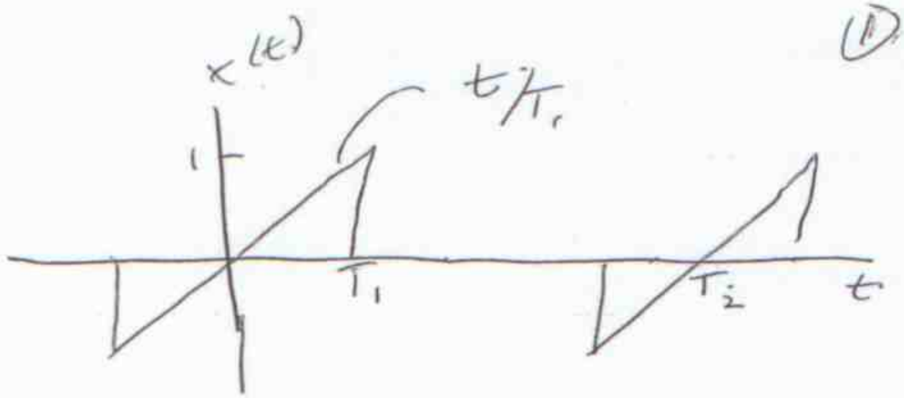
3.22

(1)

1st Consider



$a_0 = 0$

$$a_k = \frac{1}{T_2}\int_{-T_1}^{T_1} \frac{t}{T_1} e^{-j\omega_0 k t}\, dt$$

$$= \frac{1}{T_1 T_2}\left[-\frac{t\, e^{-j\omega_0 k t}}{(j\omega_0 k)} - \frac{e^{-j\omega_0 k t}}{(j\omega_0 k)^2}\right]_{-T_1}^{T_1}$$

$$= \frac{1}{T_1 T_2}\left[\frac{-T_1 e^{-j\omega_0 k T_1}}{j\omega_0 k} - \frac{e^{-j\omega_0 k T_1}}{(j\omega_0 k)^2} - \frac{T_1 e^{j\omega_0 k T_1}}{j\omega_0 k} + \frac{e^{+j\omega_0 k T_1}}{(j\omega_0 k)^2}\right.$$

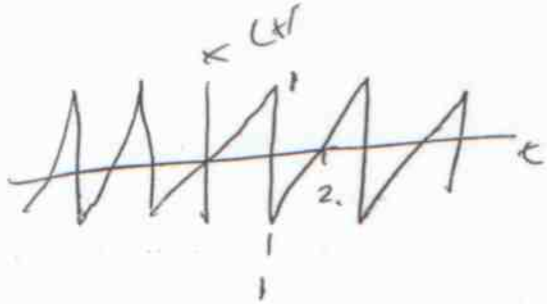
$$= \frac{1}{T_1 T_2}\left[-\frac{T_1}{j\omega_0 k} 2\cos\omega_0 k T_1 + \frac{2j\sin\omega_0 k T_1}{(j\omega_0 k)^2}\right]$$

$$= \frac{1}{T_1 T_2}\left[\frac{2jT_1}{\omega_0 k} \cancel{2}\cos\omega_0 k T_1 - \frac{2j\sin\omega_0 k T_1}{(\omega_0 k)^2}\right]$$

← purely imag.
note odd function

3.22

a)



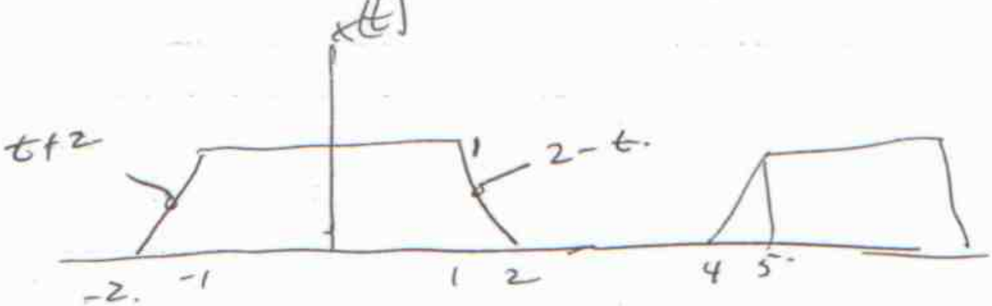
$T_1 = 1$
$T_2 = 2$
$\omega_0 = \frac{2\pi}{T_2} = \pi$

