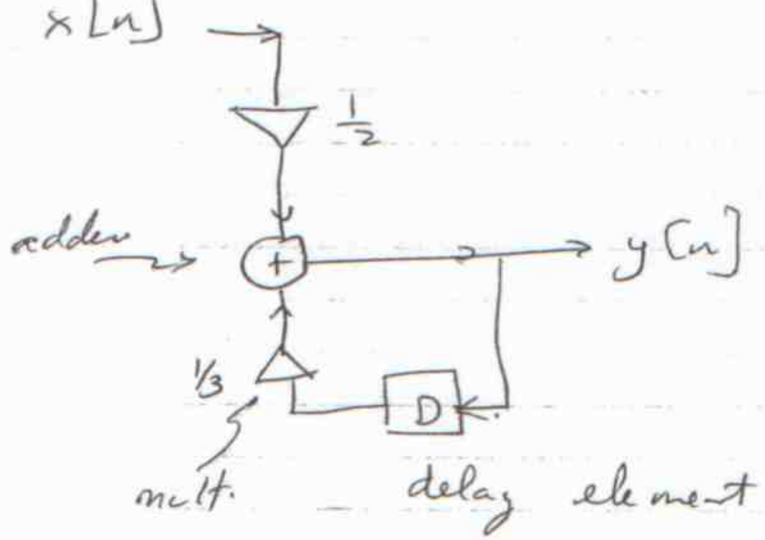
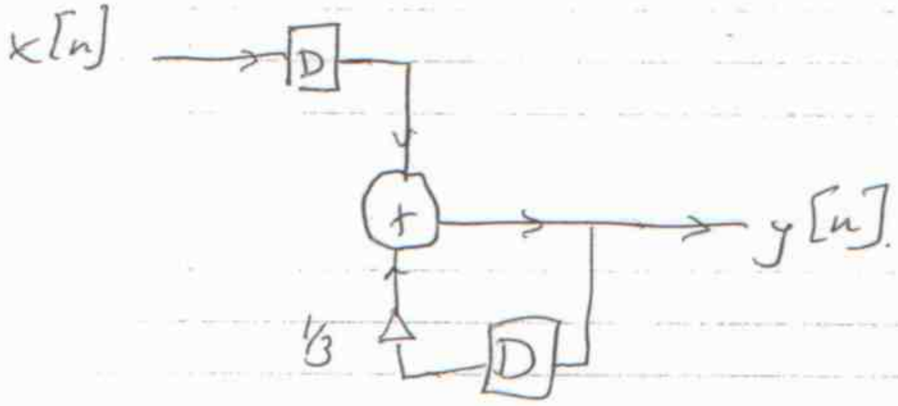
$0 \leq \omega$

1

2.38 a) $y[n] = \frac{1}{3} y[n-1] + \frac{1}{2} x[n]$



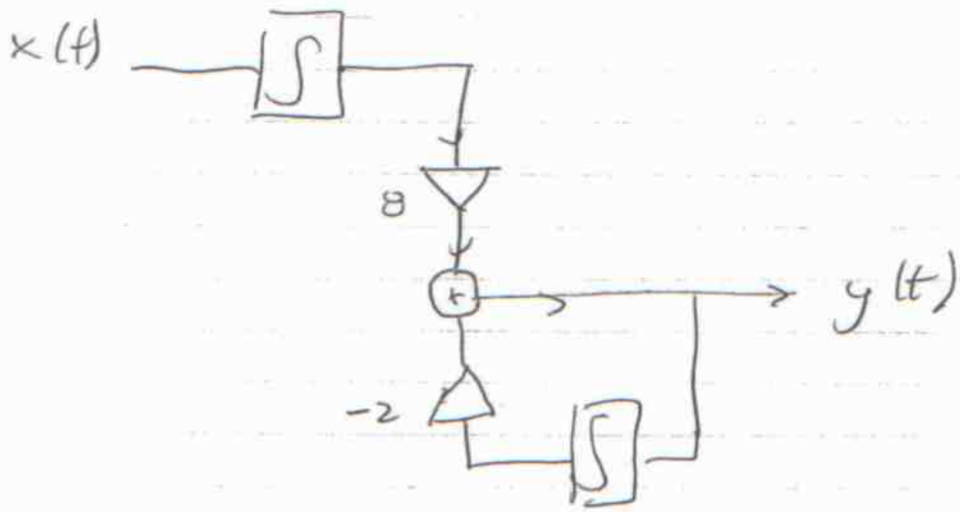
b) $y[n] = \frac{1}{3} y[n-1] + x[n-1]$



2.39 a) $y(t) = -\frac{1}{2}\frac{dy(t)}{dt} + 4x(t)$

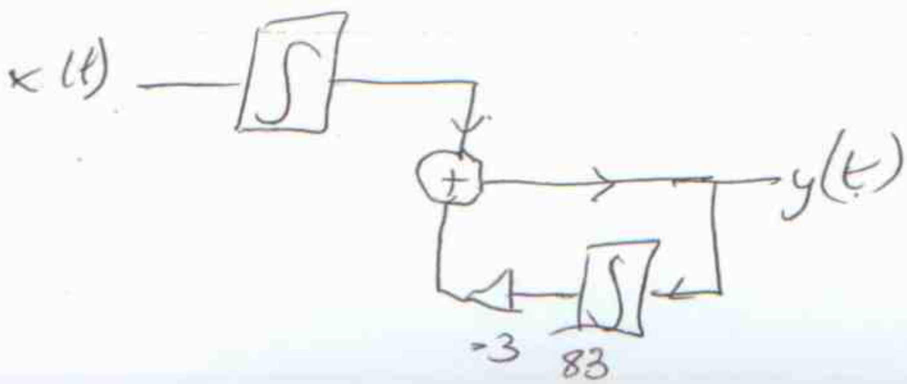
$$\frac{dy(t)}{dt} = 2\left[4x(t) - y(t)\right]$$

