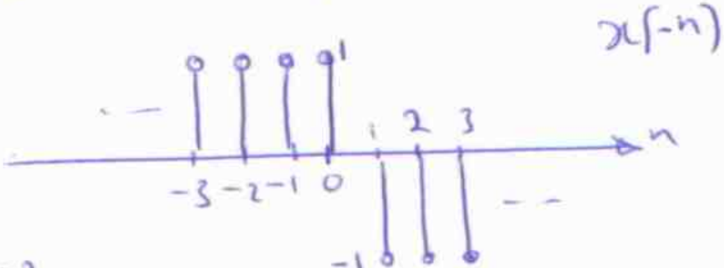
1.24

(a)

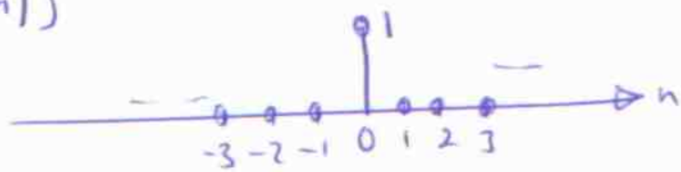


$Ev\{x[n]\} = \frac{1}{2}\{x[n] + x[-n]\}$

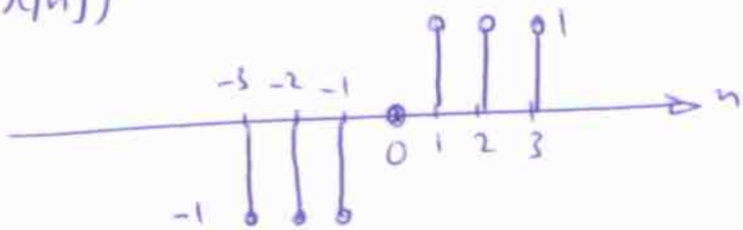
$odd\{x[n]\} = \frac{1}{2}\{x[n] - x[-n]\}$



$Ev\{x[n]\}$

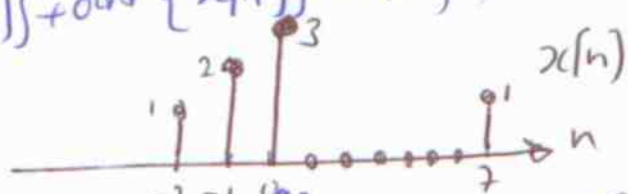


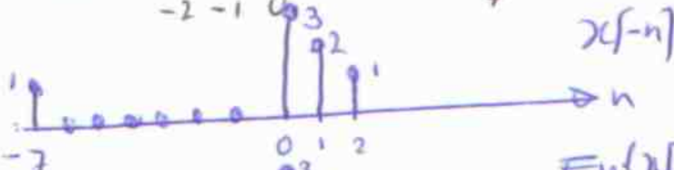
$odd\{x[n]\}$

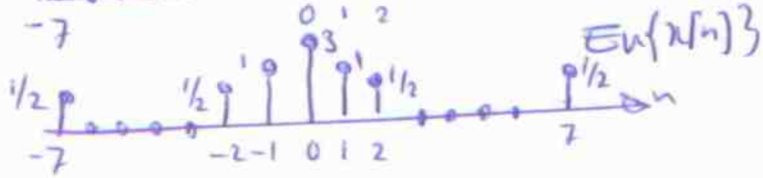


(For test:
$Ev\{x[n]\} + odd\{x[n]\} = x[n]$)

(b)

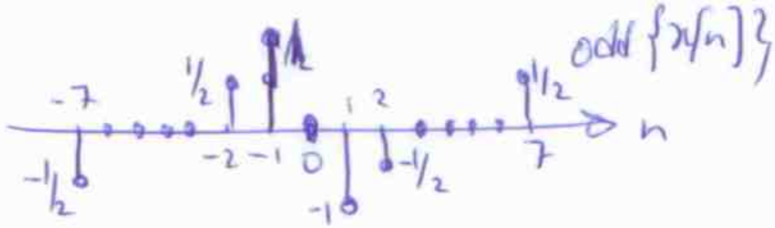






1.24

(b) Cont'd



1.25

(a) $x(t) = 3\cos\left(4t + \frac{\pi}{3}\right)$

This signal is periodic.