$\therefore \quad a_0 = 0$

$$a_k = \frac{1}{2}\left[\frac{2j}{\pi k}\cos \pi k - \cancel{\frac{2j \sin \pi k}{(\pi k)^2}}^{\,0}\right]$$

$$= \frac{j(-1)^k}{\pi k}$$

b)



Too messy to do with properties do directly

$T = 6, \quad \omega_0 = \frac{2\pi}{6} = \pi/3$

$$a_0 = \frac{1}{6}(2+1) = \underline{\underline{1/2}}$$

$$a_k = \frac{1}{6}\left[\int_{-2}^{-1}(t+2)e^{-j\omega_0 kt}\,dt + \int_{-1}^{1} e^{-j\omega_0 kt}\,dt + \int_{1}^{2}(2-t)e^{-j\omega_0 kt}\,dt\right]$$

$$= \frac{1}{6}\left\{\left[\frac{te^{-j\omega_0 kt}}{j\omega_0 k} - \frac{e^{-j\omega_0 kt}}{(j\omega_0 k)^2} - \frac{2e^{-j\omega_0 kt}}{j\omega_0 k}\right]_{-2}^{-1}\right.$$

$$\left. + \left[-\frac{e^{-j\omega_0 kt}}{j\omega_0 k}\right]_{-1}^{1} + \left[-\frac{2e^{-j\omega_0 kt}}{j\omega_0 k} + \frac{te^{-j\omega_0 kt}}{j\omega_0 k} + \frac{e^{-j\omega_0 kt}}{(j\omega_0 k)^2}\right]_{1}^{2}\right\}$$

3.22

③

$$a_k = \frac{1}{6}\Bigg\{\left[\cancel{\frac{e^{j\omega_0 k}}{j\omega_0 k}} - \frac{e^{j\omega_0 k}}{(j\omega_0 k)^2} \cancel{-2\frac{e^{j\omega_0 k}}{j\omega_0 k}} \cancel{- \frac{2e^{j\omega_0 k 2}}{j\omega_0 k}}\right.$$

$$\left. + \frac{e^{2j\omega_0 k}}{(j\omega_0 k)^2} \cancel{+ 2\frac{e^{2j\omega_0 k}}{j\omega_0 k}}\right] + \left[\cancel{-\frac{e^{-j\omega_0 k}}{j\omega_0 k}} \cancel{+ \frac{e^{j\omega_0 k}}{j\omega_0 k}}\right]$$

$$+ \left[\cancel{\frac{-2e^{-2j\omega_0 k}}{j\omega_0 k}} \cancel{+ \frac{2e^{-2j\omega_0 k}}{j\omega_0 k}} + \frac{e^{-2j\omega_0 k}}{(j\omega_0 k)^2} \cancel{+ \frac{2e^{-j\omega_0 k}}{j\omega_0 k}} \cancel{- \frac{e^{-j\omega_0 k}}{j\omega_0 k}}\right.$$

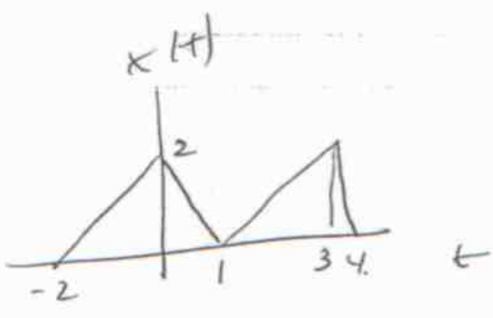
$$\left. - \frac{e^{-j\omega_0 k}}{(j\omega_0 k)^2}\right]\Bigg\}$$

$$= \frac{1}{6(-1)\left(\frac{\pi}{3}\right)^2 k^2}\left[-2\cos\frac{\pi}{3}k + 2\cos\frac{2\pi}{3}k\right]$$

$$= \frac{3}{\pi^2 k^2}\left[-\cos\frac{2\pi}{3}k + \cos\frac{\pi}{3}k\right]$$

└ zero for k even
note purely real makes sense
since orig. function even

c)