$$y(t) = 8\int x(t)\,dt - 2\int y(t)\,dt$$

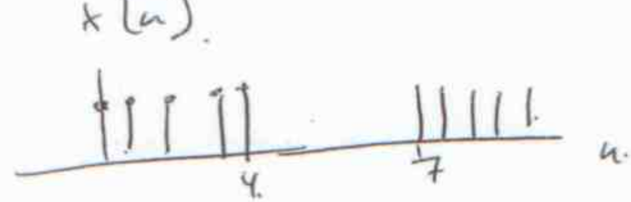


b) $\frac{dy}{dt} + 3y(t) = x(t)$

$$y(t) = \int x(t)\,dt - 3\int y(t)\,dt$$



3.28 a) a).



$a_0 = \frac{5}{7}$

$N = 7$, $\omega_0 = \frac{2\pi}{7}$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n]\, e^{-j\omega_0 k n}$$

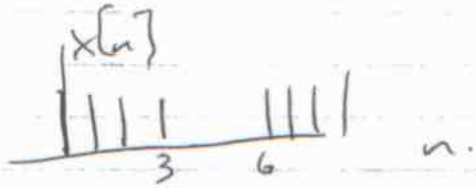
$$= \frac{1}{7} \sum_{n=0}^{4} e^{-j\omega_0 k n}$$

$$= \frac{1}{7} \frac{1 - e^{-5j\omega_0 k}}{1 - e^{-j\omega_0 k}}$$

$$= \frac{1}{7} \frac{e^{-2.5j\omega_0 k}}{e^{-0.5j\omega_0 k}} \quad \frac{2j \sin 2.5\omega_0 k}{2j \sin 0.5\omega_0 k}$$

$$= \frac{1}{7} e^{-2j\omega_0 k} \quad \frac{\sin \frac{5}{2}\omega_0 k}{\sin \frac{1}{2}\omega_0 k} \qquad k = 1, 2, \ldots N-1 = 1, 2, \ldots 6$$

a b)



$N = 6$

$\omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}$

$a_0 = \frac{4}{6} = \frac{2}{3}$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n]\, e^{-j\omega_0 k n}$$

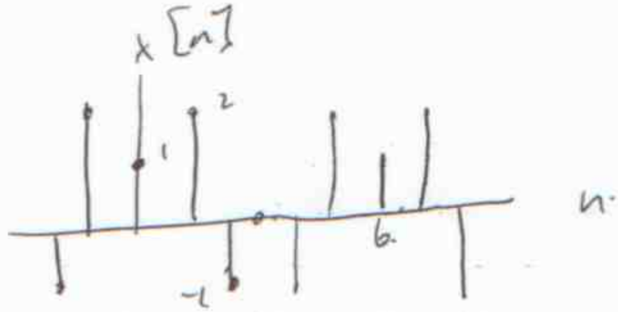
$$= \frac{1}{6} \sum_{n=0}^{3} e^{-j\omega_0 k n} = \frac{1}{6} \frac{1 - e^{-4j\omega_0 k}}{1 - e^{-j\omega_0 k}} = \frac{1}{6} e^{-\frac{3}{2} j \frac{\pi}{3} k} \frac{\sin 2\frac{\pi}{3}k}{\sin \frac{1}{2}\frac{\pi}{3}k}$$

$$= \frac{1}{6} (-j)^k \frac{\sin \frac{2\pi}{3}k}{\sin \frac{\pi}{6}k} \qquad k = 1, 2 \ldots 5$$

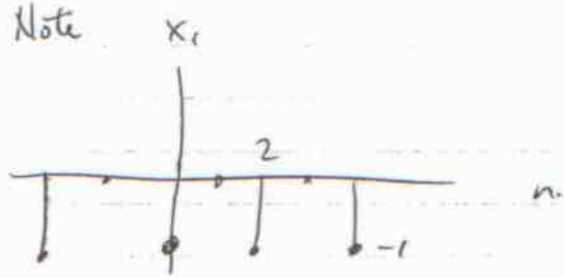
3.28

②

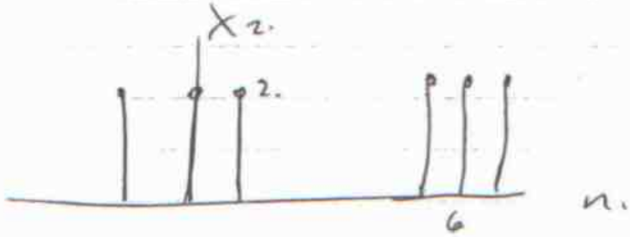
a c)



Note



and



$x = x_1 + x_2$

x_2 first.

