1

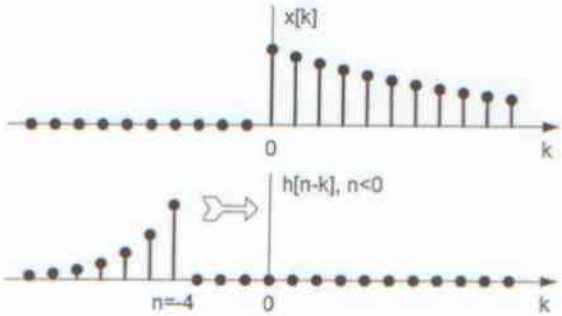
2.21

a)

$$x[n]=\alpha^n u[n], h[n]=\beta^n u[n], \alpha \neq \beta$$

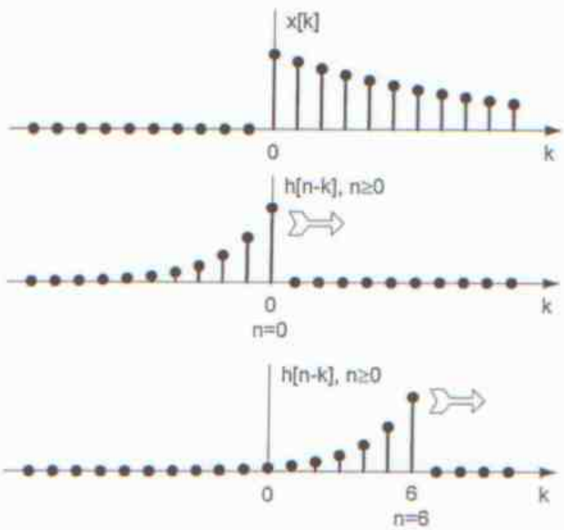
$$y[n]=x[n]*h[n]=\sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

- For $n<0$:



$$y[n]=\sum_{k=-\infty}^{\infty} x[k]h[n-k]=\sum_{k=-\infty}^{\infty} 0\cdot\beta^{n-k}=0$$

- For $n\geq 0$:



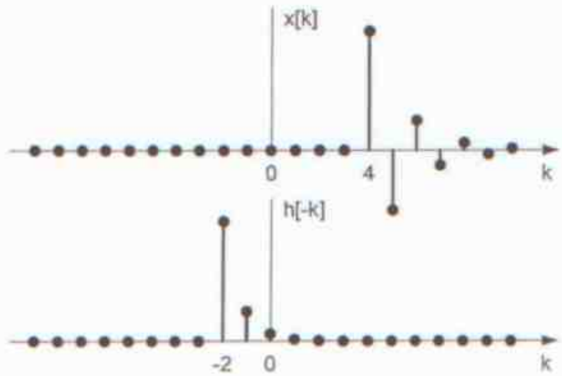
$$y[n]=\sum_{k=0}^{n}\alpha^k\cdot\beta^{n-k}=\beta^n\sum_{k=0}^{n}\left(\frac{\alpha}{\beta}\right)^k=\beta^n\frac{1-\left(\frac{\alpha}{\beta}\right)^{n+1}}{1-\frac{\alpha}{\beta}}=\frac{\beta^{n+1}-\alpha^{n+1}}{\beta-\alpha}$$

To summarize:

$$y[n]=\frac{\beta^{n+1}-\alpha^{n+1}}{\beta-\alpha}u[n]$$

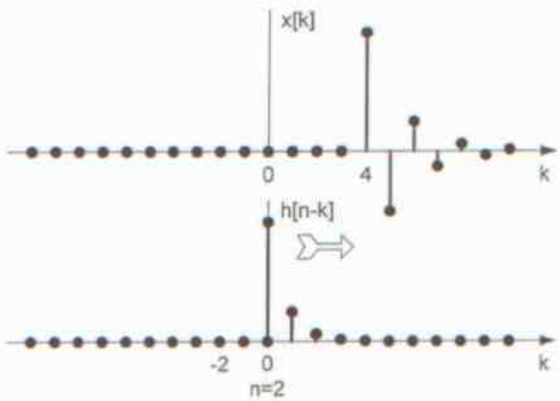
2.21 (Continue)