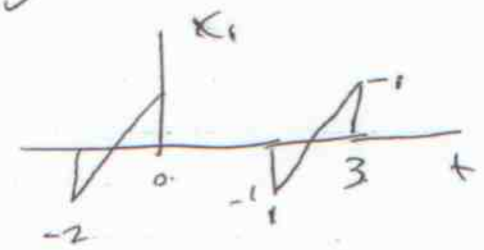


3.22 (4)

1st

$$a_0 = \frac{1}{3}(3)(2)\left(\frac{1}{2}\right) = 1$$

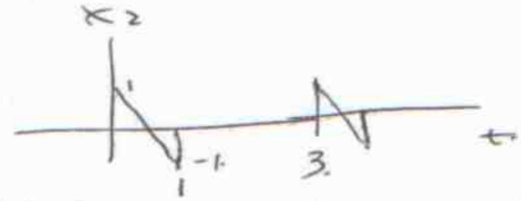
Neglecting DC we have.



$T = 3$

$\omega_0 = \frac{2\pi}{3}$

for both



$x = x_1 + x_2 + 1 \leftarrow$ DC value

x_1 is like what we have on 1st page of this question. $T_1 = 1$

~~delay~~. advance. 1

$$a_{k1} = \frac{1}{3}\left[\frac{2j}{\frac{2\pi}{3}k}\cos\frac{2\pi}{3}k - \frac{2j\sin\frac{2\pi}{3}k}{\left(\frac{2\pi}{3}k\right)^2}\right] e^{j\frac{2\pi}{3}k}$$

$$= \left(\frac{j}{\pi k}\cos\frac{2\pi}{3}k - \frac{3}{2}j\frac{\sin\frac{2\pi}{3}k}{\pi^2 k^2}\right) e^{j\frac{2\pi}{3}k}$$

for x_2 is like 1st page $T_1 = 1/2$.

delay by 1/2.

$$a_{k2} = -\frac{2}{3}\left[\frac{2j\,\frac{1}{2}}{\frac{2\pi}{3}k}\cos\left(\frac{\cancel{2}\pi}{3}k\frac{1}{\cancel{2}}\right) - \frac{2j}{\left(\frac{2\pi}{3}k\right)^2}\sin\left(\frac{\cancel{2}\pi}{3}k\frac{1}{\cancel{2}}\right)\right] e^{-j\frac{\cancel{2}\pi}{3}\frac{1}{\cancel{2}}k}$$

3.22 (5)

$$a_{k2} : \left[\frac{-j}{\pi k} \cos \frac{\pi}{3} k + \frac{3j}{\pi^2 k^2} \sin \frac{\pi}{3} k \right] e^{-j\frac{\pi}{3}k}$$

$k \neq 0$ $\quad a_k = a_{k1} + a_{k2}$

$$= \frac{j}{\pi k} \left[\cos \frac{2\pi}{3} k \left(\cos \frac{2\pi}{3} k + j \sin \frac{2\pi}{3} k \right) - \cos \frac{\pi}{3} k \left(\cos \frac{\pi}{3} k - j \sin \frac{\pi}{3} k \right) \right]$$

$$+ \frac{3j}{2\pi^2 k^2} \left[- e^{j 2\pi/3 k} \sin \frac{2\pi}{3} k + 2 \sin \frac{\pi}{3} k \, e^{-j \pi/3 k} \right]$$