$$a_{20} = \frac{6}{6} = 1 \qquad N = 6, \quad \omega_0 = \frac{2\pi}{6} = \pi/3$$

$$a_{2k} = \frac{1}{6} \sum_{n=\langle N \rangle} x_2[n] e^{-j\omega_0 n k}$$

$$= \frac{1}{6} \sum_{n=-1}^{1} 2\, e^{-j\omega_0 n k}$$

$$= \frac{1}{3} e^{j\omega_0 k} \sum_{n=0}^{2} e^{-j\omega_0 n k}$$

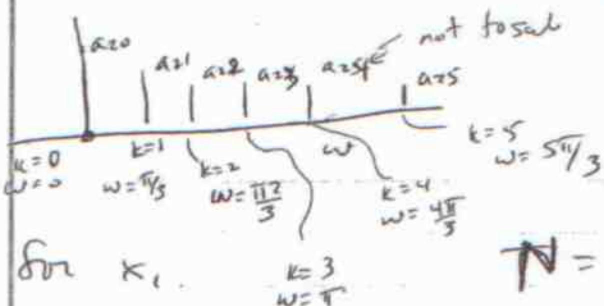
3.28

(3)

$$a_{2k} = \frac{1}{3} e^{j\omega_0 k} \frac{1 - e^{-j\omega_0 3k}}{1 - e^{-j\omega_0 k}}$$

$$= \frac{1}{3} \cancel{\frac{e^{j\omega_0 k} \; e^{-1.5 e j \omega_0 k}}{e^{-0.5 j \omega_0 k}}} \quad \frac{\sin 1.5\,\omega_0 k}{\sin 0.5\,\omega_0 k}$$

$$= \frac{1}{3} \frac{\sin \frac{\pi}{2} k}{\sin \pi/6\, k} \qquad k = 1, \ldots 5.$$



purely real
as expected since
time signal was even.

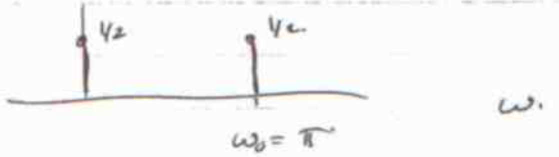
for x_1 $\quad N = 2.$

$$\omega_0 = \frac{2\pi}{2} = \pi$$

$$a_0 = -\frac{1}{2}.$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x_2[n] e^{-j\omega_0 nk}.$$

$$= -\frac{1}{2} e^{-j\omega_0 (0)k} = -1/2 \qquad k=1$$



3.28 (4)

look at graphs.

$a_0 = a_{10} + a_{20} = -\frac{1}{2} + 1 = 1/2.$

$a_i = a_{2i} = \frac{1}{3} \frac{\sin \frac{\pi}{2} k}{\sin \frac{\pi}{6} k}$ for $k = 1, 2, 4, 5$.

$a_3 = a_{23} + a_{13} = -\frac{1}{2} + \frac{1}{3} \frac{\sin \pi/2 k}{\sin \pi/6 k}.$

b) $x[n] = \sin \frac{2\pi n}{3} \cos \frac{\pi n}{2}$

$\omega = \frac{2\pi}{3}$ $\omega = \pi/2$

period $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi/2} = 4$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi/3} = 3$

When we mult 2 periodic functions together the result is probably periodic

with period $\text{lcm}(T_1, T_2)$

least common mult

T_1 period of 1

T_2 " " other

recall work in grades 4 & 5 to get "smallest common denominator" you were getting lcm.

Note if there is no lcm. then product is not periodic [like lcm $(2, \pi)$. I can think of no number that 2 goes in to evenly (integer) and π goes into evenly]

$$3.28$$

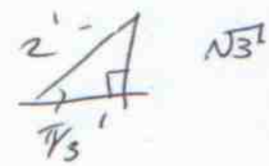
(5)

here. $\mathrm{lcm}(3,4) = \underline{\underline{12}} = N$ ↑ period

$$\omega = \frac{2\pi}{N} = \pi/6$$

a

n	$x[n]$
0	0
1	$\sin\frac{2\pi}{3}\cos\frac{\pi}{2} = 0$
2	$\sin\frac{4\pi}{3}\cos\pi = + \frac{\sqrt{3}}{2}$
3	$\sin 2\pi \cos\frac{3\pi}{2} = 0$
4	$\sin\frac{8\pi}{3}\cos 2\pi = \frac{\sqrt{3}}{2}$
5	$\sin\frac{10\pi}{3}\cos\frac{5\pi}{2} = 0$
6	$\sin 4\pi \cos 3\pi = 0$
7	$\sin\frac{14\pi}{3}\cos\frac{7\pi}{2} = 0.$
8	$\sin\frac{16\pi}{3}\cos 4\pi = -\frac{\sqrt{3}}{2}$
9	$\sin\frac{18\pi}{3}\cos\frac{9\pi}{2} = 0$
10	$\sin\frac{20\pi}{3}\cos 5\pi = -\frac{\sqrt{3}}{2}$
11	$\sin\frac{22\pi}{3}\cos\frac{11\pi}{2} = 0.$



S A
T C

not zero problems

$a_0 = 0$

$$a_1 = \frac{1}{12}\frac{\sqrt{3}}{2}\left[e^{-j\frac{\pi}{6}2} + e^{-4j\frac{\pi}{6}} - e^{-8j\frac{\pi}{6}} - e^{-10j\frac{\pi}{6}}\right]$$

$$= \frac{1}{12}\frac{\sqrt{3}}{2}\left[- j\left[\sin\frac{\pi}{3} + \sin\frac{2\pi}{3}\right]\right]$$

$$= \frac{1}{12}\frac{\sqrt{3}}{2} \; -2j\,2\,\frac{\sqrt{3}}{2} = -\frac{1}{12}\frac{3}{2}j = -\frac{1}{4}j$$