c) $$x[n]=\left(-\frac{1}{2}\right)^n u[n-4]$$
$$h[n]=4^n u[2-n]$$



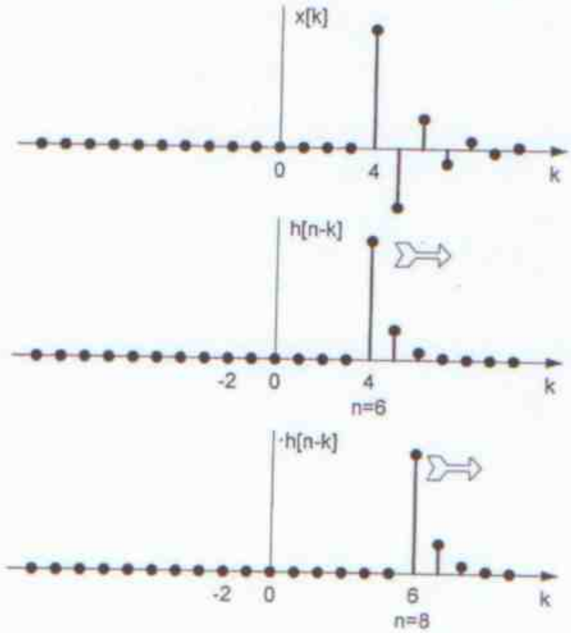
7

- For $n<6$



$$y[n]=\sum_{k=4}^{\infty}\left(-\frac{1}{2}\right)^k 4^{n-k}=4^n\sum_{k=4}^{\infty}\left(-\frac{1}{8}\right)^k=4^n\frac{\left(-\frac{1}{8}\right)^4}{1-\left(-\frac{1}{8}\right)}=4^n\frac{8}{9}\left(-\frac{1}{8}\right)^4$$

- For $n \geq 6$:



$$y[n]=\sum_{k=n-2}^{\infty}\left(-\frac{1}{2}\right)^{k}4^{n-k}=4^{n}\sum_{k=n-2}^{\infty}\left(-\frac{1}{8}\right)^{k}=4^{n}\frac{\left(-\frac{1}{8}\right)^{n-2}}{1-\left(-\frac{1}{8}\right)}=4^{n}\frac{8}{9}\left(-\frac{1}{8}\right)^{n-2}=\frac{\overset{3}{8}}{9}\left(-\frac{1}{2}\right)^{n}$$

To summarize:

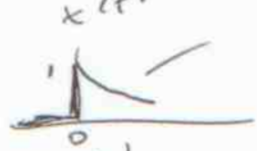
$$y[n]=4^{n}\frac{8}{9}\left(-\frac{1}{8}\right)^{4}u[-n+5]+\frac{\overset{3}{8}}{9}\left(-\frac{1}{2}\right)^{n}u[n-6]$$

OεW

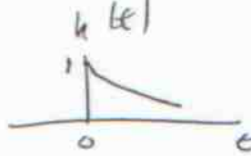
2.22.

①

2.22 a) $x(t) = e^{-\alpha t}\, u(t)$
$h(t) = e^{-\beta t}\, u(t)$



drawn as if $\alpha > 0$

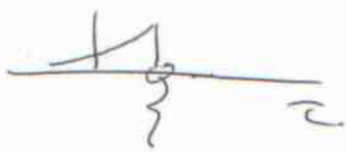


as if $\beta > 0$.

$$y(t) = x(t) * h(t)$$
$$= \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau.$$

doesn't matter which to flip flip x

$x(t-\tau)$



$t - \tau = 0$
$\tau = t.$

Interval 1

$t < 0$

clearly $y(t) = 0$.

Interval 2.
$t \geq 0.$

$$y(t) = \int_0^t e^{-\alpha(t-\tau)}\, e^{-\beta\tau}\, d\tau.$$

$$= e^{-\alpha t} \int_0^t e^{\tau(\alpha-\beta)}\, d\tau.$$

$*\alpha$

for $\alpha \neq \beta$.
$$= e^{-\alpha t}\, \frac{1}{\alpha-\beta} \left[e^{\tau(\alpha-\beta)} \right]_0^t$$

شغل

2.22.

2

$$y(t) = e^{-\alpha t} \frac{1}{\alpha-\beta}\left[e^{t(\alpha-\beta)} - 1 \right]$$

$$= \frac{1}{\alpha-\beta}\left[e^{-\beta t} - e^{-\alpha t} \right]$$

$$\therefore y(t) = \begin{cases} \frac{1}{\alpha-\beta}\left(e^{-\beta t} - e^{-\alpha t} \right) & t \geq 0 \\ 0 & \text{o.w} \end{cases}$$

$$= \frac{1}{\alpha-\beta}\left(e^{-\beta t} - e^{-\alpha t} \right) u(t)$$

for $\alpha = \beta$ continue from b.k

$$y(t) = e^{-\alpha t} \int_0^t d\tau.$$

$$= t\, e^{-\alpha t}$$

$$\therefore y(t) = \begin{cases} t\, e^{-\alpha t} & t \geq 0 \\ 0 & \text{o.w.} \end{cases}$$

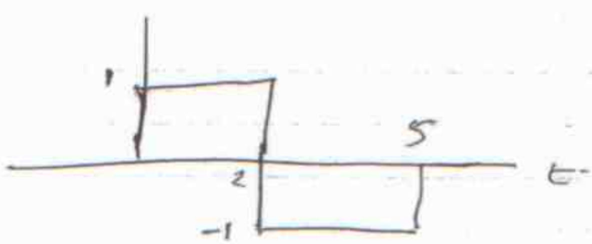
$$= t\, e^{-\alpha t} u(t)$$

2.22 O & W

③

b) $x(t) = u(t) - 2u(t-2) + u(t-5)$

$h(t) = e^{2t} \; u(1-t)$

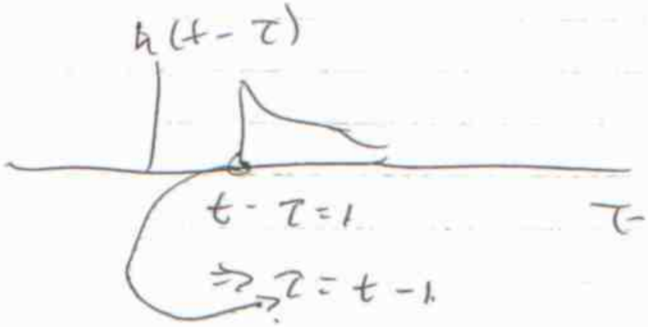


up 1 at 0
down 2 at 2
up 1 at 5.

flip h.