1st. Bracket $\rightsquigarrow$ the $\cos 2\pi$

S	A
T	C

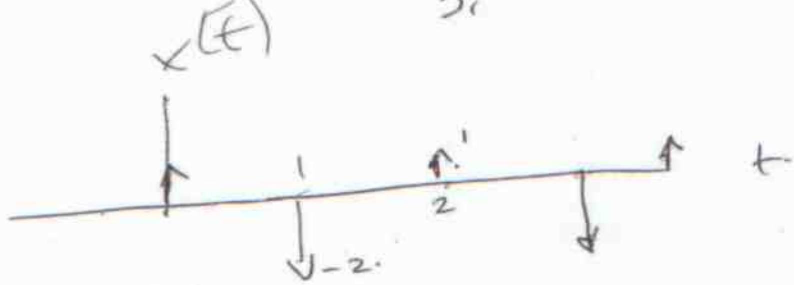
$$k=0 \quad \left[1(1+0) - 1(1-0) = 0 \right]$$
$$k=1 : \quad -\frac{1}{2}\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) - \frac{1}{2}\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = 0$$
$$k=2 \quad -\frac{1}{2}\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + \frac{1}{2}\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = 0$$
$$k=3 \quad 1(1+0) + 1(-1+0) = 0$$
$$k=4 \quad -\frac{1}{2}\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + \frac{1}{2}\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = 0$$
$$k=5 \quad -\frac{1}{2}\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) - \frac{1}{2}\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = 0$$

1st bracket always 0.

$$a_k = \frac{3j}{2\pi^2 k^2} \left[2 \sin \frac{\pi k}{3} e^{-j\frac{\pi}{3}k} - e^{j\frac{2\pi}{3}k} \sin \frac{2\pi}{3} k \right]$$

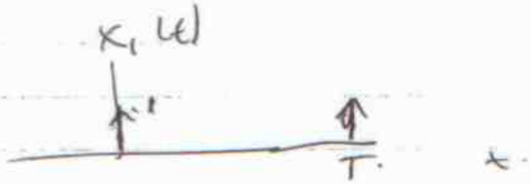
3.22

d)



$$T = 2 \qquad \omega_0 = \frac{2\pi}{2} = \pi$$

1st:



$$\omega_0 = \frac{2\pi}{T}$$

$$a_{01} = \frac{1}{T}$$

$$a_{k1} = \frac{1}{T}\int_T \delta(t)\, e^{-j\omega_0 k t}\, dt$$

$$= \frac{1}{T}$$

∴ for above

$$x(t) = x(t) - 2x(t-1)$$

$$\boxed{a_0 = -1/2}$$

$$a_k = a_{k1} - 2a_{k1} e^{-j\omega_0 k}$$

$$= \frac{1}{T} - 2\frac{1}{T} e^{-j\pi k}$$

$$= \frac{1}{2} - (-1)^k$$

3.22 ⑦

e) Use ex. 3.5 in book

call ex. 3.5 x_1

here. $x(t) = x_1(t + 3/2) - x_1(t - 3/2)$

$\omega_0 = \frac{2\pi}{6} = \pi/3 \quad T = 6$

$T_1 = 1/2$

$a_0 = 0$

$$a_k = a_{k1} e^{j\omega_0 k \frac{3}{2}} - a_{k1} e^{-j\omega_0 k \frac{3}{2}}$$

$$= \frac{\sin \frac{\pi}{3} k \frac{1}{2}}{\pi k} e^{j\frac{\pi}{2}k} - \frac{\sin \frac{\pi}{3} k \frac{1}{2}}{\pi k} e^{-j\pi/2 k}$$

$$= \frac{\sin \frac{\pi}{6} k}{\pi k} \left(j^k - (-j)^k \right)$$

$$= \frac{\sin \pi/6\, k}{\pi k} \left(j^k - (-1)^k j^k \right)$$

$\}$ clearly zero when k is even.

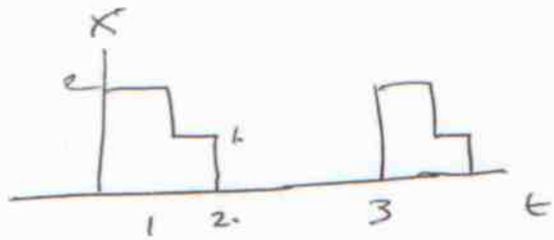
when k odd

$$= \frac{2j^k \sin \pi/6\, k}{\pi k}$$

k odd $\therefore$ purely imag. makes sense since $x(t)$ is odd!