155

3.28

⑥

$$a_2 = \frac{1}{12}\frac{\sqrt{3}}{2}\left[e^{-j\frac{\pi}{6}4} + e^{-j\frac{\pi}{6}4(2)} - e^{-8j\frac{\pi}{6}2} - e^{-10j\frac{\pi}{6}2}\right]$$

$$= \frac{\sqrt{3}}{24}\left[e^{-j\frac{2\pi}{3}} + e^{-j\frac{4\pi}{3}} - e^{-\frac{8\pi}{3}j} - e^{-\frac{10\pi j}{3}}\right]$$

$$= \frac{\sqrt{3}}{24}\left[e^{-j\frac{2\pi}{3}} + e^{-j4\pi/3} - e^{-2\pi/3} - e^{-4\pi/3}\right]$$

$$= 0.$$

$$a_3 = \frac{1}{12}\frac{\sqrt{3}}{2}\left[e^{-j\frac{\pi}{6}2(3)} + e^{-j\frac{\pi}{6}4(3)} - e^{-8j\frac{\pi}{6}(3)} - e^{-10\frac{\pi}{6}j(3)}\right]$$

$$= \frac{\sqrt{3}}{24}\left[e^{-j\pi} + e^{-2\pi j} - e^{-j4\pi} - e^{-5\pi j}\right]$$

$$= \frac{\sqrt{3}}{24}\left[-1 + 1 - 1 + 1\right] = 0$$

$$a_4 = \frac{1}{12}\frac{\sqrt{3}}{2}\left[e^{-j\frac{\pi}{6}2(4)} + e^{-4j\frac{\pi}{6}4} - e^{-8j\frac{\pi}{6}4} - e^{-10j\frac{\pi}{6}4}\right]$$

$$= \frac{\sqrt{3}}{24}\left[e^{-\frac{4\pi j}{3}} + e^{-8\pi j/3} - e^{-16\pi j/3} - e^{-20\pi j/3}\right]$$

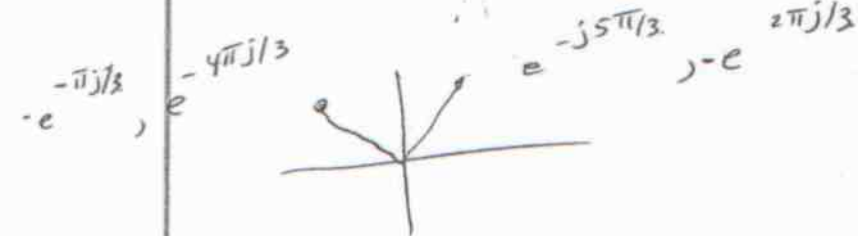
$$= \frac{\sqrt{3}}{24}\left[e^{-\frac{4\pi j}{3}} + e^{-2\pi j/3} - e^{-4\pi j/3} - e^{-2\pi j/3}\right]$$

$$= 0.$$

$$a_5 = \frac{1}{12}\frac{\sqrt{3}}{2}\left[e^{-j\frac{\pi}{6}(2)(5)} + e^{-4j\frac{\pi}{6}5} - e^{-8j\frac{\pi}{6}5} - e^{-10j\frac{\pi}{6}5}\right]$$

$$a_5 = \frac{\sqrt{3}}{24}\left[e^{-j\frac{5\pi}{3}} + e^{-\frac{10\pi j}{3}} - e^{-20\pi j/3} - e^{-25\pi/3 j} \right.$$

$$= \frac{\sqrt{3}}{24}\left[e^{-\frac{5\pi j}{3}} + e^{-4\pi j/3} - e^{-2\pi j/3} - e^{-\pi j/3} \right]$$



not zero
only imag part

$$= \frac{\sqrt{3}}{24}\left[2j \sin\frac{\pi}{3} \right] = \frac{\sqrt{3}}{24}\, 2j\, 2\frac{\sqrt{3}}{2} = \frac{1}{4} j$$

$$a_6 = \frac{1}{12}\frac{\sqrt{3}}{2}\left[e^{-j\frac{\pi}{6}2(6)} + e^{-4j\frac{\pi}{6}6} - e^{-j8\frac{\pi}{6}6} - e^{-j10\frac{\pi}{6}6} \right]$$

$$= \frac{\sqrt{3}}{24}\left[e^{-2\pi j} + e^{-4\pi j} - e^{-8\pi j} - e^{-10\pi j} \right]$$

$$= 0$$

Now. Note that the time signal is real
$\therefore$ FS. coeff are conj symmetric

i.e. $a_k = a_{-k}^*$

also since we have discrete time the F.S. coeff are periodic in $N=12$.

i.e. $a_{11} = a_{-1}$
$a_{10} = a_{-2}$ etc

3.28 ⑧

$\therefore$ we have $a_1 = -\frac{1}{4}j$

and $a_{11} = a_{-1} = a_1^* = \frac{1}{4}j$

and $a_5 = \frac{1}{4}j$

and $a_7 = a_{-5} = a_{+5}^* = -\frac{1}{4}j$

all others are zero.

c) $x[n]$ periodic period $N=4$, $\omega_0 = \frac{2\pi}{4} = \pi/2$.