$y(t) = x(t) * h(t)$

$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau.$

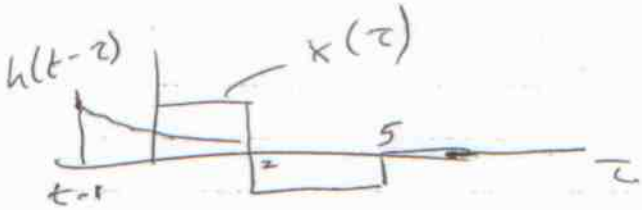


2.22 O&W (4)

Int 1

~~case.~~ $t-1<0$

$t<1$



$$y(t) = \int_0^2 e^{2(t-\tau)}\,d\tau + \int_2^5 -e^{2(t-\tau)}\,d\tau$$

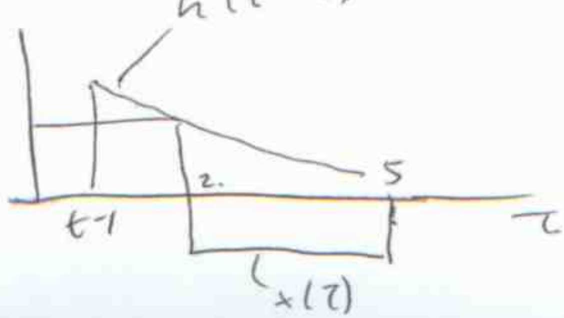
$$= -\frac{1}{2}\left[e^{2(t-\tau)}\right]_0^2 + \frac{1}{(-2)}\left[-e^{2(t-\tau)}\right]_2^5$$

$$= -\frac{1}{2}\left[e^{2(t-2)} - e^{2t} - e^{2(t-5)} + e^{2(t-\tau)}\right]$$

$$= \frac{1}{2}\left[e^{2t} - 2e^{2(t-2)} + e^{2(t-5)}\right]$$

Int. 2. $0<t-1<2$.

$1<t<3$



2.22 $\quad 0 \leq i \leq w$ $\quad$ (5)

$$y(t) = \int_{t-1}^{2} e^{2(t-\tau)}\,d\tau + \int_{2}^{5} -e^{2(t-\tau)}\,d\tau$$

$$= -\frac{1}{2}\left[e^{2(t-\tau)}\right]_{t-1}^{2} + \left(-\frac{1}{2}\right)\left[-e^{2(t-\tau)}\right]_{2}^{5}$$

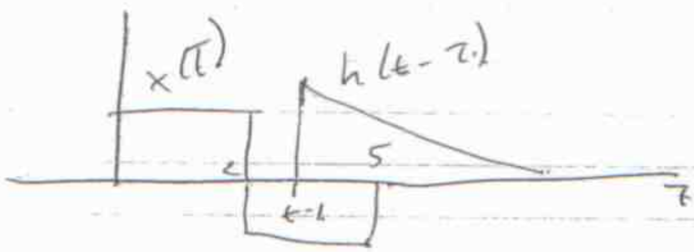
$$= -\frac{1}{2}\left\{\left[e^{2(t-2)} - e^{2(t-(t-1))}\right] + \left[-e^{2(t-5)} + e^{2(t-2)}\right]\right\}$$

$$= \frac{1}{2}\left[e^{2} - 2e^{2(t-2)} + e^{2(t-5)}\right]$$

Int 3.

$2 < t-1 < 5$

$3 < t < 6$



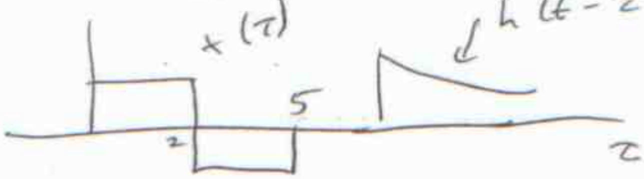
$$y(t) = \int_{t-1}^{5} -e^{2(t-\tau)}\,d\tau$$

$$= \frac{1}{2}\left[e^{2(t-\tau)}\right]_{t-1}^{5} = \frac{1}{2}\left[e^{2(t-5)} - e^{2}\right]$$

