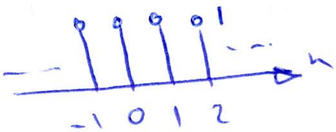
Solutions for Tutorial week 2

1.11

$$x[n] = 1 + e^{j4\pi n/7} - e^{j2\pi n/5}$$

Period of the first term is 1



Period of the second term is $m\left(\frac{2\pi}{4\pi/7}\right) = 7$ (when $m = 2$)
(Since period should be an integer)

Period of the third term is $m\left(\frac{2\pi}{2\pi/5}\right) = 5$ (when $m=1$)

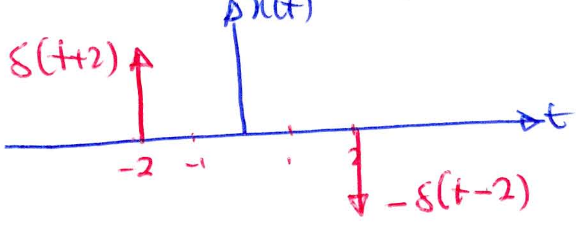
Therefore, the overall signal $x[n]$ is periodic with a period which is the least common multiple of the periods of the three term is $x[n]$. This is equal to 35.

1.13

$$x(t) = \delta(t+2) - \delta(t-2)$$

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

Calculate E_∞ for the signal.



$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau = \int_{-\infty}^{t} [\delta(\tau+2) - \delta(\tau-2)]\,d\tau = \begin{cases} 0, & t < -2 \\ 1, & -2 \leq t \leq 2 \\ 0, & t > 2 \end{cases}$$

Therefore, $E_\infty = \int_{-2}^{2} dt = \boxed{4}$ $\left(\text{Since } E_\infty = \int_{-\infty}^{+\infty} |y(t)|^2\, dt\right)$

1.26

(a) $x[n] = \sin\left(\frac{6\pi}{7}n+1\right) \leftarrow$ Periodic

Period $= m\left(\frac{2\pi}{6\pi/7}\right) = 7$ (For $m=3$)

(b) $x[n] = \cos\left(\frac{n}{8}-\pi\right) \leftarrow$ Not Periodic

Since $m\left(\frac{2\pi}{1/8}\right)$ cannot be an integer considering integer m.

(c) $x[n] = \cos\left(\frac{\pi}{8}n^2\right) \leftarrow$ Periodic

Period $= 8$

(d) $x[n] = \cos\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{4}n\right)$

$x[n] = \frac{1}{2}\left[\cos\left(\frac{3\pi}{4}n\right) + \cos\left(\frac{\pi n}{4}\right)\right] \leftarrow$ Periodic

Period $= 8$

(e) $x[n] = 2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \pi/6\right) \leftarrow$ Periodic

Period $= 16$

(Period for the first term is 8, for the second 16, and for the third

1.39

$$x(t)\,\delta(t) = x(0)\,\delta(t)$$

$$\lim_{\Delta \to 0} [u_\Delta(t)\,\delta(t)] = 0,$$

$$\lim_{\Delta \to 0} [u_\Delta(t)\,\delta_\Delta(t)] = \frac{1}{2}\,\delta(t)$$

We have

$$\lim_{\Delta \to 0} u_\Delta(t)\,\delta(t) = \lim_{\Delta \to 0} u_\Delta(0)\,\delta(t) = 0$$

$$\lim_{\Delta \to 0} u_\Delta(t)\,\delta_\Delta(t) = \frac{1}{2}\,\delta(t)$$

we have $g(t) = \int_{-\infty}^{+\infty} u(\tau)\,\delta(t-\tau)\,d\tau = \int_{0}^{\infty} u(\tau)\,\delta(t-\tau)\,d\tau$

Therefore,

$$g(t) = \begin{cases} 0, & t<0 \\ 1, & t>0 \\ \text{undifined}, & \text{for } t=0 \end{cases}$$

$\because\ \delta(t-\tau)=0$

$\therefore\ u(\tau)\,\delta(t-\tau) = \delta(t-\tau)$

1.26 c)

$$\cos \frac{\pi}{8} n^2 = \cos\left(\frac{\pi}{8} (n+n_0)^2 \right)$$

If we can find an integer of n_0, then n_0 is the Period.

$$\Rightarrow \quad \frac{\pi}{8}(n+n_0)^2 = \frac{\pi}{8} n^2 + 2k\pi \qquad (k \text{ integer})$$

$$\frac{\pi}{8} n^2 + \frac{\pi}{8} n_0^2 + \frac{\pi}{4} n n_0 = \frac{\pi}{8} n^2 + 2k\pi$$

$$\frac{\pi}{8} n_0^2 + \frac{\pi}{4} n n_0 = 2k\pi$$

If $n_0 = 8$ then left hand of above equation will be multiple of 2π.

Therefore Period is "8".

No integer less than "8" satisfies, therefore "8" is the fundamental period.