$x[n] = 1 - \sin \frac{\pi n}{4}$

n	$x[n]$
0	1
1	$1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{2}$
2	0
3	$1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{2}$

$\begin{array}{c|c} S & A \\ \hline T & C \end{array}$

$$a_0 = \frac{1}{4}\left[1 + 1 + 1 - \frac{2}{\sqrt{2}}\right]$$

$$= \frac{1}{4}\left[3 - \sqrt{2}\right]$$

$$a_1 = \frac{1}{4}\left[1 + \left(1 - \frac{1}{\sqrt{2}}\right)\left(e^{-j\pi/2} + e^{-j\frac{3\pi}{2}}\right)\right]$$

$$= \frac{1}{4}\left[1 + \left(1 - \frac{1}{\sqrt{2}}\right)\left(-j + j\right)\right] = \frac{1}{4}$$

3.28

9

$$a_2 = \frac{1}{4}\left[1 + \left(1 - \frac{1}{\sqrt{2}}\right)\left[e^{-j\frac{\pi}{2}2} + e^{-j\frac{3\pi}{2}2}\right]\right]$$

$$= \frac{1}{4}\left[1 + \left(1 - \frac{1}{\sqrt{2}}\right)\left[-1 \; -1\right]\right]$$

$$= \frac{1}{4}\left(1 - 2 + \frac{2}{\sqrt{2}}\right) = \frac{1}{4}\left(-1 + \sqrt{2}\right)$$

since time signal is real.

$$a_3 = a_{-1} = a_1^* = \frac{1}{4}.$$

$$a_0 = \frac{1}{4}\left(3 - \sqrt{2}\right)$$

$$a_1 = 1/4 = a_3$$

$$a_2 = \frac{1}{4}\left(-1 + \sqrt{2}\right)$$

d) $x[n] = 1 - \sin\frac{\pi n}{4}$

$N = 12$

$\omega_0 = \frac{2\pi}{12} = \frac{\pi}{6}$

(period of sinusoid is $\frac{2\pi}{\pi/4} = 8$.

$\therefore$ cycle & half

not beautiful

(10)

$$a_k = \frac{1}{12} \sum_{n=0}^{11} x[n] \; e^{-j\omega_0 kn}$$

$$= \frac{1}{12} \sum_{n=0}^{11} \left(1 - \sin\frac{\pi}{4}n\right) e^{-j\omega_0 kn}$$

$$= \frac{1}{12}\left[\sum_{n=0}^{11} e^{-j\omega_0 kn} - \frac{1}{2j}\sum_{n=0}^{11}\left(e^{j(\frac{\pi}{4}n - \frac{\pi}{6}kn)} - e^{-j(\frac{\pi}{4}n + \frac{\pi}{6}kn)}\right)\right.$$

$$= \frac{1}{12} \; \frac{1 - e^{-j\omega_0 k 12}}{1 - e^{-j\omega_0 k}} - \frac{1}{24j}\left[\frac{1 - e^{j(\frac{\pi}{4} - \frac{\pi}{6}k)12}}{1 - e^{j(\pi/4 - \pi/6 k)}}\right.$$

$$\left. - \frac{1 - e^{-j(\frac{\pi}{4} + \frac{\pi}{6}k)12}}{1 - e^{-j(\pi/4 + \pi/6 k)}}\right]$$

$$= \frac{1}{12} \; \frac{1 - e^{-j2\pi k}}{1 - e^{-j\pi/6 k}} + \frac{j}{24}\left[\frac{1 - e^{j3\pi - 2\pi k j}}{1 - e^{j[\pi/4 - \pi/6 k]}}\right.$$

$$\left. - \frac{1 - e^{-3\pi j - 2\pi kj}}{1 - e^{-j(\pi/4 + \pi/6 k)}}\right]$$

$$= \frac{1}{12} \; \frac{1 - e^{-2\pi jk}}{1 - e^{-j\pi/6 k}} + \frac{j}{24}\left[\frac{2}{1 - e^{j(\pi/4 - \pi/6 k)}} - \frac{2}{1 - e^{-j(\frac{\pi}{4} + \frac{\pi}{6}k)}}\right]$$

$$= \frac{1}{12} \; \frac{1 - e^{-2\pi jk}}{1 - e^{-j\pi/6 k}} + \frac{j}{12} \; \frac{1 - e^{-j(\pi/4 + \pi/6 k)} - 1 + e^{j(\pi/4 - \pi/6 k)}}{1 - e^{-j(\pi/4 + \pi/6 k)} - e^{j(\pi/4 - \pi/6 k)} + e^{\pi/3 k}}$$

$$= \frac{1}{12} \; \frac{1 - e^{-2\pi jk}}{1 - e^{-j\pi/6 k}} + \frac{j}{12} \; \frac{e^{j\pi/4} - e^{-j\pi/4}}{-e^{j\pi/4} - e^{-j\pi/4} + e^{j\pi/6 k} + e^{j\pi/6 k}}$$

3.28 (11)

$$a_k = \frac{1}{12} \frac{1-e^{-2\pi jk}}{1-e^{-j\pi/6k}} + \frac{j}{\cancel{12}6} \frac{\cancel{2}j \sin \pi/4}{-2\cos\pi/4 + e^{j\pi/6k} + e^{j\pi/2k}}$$

$$= \frac{1}{12} \frac{1-e^{-2\pi jk}}{1-e^{-j\pi/6k}} - \frac{1}{6\sqrt{2}} \frac{1}{-\sqrt{2}+e^{j\pi/6k}+e^{j\pi/2k}}$$