O.W 2.22

1

Int 4. t > 6

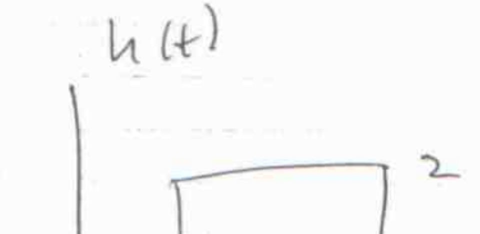


$y(t) = 0$

cts put end pts where you like

$$\therefore y(t) = \begin{cases} \frac{1}{2}\left(e^{2t} - 2e^{2(t-2)} + e^{2(t-5)}\right) & t \le 1 \\ \frac{1}{2}\left(e^{2} - 2e^{2(t-2)} + e^{2(t-5)}\right) & 1 < t \le 3 \\ \frac{1}{2}\left(e^{2(t-5)} - e^{2}\right) & 3 < t \le 6 \\ 0 & \text{o.w} \end{cases}$$

c) $x(t) = \begin{cases} \sin \pi t & 0 < t < 2 \\ 0 & \text{o.w} \end{cases}$



O & W
2.22.

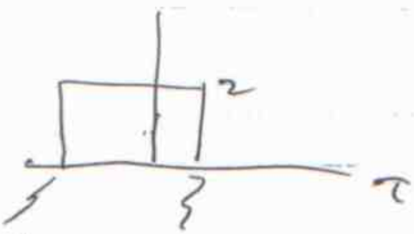
(7)

flip h

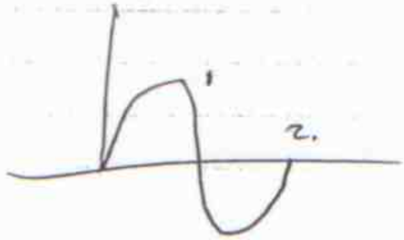
$\therefore\ y(t) = x(t) * h(t)$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau.$$

$h(t-\tau)$



$x(\tau)$



$t-\tau = 3$
$\tau = t-3$

$t - \tau = 1.$
$\tau = t-1$

Int. 1 $\quad t - 1 < 0$
$\quad t < 1$

$y(t) = 0.$

Int 2

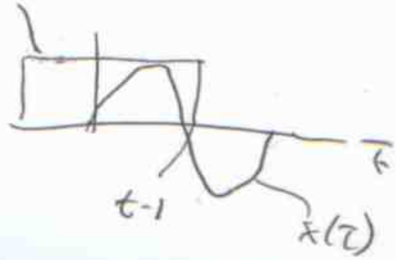
Note trailing ~~en~~ breakpt. of. $h(t-\tau)$ hits. 1st breakpoint of $x(\tau)$ at same t as leading breakpoint of $h(t-\tau)$ hits last breakpoint of $x(\tau)$

that is when $\quad t - 1 = 2.$
$\quad t = 3$

$\therefore$ int 2 is $\quad 1 < t < 3$

$h(t-\tau)$

$$y(t) = \int_0^{t-1} 2 \sin \pi\tau \, d\tau.$$



O & W.

2.22.

⑧

$$y(t) = \left[-\frac{2}{\pi}\cos\pi\tau\right]_0^{t-1}$$

$$= -\frac{2}{\pi}\left[\cos\pi(t-1) - 1\right]$$

$$= -\frac{2}{\pi}\left[\cos\pi t \cancel{\cos\pi}^{-1} + \sin\pi t \cancel{\sin\pi}^{0} - 1\right]$$

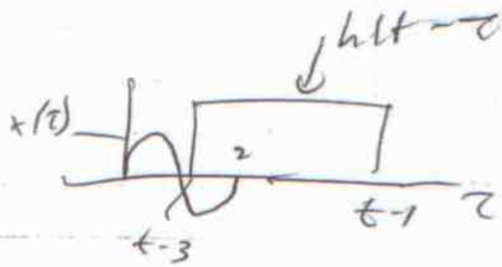
$$= \frac{2}{\pi}\left[1 + \cos\pi t\right]$$

Int 3.

τ ends when trailing breakpt. of $h(t-\tau)$ hits 2nd breakpoint of $x(\tau)$

i.e.

$$t-3 = 2$$
$$t = 5$$



$$3 < t < 5.$$

$$y(t) = \int_{t-3}^{2} 2\sin\pi\tau \, d\tau.$$

$$= \left[-\frac{2}{\pi}\cos\pi\tau\right]_{t-3}^{2}$$

$$= -\frac{2}{\pi}\left[\cancel{\cos 2\pi}^{1} - \cos\pi(t-3)\right]$$

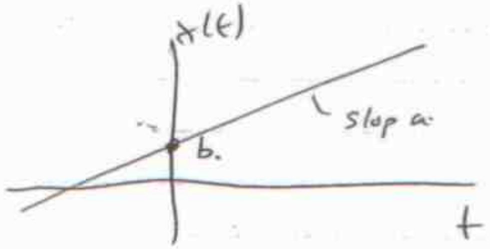
$$= -\frac{2}{\pi}\left[1 - \left[\cos\pi t \cancel{\cos 3\pi t}^{-1} + \sin\pi t \cancel{\sin 3\pi}^{0}\right]\right]$$