$$T = \frac{2\pi}{4} = \frac{\pi}{2}$$

(b) $x(t) = e^{j(\pi t - 1)}$

$$= \cos(\pi t - 1) + j\sin(\pi t - 1)$$

This signal is periodic

$$T = \frac{2\pi}{\pi} = 2$$

(c) $x(t) = \left[\cos\left(2t - \frac{\pi}{3}\right)\right]^2$ $\left(\cos^2(t) = \frac{1+\cos(2t)}{2}\right)$

$$= \frac{1 + \cos 2\left(2t - \frac{\pi}{3}\right)}{2} = \frac{1 + \cos\left(4t - \frac{2\pi}{3}\right)}{2}$$

This signal is periodic with $T = \frac{2\pi}{4} = \frac{\pi}{2}$

1.49

(a) $1+j\sqrt{3} = \boxed{2e^{j\frac{\pi}{3}}}$

$r = \sqrt{1+3} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$

$\left(\begin{array}{l} a+jb = re^{j\theta} \\ r = \sqrt{a^2+b^2}, \ \theta = \tan^{-1}\left(\frac{b}{a}\right) \end{array} \right)$

(e) $(1-j\sqrt{3})^3$

$1-j\sqrt{3} = 2e^{-\pi/3}$

$r = \sqrt{1+3} = 2$

$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = \frac{-\pi}{3}$

$(1-j\sqrt{3})^3 = \left(2e^{-\pi/3}\right)^3 = \boxed{8e^{-\pi}}$

(i) $\frac{1+j\sqrt{3}}{\sqrt{3}+j} = \frac{\cancel{2}e^{j\pi/3}}{\cancel{2}e^{j\pi/6}} = e^{j\frac{\pi}{3}} \cdot e^{-j\pi/6} = e^{j\left(\frac{\pi}{3}-\frac{\pi}{6}\right)} = \boxed{e^{j\pi/6}}$

$1+j\sqrt{3} = 2e^{j\pi/3}$

$r = \sqrt{1+3} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$

$\sqrt{3}+j = 2e^{j\pi/6}$

$r = \sqrt{3+1} = 2$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi/6$

(j) $j(1+j)e^{j\pi/6} = e^{j\pi/2}\sqrt{2}e^{j\pi/4} \cdot e^{j\pi/6} = \sqrt{2}\,e^{j\left(\frac{\pi}{2}+\frac{\pi}{4}+\pi/6\right)} = \boxed{\sqrt{2}\,e^{j\frac{11\pi}{12}}}$

$j = e^{j\pi/2}$, $(1+j) = \sqrt{2}e^{j\pi/4}$

$r = \sqrt{0+1} = 1$ | $r = \sqrt{1+1} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{1}{0}\right) = \pi/2$ | $\theta = \tan^{-1}\left(\frac{1}{1}\right) = \pi/4$

1.52

(a) $zz^* = r^2$

$$zz^* = re^{j\theta} \cdot re^{-j\theta} = r^2$$

(b) $$\frac{z}{z^*} = \frac{\cancel{r}e^{j\theta}}{\cancel{r}e^{-j\theta}} = e^{j\theta} \cdot e^{j\theta} = e^{j2\theta}$$

(c) $$z + z^* = (x + jy) + (x - jy) = 2x = 2Re\{z\}$$

(d) $$(z_1 + z_2)^* = [(x_1 + jy_1) + (x_2 + jy_2)]^* = [(x_1 + x_2) + j(y_1 + y_2)]^*$$

$$= (x_1 + x_2) - j(y_1 + y_2) = (x_1 - jy_1) + (x_2 - jy_2)$$

$$= z_1^* + z_2^*$$