Now when $k=0$. 1st term is a problem

$$\lim_{k\to 0} \frac{1-e^{-2\pi jk}}{1-e^{-j\pi/6k}} = \lim_{k\to 0} \frac{2\pi j\, e^{-2\pi jk}}{j\frac{\pi}{6}\, e^{-j\pi/6k}} = 12$$

$$\therefore\ a_0 = 1 - \frac{1}{6\sqrt{2}} \frac{1}{2-\sqrt{2}}$$

for $k \neq 0$ 1st term is zero

$$a_k = \frac{1}{6\sqrt{2}} \frac{1}{-\sqrt{2}+e^{j\pi/6k}+e^{j\pi/2k}}$$

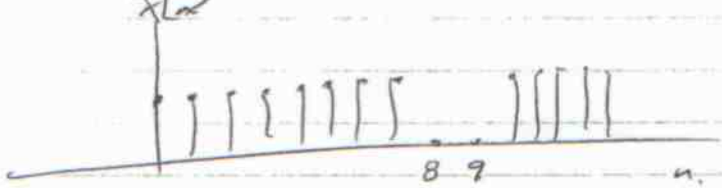
3.31 $x[n] = \begin{cases} 1 & 0 \le n \le 7 \\ 0 & 8 \le n \le 9 \end{cases}$

$N = 10$

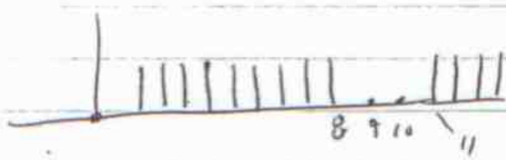
$g[n] \triangleq x[n] - x[n-1]$

Now

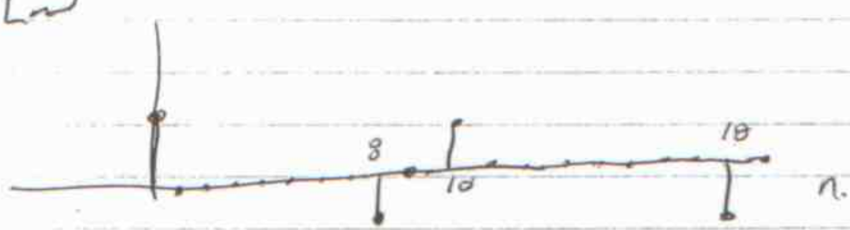
$g[n]$
$x[n]$



$x[n-1]$



$\therefore g[n]$



$\underline{\underline{N = 10}}$ $\qquad \omega_0 = \frac{2\pi}{N} = \frac{\pi}{5}$

call F.S. of $g[n]$ b_k

b) $b_0 = 0.$

$$b_k = \frac{1}{N} \sum_{n=\langle N \rangle} g[n] e^{-j\omega_0 nk} = \underline{\underline{\frac{1}{10}\left[1 - e^{-j\frac{\pi}{5}8k}\right]}}$$

3.31

(2)

c) get a_k of $x[n]$

by table.

$$b_k = \left(1 - e^{-jk\frac{\pi}{5}}\right) a_k.$$

$$a_k = \frac{b_k}{1 - e^{-jk\pi/5}}$$

$$= \frac{1}{10} \frac{1 - e^{-j\frac{\pi}{5}8k}}{1 - e^{-jk\pi/5}}$$

3.38

$$h[n] = \begin{cases} 1 & 0 \le n \le 2 \\ -1 & -2 \le n \le -1 \\ 0 & \text{o.w.} \end{cases}$$

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k].$$

get F.S. of $y[n]$ output.

[Note we need ch 5 to do this].

get freq Response of system.

$$H(e^{j\omega}) = 1 + e^{-j\omega} + e^{-2j\omega} - e^{+j\omega} - e^{+2j\omega}$$

$$= 1 - 2j\sin\omega - 2j\sin 2\omega$$

F.S. of $x[n]$ $\qquad T = 4 \qquad \omega_0 = \frac{2\pi}{4} = \pi/2$

$a_0 = 1/4$

$a_1 = 1/4$

$a_2 = 1/4$

$a_3 = 1/4$

$\therefore$ for b_k of $y(n)$

$$b_k = a_k \, H(e^{j\omega})\Big|_{\omega = \frac{2\pi}{N}k.}$$

$$b_k = \frac{1}{4}\left(1 - 2j\sin\frac{\pi}{2}k - \cancel{2j\sin 2\frac{\pi}{2}k}^{\,0}\right) = \frac{1}{4}\left(1 - 2j\sin\frac{\pi}{2}k\right)$$

①

3.48 $x[n]$ periodic period N.

$$x[n] = \sum_{k=\langle N\rangle} a_k e^{j\frac{2\pi}{N}kn}$$

a) F.S. of $x[n-n_0]$

$$b_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n-n_0]\, e^{-j\omega_0 kn}$$

let $m = n-n_0$

$$= \frac{1}{N}\sum_{m=\langle N\rangle} x[m]\, e^{-j\omega_0 k(m+n_0)}$$

$$= e^{-j\omega_0 kn_0}\left(\frac{1}{N}\sum_{m=\langle N\rangle} x[m]\, e^{-j\omega_0 km}\right)$$

$$= e^{-j\omega_0 kn_0}\, a_k$$

as expected

b) $x[n] - x[n-1]$.

$$b_k = \frac{1}{N}\sum_{n=\langle N\rangle}\left(x[n] - x[n-1]\right) e^{-j\omega_0 kn}$$

$$= \frac{1}{N}\sum_{n=\langle N\rangle} x[n]\, e^{-j\omega_0 kn} - \frac{1}{N}\sum_{n=\langle N\rangle} x[n-1]\, e^{-j\omega_0 kn}$$