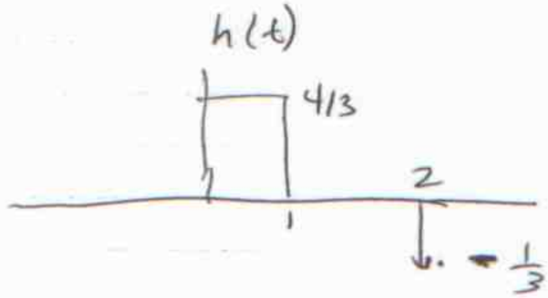
$$= -\frac{2}{\pi}\left[1 + \cos\pi t\right]$$

for $t > 5$.

$$y(t) = 0$$

$$\therefore y(t) = \begin{cases} \frac{2}{\pi}\left[1 + \cos \pi t\right] & 1 < t \le 3 \\ -\frac{2}{\pi}\left[1 + \cos \pi t\right] & 3 < t \le 5 \\ 0 & \text{o.w.} \end{cases}$$

d) 



$$x(t) = at + b$$

$$h(t) = \underbrace{\frac{4}{3}\left[u(t) - u(t-1)\right]}_{h_1} - \frac{1}{3}\,\delta(t-2) = h_1(t) - \frac{1}{3}\delta(t-2)$$

$$y(t) = x(t) * h(t)$$

$$= x(t) * \left[h_1(t) - \frac{1}{3}\,\delta(t-2)\right]$$

$$= x(t) * h_1(t) - \frac{1}{3}\,x(t) * \delta(t-2)$$

$$= x(t) * h_1(t) - \frac{1}{3}\left(a(t-2) + b\right)$$

~ convolution with $\delta(t-t_0)$ is same thing over again shifted by t_0

Calc. $x(t) * h_1(t)$

Note x has no break pts.

flip x then integrate between $(0;1)$

$$y(t) = \int_0^1 \frac{4}{3}\left[a(t-\tau)+b\right]d\tau - \frac{1}{3}\left(at-2a+b\right)$$

$$= \frac{4}{3}\left[at\tau - \frac{a}{2}\tau^2 + b\tau\right]_0^1 - \frac{at}{3} + \frac{2}{3}a - \frac{b}{3}$$

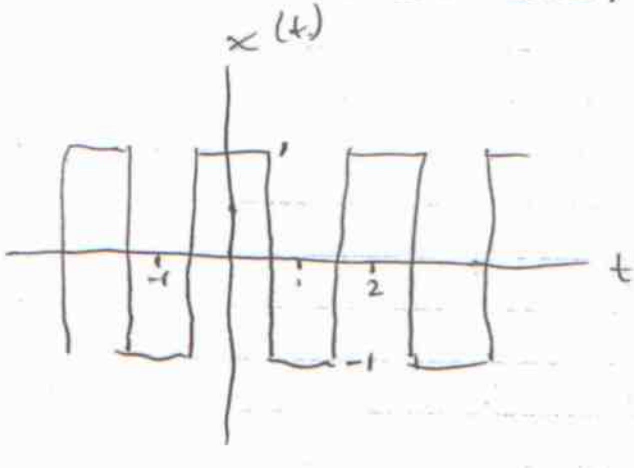
$$= \frac{4}{3}\left[at - \frac{a}{2} + b\right] - \frac{at}{3} + \frac{2}{3}a - \frac{b}{3}$$

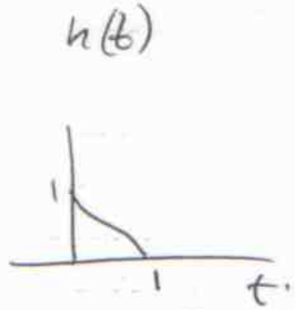
$$= \frac{4}{3}at - \frac{at}{3} - \frac{2}{3}a + \frac{2}{3}a + \frac{4}{3}b - \frac{b}{3}$$

$$\underline{\underline{y(t) = at + b}}$$

Same as x !!
(this is a specially cooked coincidence)

e)





$$h(t) = \begin{cases} -t+1 & 0<t<1 \\ 0 & \text{o.w.} \end{cases}$$

Note $x(t)$ periodic with period $T = 2$.

Examine $y(t) = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau$

and $y(t+T) = \int_{-\infty}^{\infty} x(t+T-\tau)\, h(\tau)\, d\tau$

O & W 2.22

(11)

$$x(t+T-\tau) = x(t-\tau) \quad \left(\underset{\text{in } T}{x \text{ periodic}} \right)$$

$$\therefore \quad y(t+T) = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau = y(t)$$

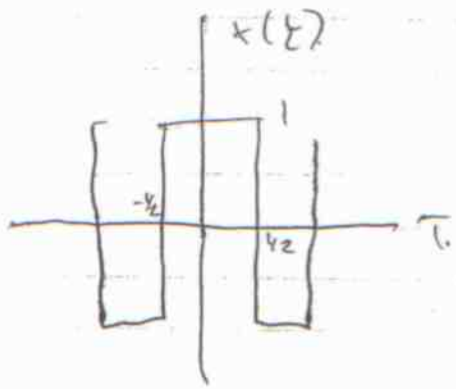
$\therefore$ x periodic in T.