3.22

f)



If the waveform in Ex. 3.5 in book is x_1 then

$T = 3, \quad T_1 = 1/2$

$\omega_0 = \frac{2\pi}{3}$

$$x(t) = 2x_1(t - 1/2) + x_1(t - 3/2)$$

$$a_0 = \frac{3}{3} = 1$$

$$a_k = 2a_{k_1} e^{-j\omega_0 k \frac{1}{2}} + a_{k_1} e^{-j\omega_0 k \, 3/2}$$

$$= 2\frac{\sin\left(\frac{2\pi}{3} k \frac{1}{2}\right)}{\pi k} e^{-j\frac{\pi}{3}k} + \frac{\sin \frac{2\pi}{3} k \frac{1}{2}}{\pi k} e^{-j\pi k}$$

$$= \frac{\sin \frac{\pi}{3} k}{\pi k}\left[2e^{-j\pi/3 k} + (-1)^k \right]$$

3.22. (a) (i) $T = 1$, $a_0 = 0$, $a_k = \frac{j(-1)^k}{k\pi}$, $k \neq 0$.

(ii) Here,

$$x(t) = \begin{cases} t+2, & -2 < t < -1 \\ 1, & -1 < t < 1 \\ 2-t, & 1 < t < 2 \end{cases}$$

$T = 6$, $a_0 = 1/2$, and

$$a_k = \begin{cases} 0, & k \text{ even} \\ \frac{4}{\pi^2 k^2} \sin(\frac{\pi k}{2}) \sin(\frac{\pi k}{6}), & k \text{ odd} \end{cases}$$

(iii) $T = 3$, $a_0 = 1$, and

$$a_k = \frac{3j}{2\pi^2 k^2}[e^{jk2\pi/3} \sin(k2\pi/3) + 2e^{jk\pi/3} \sin(k\pi/3)], \qquad k \neq 0.$$

(iv) $T = 2$, $a_0 = -1/2$, $a_k = \frac{1}{2} - (-1)^k$, $k \neq 0$.

(v) $T = 6$, $\omega_0 = \pi/3$, and

$$a_k = \frac{\cos(2k\pi/3) - \cos(k\pi/3)}{jk\pi/3}.$$

Note that $a_0 = 0$ and $a_{k\ \text{even}} = 0$.

(vi) $T = 4$, $\omega_0 = \pi/2$, $a_0 = 3/4$ and

$$a_k = \frac{e^{-jk\pi/2} \sin(k\pi/2) + e^{-jk\pi/4} \sin(k\pi/4)}{k\pi}, \qquad \forall k.$$

(b) $T = 2$, $a_k = \frac{(-1)^k}{2(1+jk\pi)}[e - e^{-1}]$ for all k.

(c) $T = 3$, $\omega_0 = 2\pi/3$, $a_0 = 1$ and

$$a_k = \frac{2e^{-j\pi k/3}}{\pi k} \sin(2\pi k/3) + \frac{e^{-j\pi k}}{\pi k} \sin(\pi k).$$

3.23 ①

a

$$a_k = \begin{cases} 0 & k=0 \\ j^k \dfrac{\sin k\pi/4}{\pi k} & \text{o.w} \end{cases}$$

$$T = 4$$

$$\therefore \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

manipulating

$$= \begin{cases} 0 & k=0 \\ e^{j\pi/2 k} \dfrac{\sin k \frac{\pi}{2} \frac{1}{2}}{\pi k} & \text{o.w.} \end{cases}$$

Now.

$$= \begin{cases} 0 & k=0 \\ e^{j\omega_0 k} \dfrac{\sin k\omega_0 1/2}{k\pi} & \text{o.w.} \end{cases}$$

for square wave Ex. 5.3

$$\frac{\sin k\omega_0 T_1}{k\pi} \qquad \therefore T_1 = 1/2$$

shift left

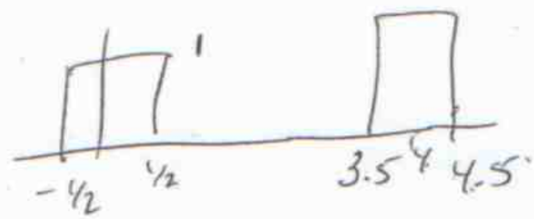
advance by 1

DC value not like Ex. 5.3
there has been a vertical shift
to give DC value 0.
without shift (vert or horiz)