Hilroy

using a part

$$b_k = a_k - e^{-j\omega_0 k} a_k$$

$$= \left(1 - e^{-j\omega_0 k}\right) a_k \qquad \text{as expected}$$

c) $x[n] - x[n - \frac{N}{2}]$

$$b_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] - x[n - \frac{N}{2}] \, e^{-j\omega_0 k n}$$

$$= \frac{1}{N} \sum_{n=\langle N\rangle} x[n] e^{-j\omega_0 k n} - \frac{1}{N} \sum_{n=\langle N \rangle} x[n - \frac{N}{2}] e^{-j\omega_0 k n}$$

$$= a_k - a_k e^{-j\omega_0 k \frac{N}{2}}$$

$$= \left(1 - e^{-j\frac{2\pi}{N} k \frac{N}{2}}\right) a_k$$

$$= \left(1 - (-1)^k\right) a_k$$

$$= \begin{cases} 2a_k & k \text{ odd} \\ 0 & k \text{ even} \end{cases}$$

Hilroy

3.48

③

d) $x[n] + x[n + N/2]$

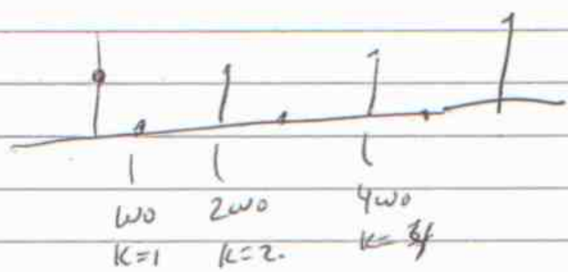
$$b_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] + x\left[n + \frac{N}{2}\right] e^{-j\omega_0 k n}$$

$$= a_k + e^{+j\omega_0 k \frac{N}{2}} a_k$$

$$= \left(1 + e^{+j\frac{2\pi}{N} k \frac{N}{2}}\right) a_k$$

$$= \left(1 + e^{j\pi k}\right) a_k = \begin{cases} 2a_k & k \text{ even} \\ 0 & \text{odd} \end{cases}$$

Note:



[illegible] is an [illegible] of all the odd [illegible] that the F.S. Series

Now only k even are non-zero
$\therefore$ we could write F.S as

$$x[n] = \sum_{\substack{k=\langle N \rangle \\ k \text{ even}}} a_k e^{j\frac{2\pi}{N} kn} = \sum_{l=\langle \frac{N}{2} \rangle} a_k e^{j\frac{2\pi}{N} 2l n}$$

$$= \sum_{l=\langle \frac{N}{2} \rangle} a_k e^{j\frac{2\pi}{N/2} ln} \quad \text{indicating a}$$

3.48 (4)

F.S. for a signal which is periodic in $\frac{N}{2}$!

We could also show $x[n] + x[n+\frac{N}{2}]$ is periodic in $\frac{N}{2}$ by direct proof.

So disappearance of all odd terms indicates period was too large by factor 2.

For Fun. Extend this to when period is too large by factor: 3, 4, L.

e) $x^*[-n]$

$$b_k = \frac{1}{N} \sum_{n=\langle N\rangle} x^*[-n]\, e^{-j\omega_0 kn}$$

$$= \left(\frac{1}{N} \sum_{n=\langle N\rangle} x[-n]\, e^{j\omega_0 kn}\right)^*$$

$m = -n.$

$$= \left(\frac{1}{N} \sum_{m=\langle N\rangle} x[m]\, e^{-j\omega_0 km}\right)^*$$

$$= a_k^*$$

f). $(-1)^n \times [n]$ N even c.o.w. discontinuity

$$b_k = \frac{1}{N} \sum_{n=\langle N \rangle} (-1)^n x[n] e^{-j\omega_0 kn}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} e^{-j\pi n} x[n] e^{-j\omega_0 kn}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jn(\omega_0 k + \pi)}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jn\left(\frac{2\pi}{N}k + \pi\right)}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jn\frac{2\pi}{N}\left(k + \frac{N}{2}\right)}$$

$$= a_{k+N/2}.$$

g) $(-1)^n x[n]$ • N odd.

when we guess wrong and make period too large (by an integer mult) the result will fall out by itself. i.e. F.S. coeff will disappear

but when we guess wrong and use too small a period we ~~have~~ get unreliable results

$\therefore$ it is important to notice that N ~~ed~~ ~~odd~~ odd will result in adjacent periods not being the same.

Note $y[n] = (-1)^n x[n]$

$y[0] = x[0]$

$y[N] = (-1)^N x[N]$ N odd

$= -x[0]$ not periodic in N

but $y[n+2N] = (-1)^{n+2N} x[n+2N]$

$= (-1)^n x[n]$

$= y[n]$ $\therefore$ periodic in 2N

Now

$\omega_0 = \frac{2\pi}{2N} = \pi/N$

$b_k = \frac{1}{2N} \sum_{n=\langle 2N \rangle} (-1)^n x[n] \, \cancel{\ } \, e^{-j\omega_0 k n}$

$= \frac{1}{2N} \left[\sum_{n=\langle N \rangle} e^{-j\pi n} x[n] e^{-j\frac{\pi}{N}kn} + \sum_{n=\langle N \rangle} e^{-j\pi(n+N)} x[n+N] e^{-j\frac{\pi}{N}k(n+N)} \right]$

$= \frac{1}{2N} \left[\sum_{n=\langle N \rangle} x[n] e^{-j\frac{2\pi}{N}n\left(\frac{k}{2}+\frac{N}{2}\right)} - \sum_{n=\langle N \rangle} x[n] e^{-j\frac{2\pi}{N}n\left(\frac{k}{2}+\frac{N}{2}\right)} e^{-j\pi k} \right]$

from $e^{-j\pi N}$

$= \frac{1}{2} \left(a_{\frac{k+N}{2}} - e^{-j\pi k} a_{\frac{k+N}{2}} \right)$

$= \begin{cases} a_{\frac{k+N}{2}} & k \text{ odd} \\ 0 & k \text{ even} \end{cases}$ (7) Note $k = 0, 1, 2, \ldots 2N-1$.