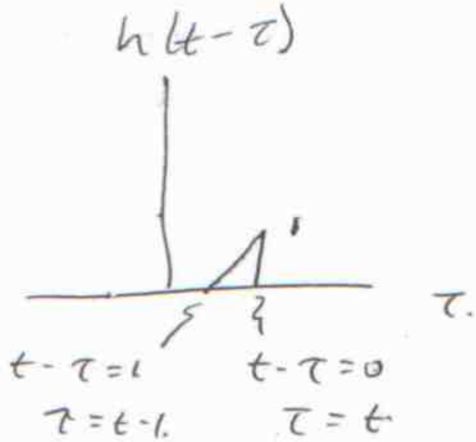
$\Rightarrow$ y periodic in T.

(Same for h)

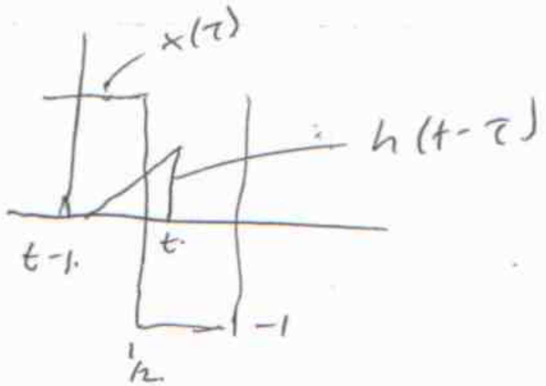
$\therefore$ need only calc. y over a period $T = 2$ then repeat.

flip. $h(t)$.





let t
start with $t = \frac{1}{2}$. (must go to $t = 2.5$)



interval is from

$$\frac{1}{2} < t < \frac{3}{2}.$$

$$y(t) = \int_{t-1}^{1/2} (1-(t-\tau))\, d\tau.$$

$$+ \int_{1/2}^{t} -\left(1-(t-\tau)\right) d\tau.$$

oziw 2.22

(12)

$$y(t) = \left[\tau - t\tau + \frac{\tau^2}{2}\right]_{t-1}^{1/2} + \left[-\tau + \tau t - \frac{\tau^2}{2}\right]_{1/2}^{t}$$

$$= \frac{1}{2} - \frac{t}{2} + \frac{1}{8} - (t-1) + t(t-1) - \frac{(t-1)^2}{2}$$

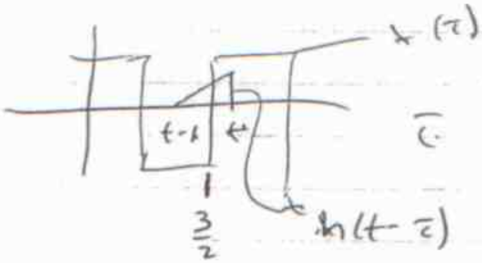
$$- t + t^2 - \frac{t^2}{2} + \frac{1}{2} - \frac{t}{2} + \frac{1}{8}$$

$$= \frac{5}{4} - 2t - t + 1 + t^2 - t - \frac{t^2}{2} + t - \frac{1}{2} + \frac{t^2}{2}$$

$$= \frac{7}{4} - 3t + t^2$$

Int 2

$\frac{3}{2} < t < 5/2$



$$y(t) = \int_{t-1}^{3/2} -(1 - (t-\tau))\, d\tau + \int_{3/2}^{t} (1 - (t-\tau))\, d\tau$$

$$= \left[-\tau + t\tau - \frac{\tau^2}{2}\right]_{t-1}^{3/2} + \left[\tau - t\tau + \frac{\tau^2}{2}\right]_{3/2}^{t}$$

$$= -\frac{3}{2} + \frac{3}{2}t - \frac{9}{8} + (t-1) - t(t-1) + \frac{(t-1)^2}{2}$$

$$+ t - t^2 + \frac{t^2}{2} - \frac{3}{2} + \frac{3}{2}t - \frac{9}{8}$$

0 § W. 222

(13)

$$y(t) = -\frac{21}{4} + 4t - \frac{t^2}{2} + t - 1 - t^2 + t + \frac{t^2}{2} - t + \frac{1}{2}$$

$$= -\frac{23}{4} + 5t - t^2.$$

$$y(t) = \begin{cases} \frac{7}{4} - 3(t - Tk) + (t - kT)^2 & kT + \frac{1}{2} < t < kT + \frac{3}{2} \\ -\frac{23}{4} + 5(t - kT) - (t - kT)^2. & kT + \frac{3}{2} < t < kT + 5/2 \end{cases}$$