3.23

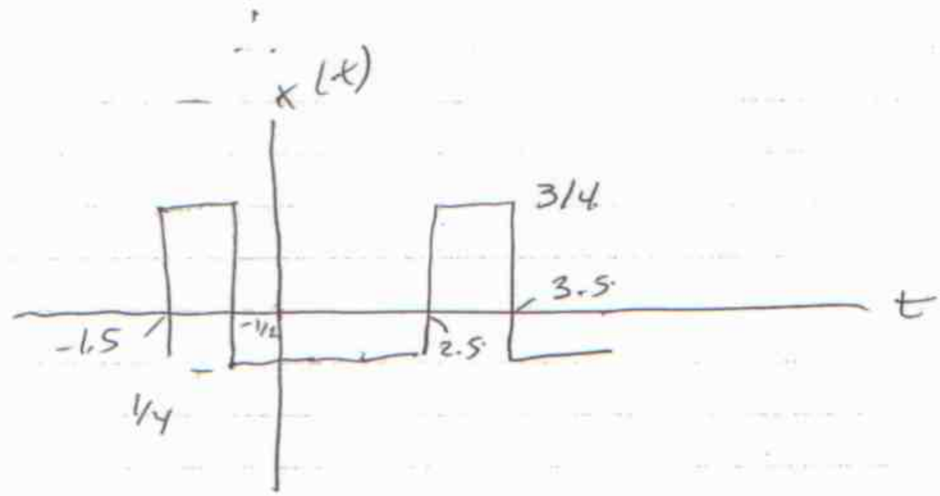
(2)

we would have.



DC value. $1/4$.

to get DC value of 0 shift down by $1/4$



b) $a_k = (-1)^k \dfrac{\sin k\pi/8}{2k\pi}$ $T = 4$, $\omega_0 = \pi/2$

again form of sq. wave.

$$a_k = \frac{1}{2} e^{j\pi k} \frac{\sin k \frac{\pi}{2} \frac{1}{4}}{k\pi}$$

$$= \frac{1}{2} \underbrace{e^{j\frac{\pi}{2}k2}}_{\text{advance i.e. shift left by 2.}} \underbrace{\frac{\sin k\,\omega_0\, 1/4}{k\pi}}_{\text{sq. wave}} \quad T_1 = 1/4.$$

size. [illegible]lt by 0.5

3.23 ③

Check. $a_0 = \frac{1}{2} \lim_{k \to 0} \frac{\sin k\omega_0 \, 1/4}{k\pi}$

$= \frac{1}{2} \lim_{k \to 0} \omega_0/4 \, \frac{\cancel{\sin}\cos k\omega_0 \, 1/4}{\pi}$

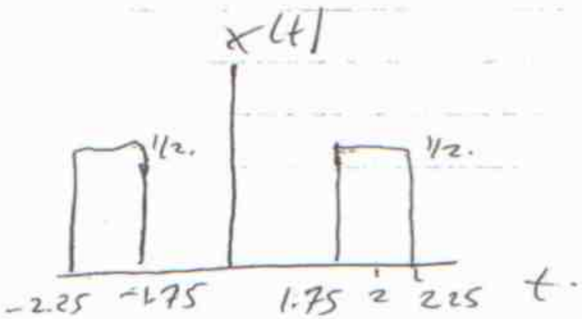
$= \frac{1}{\cancel{2}} \, \frac{\cancel{2\pi}}{4} \, \frac{1}{4} \, \frac{1}{\cancel{\pi}}$

$= \frac{1}{16}$

In 5.3 $a_0 = \frac{2T_1}{T} = \frac{1/2}{4} = \frac{1}{8}$ but this is for height 1 on sq wave we have height 1/2.

∴ ~~should be~~ for zero vert shift. a_0 should be ½ this i.e. $\frac{1}{16}$ which it is.

∴ no vert shift.



4

3.23 (Continue)

c)

$$a_k = \begin{cases} jk & |k| < 3 \\ 0 & \text{otherwise} \end{cases}, \ T = 4, \ \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{2\pi}{2}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{k\omega_0 t} = \sum_{k=-2}^{2} jke^{k\omega_0 t} = \overbrace{-2je^{-2\omega_0 t} \underbrace{-je^{-\omega_0 t} + je^{\omega_0 t}}_{-2\sin\omega_0 t} + 2je^{2\omega_0 t}}^{-4\sin 2\omega_0 t}$$

$$= -2\sin\omega_0 t - 4\sin 2\omega_0 t$$

Note:

- ***sin*** is odd symmetric, so $x(t)$ is odd symmetric as well.
- This make sense as a_k was purely imaginary!

d)

$$a_k = \begin{cases} 1 & k \text{ even.} \\ 2 & k \text{ odd} \end{cases} \qquad T = 4, \quad \omega_0 = \pi/2.$$

Flat makes us think of δ's. (see 3.22. d).