h) $y[n] = \begin{cases} x[n] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$

Note $y[n] = \frac{1}{2}\left(\underbrace{x[n]}_{\text{periodic in } N.} + \underbrace{(-1)^n x[n]}_{} \right)$

periodic in N. if N even
periodic in $2N$ if N odd

$\therefore$ N even $y[n]$ periodic in N.

$$b_k = \frac{1}{2}\left(a_k + a_{k+\frac{N}{2}}\right) \qquad k = 0, 1, \ldots N-1$$

if N odd $y[n]$ periodic in $2N$

$$b_k = \begin{cases} \frac{1}{2}\left(a_{k/2} + a_{(k+N)/2}\right) & k \text{ even.} \\ \frac{1}{2} a_{k/2} \end{cases}$$

two problems
$x[n]$ part is periodic in N but we want coeff in $2N$. Recall d) part.
all odd values will be 0
" even values will be the orig a_k

for $(-1)^n x[n]$ use result from g (Note even values 0

$$b_k = \begin{cases} \frac{1}{2} a_{\frac{k+N}{2}} & k \text{ odd} \quad (g) \text{ part} \\ \frac{1}{2} a_{\frac{k}{2}} & k \text{ even.} \end{cases}$$

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(c) $N = 8$. Over one period ($0 \le n \le 7$),

$$x[n] = 1 + (-1)^n + 2\cos\left(\frac{\pi n}{4}\right) + 2\cos\left(\frac{3\pi n}{4}\right).$$

(d) $N = 8$. Over one period ($0 \le n \le 7$),

$$x[n] = 2 + 2\cos\left(\frac{\pi n}{4}\right) + \cos\left(\frac{\pi n}{2}\right) + \frac{1}{2}\cos\left(\frac{3\pi n}{4}\right).$$

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(b) Here, $T = 2$ and $\omega_0 = \pi$ and

$$a_k = \begin{cases} 0, & k \text{ even} \\ 1, & k \text{ odd} \end{cases}.$$

Therefore, the FS coefficients of the output are

$$b_k = a_k H(jk\omega_0) = \begin{cases} 0, & k \text{ even} \\ \frac{1}{4+j\pi k} + \frac{1}{4-j\pi k}, & k \text{ odd} \end{cases}.$$

(c) Here, $T = 1$, $\omega_0 = 2\pi$ and

$$a_k = \begin{cases} 1/2, & k = 0 \\ 0, & k \text{ even}, k \neq 0 \\ \frac{\sin(\pi k/2)}{\pi k}, & k \text{ odd} \end{cases}.$$

Therefore, the FS coefficients of the output are

$$b_k = a_k H(jk\omega_0) = \begin{cases} 1/4, & k = 0 \\ 0, & k \text{ even}, k \neq 0 \\ \frac{\sin(\pi k/2)}{\pi k}\left[\frac{1}{4+j2\pi k} + \frac{1}{4-j2\pi k}\right], & k \text{ odd} \end{cases}.$$

3.36. We will first evaluate the frequency response of the system. Consider an input $x[n]$ of the form $e^{j\omega n}$. From the discussion in Section 3.9 we know that the response to this input will be $y[n] = H(e^{j\omega})e^{j\omega n}$. Therefore, substituting these in the given difference equation, we get

$$H(e^{j\omega})e^{j\omega n} - \frac{1}{4}e^{-j\omega}e^{j\omega n}H(e^{j\omega}) = e^{j\omega n}.$$

Therefore,

$$H(j\omega) = \frac{1}{1 - \frac{1}{4}e^{-j\omega}}.$$

From eq. (3.131), we know that

$$y[n] = \sum_{k=<N>} a_k H(e^{j2\pi k/N}) e^{jk(2\pi/N)n}$$

when the input is $x[n]$. $x[n]$ has the Fourier series coefficients a_k and fundamental frequency $2\pi/N$. Therefore, the Fourier series coefficients of $y[n]$ are $a_k H(e^{j2\pi k/N})$.

(a) Here, $N = 4$ and the nonzero FS coefficients of $x[n]$ are $a_3 = a_{-3}^* = 1/2j$. Therefore, the nonzero FS coefficients of $y[n]$ are

$$b_3 = a_3 H(e^{3j\pi/4}) = \frac{1}{2j(1 - (1/4)e^{-j3\pi/4})}, \qquad b_{-3} = a_{-3} H(e^{-3j\pi/4}) = \frac{-1}{2j(1 - (1/4)e^{j3\pi/4})}.$$

(b) Here, $N = 8$ and the nonzero FS coefficients of $x[n]$ are $a_1 = a_{-1} = 1/2$ and $a_2 = a_{-2} = 1$. Therefore, the nonzero FS coefficients of $y(t)$ are

$$b_1 = a_1 H(e^{j\pi/4}) = \frac{1}{2(1 - (1/4)e^{-j\pi/4})}, \qquad b_{-1} = a_{-1} H(e^{-j\pi/4}) = \frac{1}{2(1 - (1/4)e^{j\pi/4})},$$

$$b_2 = a_2 H(e^{j\pi/2}) = \frac{1}{(1 - (1/4)e^{-j\pi/2})}, \qquad b_{-2} = a_{-2} H(e^{-j\pi/2}) = \frac{1}{(1 - (1/4)e^{j\pi/2})}.$$