$k \in \mathbb{Z}$

NOTE here. $T = 2$

to handle periodic

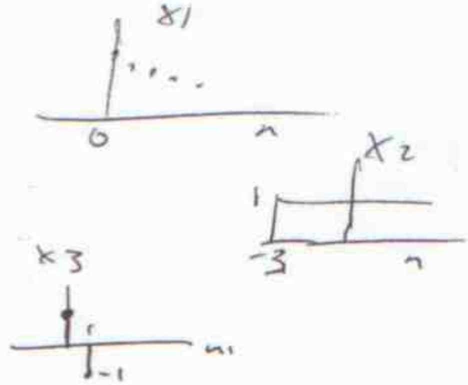
O ξίω

2.26 $\quad y[n] = x_1[n] * x_2[n] * x_3[n]$

$x_1[n] = (0.5)^n \, u[n]$

$x_2[n] = u[n+3]$

$x_3[n] = \delta[n] - \delta[n-1]$



a) $y_1[n] = x_1[n] * x_2[n]$

flip x_2 $\quad x_2[n-k]$

$n-k = -3$

$k = n+3$

$\therefore$ Int 1 $\qquad n+3 < 0 \quad \Rightarrow \quad n < -3$

$$y_1[n] = 0$$

Int 2 $\qquad n \geq -3$

$$y_1[n] = \sum_{k=0}^{n+3} (0.5)^k$$

~ geometric series 1st term is 1

$$= \frac{1-(0.5)^{n+3+1}}{1-0.5}$$

$$= 2\left(1-\left(\tfrac{1}{2}\right)^{n+4}\right)$$

$\therefore y_1[n] = 2\left(1-\left(\tfrac{1}{2}\right)^{n+4}\right) u[n+3]$ //

2.26

$0 \xi \omega$ ②

b) $y[n] = y_1[n] * x_3[n] = y_1[n] * [\delta[n] - \delta[n-1]] = y_1[n] - y_1[n-1]$

$$= 2\left(1 - \left(\tfrac{1}{2}\right)^{n+4}\right) u[n+3] - 2\left(1 - \left(\tfrac{1}{2}\right)^{(n-1)+4}\right) u[n-1+3]$$

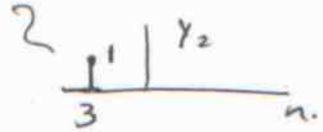
$$= 2\left[1 - \left(\tfrac{1}{2}\right)^{n+4}\right] u[n+3] - 2\left(1 - \left(\tfrac{1}{2}\right)^{n+3}\right) u[n+2].$$

c) define $y_2[n] = x_2[n] * x_3[n]$

$$= x_2[n] * [\delta[n] - \delta[n-1]]$$

$$= x_2[n] - x_2[n-1]$$

$$= u[n+3] - u[n+2] = \delta[n+3]$$



d) $y'[n] = x_1[n] * y_2[n]$

$$= x_1[n] * \delta[n+3]$$

$$= x_1[n+3]$$

$$= (0.5)^{n+3} u[n+3].$$

is this equal to $y[n]$ from b ?

$y[n] = y'[n] = 0 \quad \forall\, n < -3$

evaluate $y[-3] = 2\left(1 - \tfrac{1}{2}\right) = 1$

$y'[-3] = 1$

$n > -3.$

$$y[n] = 2\left(1-\left(\tfrac{1}{2}\right)^{n+4}\right) - 2\left(1-\left(\tfrac{1}{2}\right)^{n+3}\right)$$

$$= -\left(\tfrac{1}{2}\right)^{n+3} + \left(\tfrac{1}{2}\right)^{n+2}.$$

$$= \left(\tfrac{1}{2}\right)^{n+2}\left[1-\tfrac{1}{2}\right] = \left(\tfrac{1}{2}\right)^{n+3} = y'[n]$$

in this range

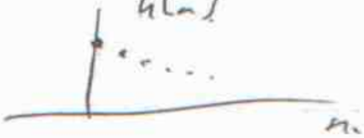
$\therefore\ y[n] = y'[n] \quad \forall\ n \in \mathbb{Z}$

Note: the $u[n]$ (or $u(t)$'s) can sometimes confuse us. Sometimes functions that look quite different are the same

$O \; \xi \; \omega$

①

2.28 a) $h[n] = \left(\frac{1}{5}\right)^n u[n]$



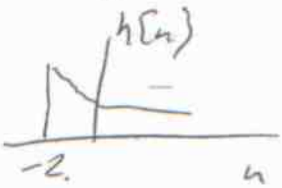
System is <u>causal</u> since $h[n] = 0 \;\; \forall n < 0$

for stability check abs. summable.

$$S = \sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=0}^{\infty} \left(\frac{1}{5}\right)^n = \frac{1}{1-1/5} = \frac{5}{4} < \infty$$

$\therefore$ system is BIBO stable

b) $h[n] = (0.8)^n \, u[n+2]$



System is <u>NOT</u> causal since

$h[-1] = (0.8)^{-1} \neq 0$

also $h[-2]$

for stability check abs. summ.

$$S = \sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=-2}^{\infty} (0.8)^k$$ ← 1st term not 0

$$= (0.8)^{-2} \sum_{k=0}^{\infty} (0.8)^k$$

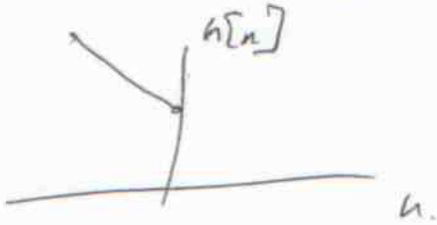
$$= (0.8)^{-2} \frac{1}{1-0.8} = 7.8125 < \infty$$

$\therefore$ system stable.