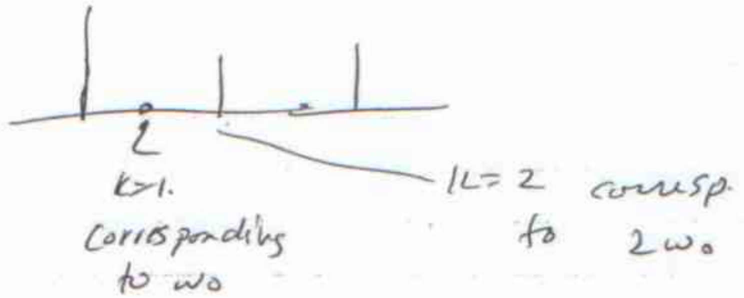
let $~~b_k~~$ =

$$b_k = \begin{cases} 1 & k \text{ even.} \\ 0 & k \text{ odd} \end{cases}$$

$$c_k = \begin{cases} 2 & k \text{ odd} \\ 0 & k \text{ even} \end{cases}$$

$$a_k = b_k + c_k.$$

for b_k we have $x_1(t)$



$\therefore$ this will be a train of imp. whose fundamental freq is $2\omega_0$

$$\therefore \quad T = \frac{2\pi}{2\omega_0} = \frac{2\pi(4)}{2\;2\pi} = 2$$

$$\omega_0 = \frac{2\pi}{4}$$

$$\text{Now} \quad x_1(t) = \sum_{i=-\infty}^{\infty} K\,\delta(t-2i)$$

would have coeff. $\frac{K}{2}$

but we have. $b_k = 1$.

$\therefore K = 2.$

$$\therefore x_1(t) = 2\sum_{i=-\infty}^{\infty} \delta(t-2i)$$

for c_k. similar to above but shift in freq by 1 term. $\therefore$ mult by $e^{j\omega_0 t}$ as above $\omega_0 = \frac{\pi}{2}$

note c_k are twice as big as B_k

⑥

$$\therefore \quad x_2(t) = 4\sum_{i=-\infty}^{\infty} e^{\frac{j\pi t}{2}}\, \delta(t-2i)$$

$$= 4\sum_{i=-\infty}^{\infty} e^{\frac{j\pi 2i}{2}}\, \delta(t-2i)$$

$$= 4\sum_{i=-\infty}^{\infty} (-1)^i\, \delta(t-2i)$$

$$\therefore \quad x(t) = 2\sum_{i=-\infty}^{\infty} \delta(t-2i)\left[1+2(-1)^i\right]$$

3.44. ① $x(t)$ real
② $x(t)$ periodic. period $T = 6$.
F.S. coeff a_k.

③ $a_k = 0$ $k = 0,\ k > 2$.

④ $x(t) = -x(t-3)$

⑤ $\frac{1}{6}\int_{-3}^{3} |x(t)|^2\, dt = 1/2$ ⟵ avg energy!

⑥ a_1 is pos real

Show $x(t) = A\cos(Bt + c)$

from ③ & ① only a_1 & a_{-1} are non zero

and $a_1 = a_{-1}^*$ by ⑥ $a_1 = a_{-1}$

$\therefore\ x(t) = a_1 e^{j\omega_0 t} + a_1 e^{-j\omega_0 t}$

$= a_1\, 2\cos \omega_0 t.$

$\omega_0 = \frac{2\pi}{T} = \pi/3$

Now by Parseval

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{n=\langle N\rangle} |a_k|^2$$

$$\frac{1}{2} = a_1^2 + a_{-1}^2$$

$$a_1^2 = 1/4$$

$$a_1 = 1/2.$$

$\boxed{\therefore x(t) = \cos\frac{\pi}{3}t}$