0 ; ω

2.28 ②

c) $h[n] = \left(\frac{1}{2}\right)^n u[-n]$



Note $h[-1] = \frac{1}{2}. \neq 0$

$\therefore$ system is <u>NOT</u> casual

Stability check. $S = \sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=-\infty}^{0} \left(\frac{1}{2}\right)^k$

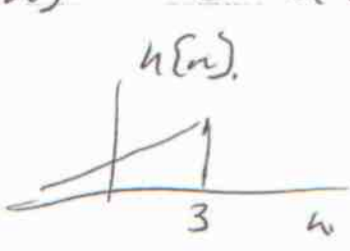
$l = -k. \quad \therefore S = \sum_{l=0}^{\infty} \left(\frac{1}{2}\right)^{-l} = \sum_{l=0}^{\infty} 2^l.$

since factor is larger than 1
this series diverges.

i.e. $S = \infty$ $\therefore$ <u>NOT</u> BIBO stable.

d) $h[n] = 5^n u[3-n]$. since $\exists\, n<0$ s.t

$\therefore$ $h[n] \neq 0$



system
NOT casal.

for stability check. $S = \sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=-\infty}^{3} 5^n.$

228 σ:ξω

(3)

let $l = -k$.

$$\therefore S = \sum_{l=3}^{\infty} 5^{-l}$$ 1st term not 0

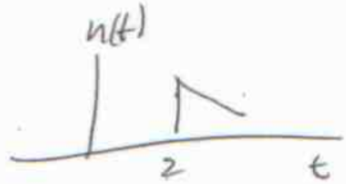
$$= 5^{-3} \sum_{k=0}^{\infty} \left(\frac{1}{5}\right)^{l}$$

$$= \frac{1}{125} \frac{1}{1-\frac{1}{5}} = \frac{1}{125} \frac{5}{4} = 0.01 < \infty$$

$\therefore$ System is **BIBO stable**

$$0 \;\&\; \mathcal{W}$$ ①

2.29 a) $h(t) = e^{-4t}\, u(t-2)$



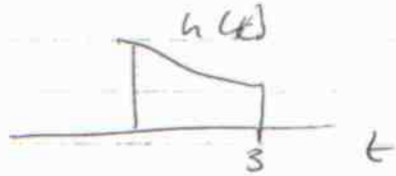
system is <u>causal</u> since $\nexists\; t < 0$ s.t. $h(t) \neq 0$.

for stability check absolutely integrable

$$S = \int_{-\infty}^{\infty} |h(t)|\, dt = \int_{2}^{\infty} e^{-4t}\, dt = \left[-\frac{1}{4} e^{-4t}\right]_{2}^{\infty}$$

$$= \frac{1}{4} e^{-8} < \infty \qquad \therefore \text{ system } \underline{\text{stable}}$$

b) $h(t) = e^{-6t}\, u(3-t)$



System is <u>not</u> causal since $\exists\; t < 0$ (eg $t=-1$) s.t. $h(t) \neq 0$

for stability check

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{3} e^{-6t}\, dt = \left[-\frac{1}{6} e^{-6t}\right]_{-\infty}^{3}$$

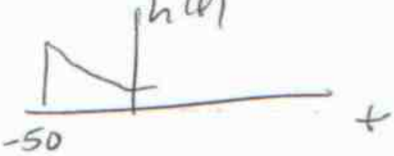
$$= \left[-\frac{1}{6} e^{-18} + \frac{1}{6} \lim_{t \to \infty} e^{6t}\right] \quad \text{no limit}$$

$\therefore$ System <u>not</u> stable

2.29. O & W

(2)

c) $h(t) = e^{-2t}\, u(t+50)$



System is <u>NOT causal</u> since $\exists\ t<0$ s.t. $h(t) \neq 0$ ex $t=-1$

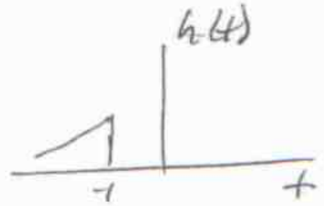
stability check

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-50}^{\infty} e^{-2t}\, dt = \left[-\frac{1}{2} e^{-2t}\right]_{-50}^{\infty}$$

$$= \frac{1}{2} e^{-100} < \infty$$

$\therefore$ System is <u>stable</u>

d) $h(t) = e^{2t}\, u(-1-t)$



System is <u>NOT causal</u> since $\exists\ t<0$ s.t. $h(t) \neq 0$ ex $t=-2$

for stability check.

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{-1} e^{2t}\, dt$$

$$= \left[\frac{1}{2} e^{2t}\right]_{-\infty}^{-1} = \frac{1}{2} e^{-2} < \infty$$

$\therefore$ system <u>stable</u>