6

4.21 a) $x(t) = e^{-\alpha t} \cos \omega_0 t \; u(t) \qquad \alpha > 0$

$$= \frac{e^{-t(\alpha + j\omega_0)} + e^{-t(\alpha - j\omega_0)}}{2} u(t)$$

using table p. 329

$$X(j\omega) = \frac{1}{2}\left(\frac{1}{\alpha + j\omega_0 + j\omega} + \frac{1}{\alpha - j\omega_0 + j\omega} \right)$$

b) $x(t) = e^{-3|t|} \sin 2t$

let

$$x_1(t) = e^{-3t} \sin 2t \; u(t)$$

Note $x(t) = x_1(t) - x_1(-t)$

$$x_1(t) = \frac{e^{-t(3-2j)} - e^{-t(3+j2)}}{2j} u(t)$$

$$X_1(j\omega) = \frac{1}{2j}\left[\frac{1}{3-2j+j\omega} - \frac{1}{3+2j+j\omega} \right]$$

$$X(j\omega) = X_1(j\omega) - X(-j\omega)$$

$$= \frac{1}{2j}\left[\frac{1}{3-2j+j\omega} - \frac{1}{3+2j+j\omega} - \frac{1}{3-2j-j\omega} + \frac{1}{3+2j-j\omega} \right]$$

$$= \frac{1}{2j}\left[\frac{6}{9+(2-\omega)^2} - \frac{6}{9+(2+\omega)^2} \right] = \frac{3j}{9+(2+\omega)^2} - \frac{3j}{9+(2-\omega)^2}$$

c) $x(t) = \begin{cases} 1 + \cos\pi t & |t| \leq 1 \\ 0 & \text{o.w} \end{cases}$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-1}^{1} (1 + \cos\pi t)\, e^{-j\omega t}\, dt$$

$$= \int_{-1}^{1} e^{-j\omega t} + \frac{1}{2}\left(e^{-tj(\omega \cancel{\pi} - \pi)} + e^{-tj(\omega+\pi)} \right) dt$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega} + \frac{1}{2}\,\frac{e^{-tj(\omega-\pi)}}{-j(\omega-\pi)} + \frac{1}{2}\,\frac{e^{-tj(\omega+\pi)}}{-j(\omega+\pi)} \right]_{-1}^{1}$$

$$= \frac{e^{-j\omega} - e^{j\omega}}{-j\omega} + \frac{1}{2}\,\frac{e^{-j(\omega-\pi)} - e^{j(\omega-\pi)}}{-j(\omega-\pi)} + \frac{1}{2}\,\frac{e^{-j(\omega+\pi)} - e^{j(\omega+\pi)}}{-j(\omega+\pi)}$$

$$= \frac{-2j\sin\omega}{-j\omega} + \frac{1}{2}\,\frac{-2j\sin(\omega-\pi)}{-j(\omega-\pi)} + \frac{1}{2}\,\frac{-2j\sin(\omega+\pi)}{-j(\omega+\pi)}$$

$$= \frac{2\sin\omega}{\omega} + \frac{\sin(\omega-\pi)}{\omega-\pi} + \frac{\sin(\omega+\pi)}{\omega+\pi}$$

$$= \frac{2\sin\omega}{\omega} + \frac{\sin\omega}{\pi-\omega} - \frac{\sin\omega}{\pi+\omega}$$

4.21

(4.21) d) $x(t) = \sum_{k=0}^{\infty} \alpha^k \delta(t-kT) \qquad |\alpha|<1$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} \sum_{k=0}^{\infty} \alpha^k \delta(t-kT) e^{-j\omega t} dt$$

$$= \sum_{k=0}^{\infty} \alpha^k \int_{-\infty}^{\infty} \delta(t-kT) e^{-j\omega t} dt$$

$$= \sum_{k=0}^{\infty} \alpha^k e^{-j\omega kT}$$

$$= \sum_{k=0}^{\infty} \underbrace{\left(\alpha e^{-j\omega T}\right)^k}_{\text{1st term 1}}$$

geom series factor $\alpha e^{-j\omega T}$

note $|\alpha e^{-j\omega T}| = |\alpha| < 1$

$$= \frac{1}{1-\alpha e^{-j\omega T}}$$

e) $\left(t e^{-2t} \sin 4t\right) u(t) = x(t)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} t e^{-2t} \sin 4t \, e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} t e^{-2t} \left(\frac{e^{4jt} - e^{-4jt}}{2j}\right) e^{-j\omega t} dt$$

$$4.21$$

(4)

$$X(j\omega) = \frac{1}{2j}\int_0^{\infty} t\left(e^{t(-2+4j-j\omega)} - e^{t(-2-4j-j\omega)}\right) dt$$

$$= \frac{1}{2j}\left[\frac{t\,e^{t(-2+4j-j\omega)}}{-2+4j-j\omega} - \frac{e^{t(-2+4j-j\omega)}}{(-2+4j-j\omega)^2} - \frac{t\,e^{t(-2-4j-j\omega)}}{-2-4j-j\omega} + \frac{e^{t(-2-4j-j\omega)}}{(-2-4j-j\omega)^2}\right]_0^{\infty}$$

$$= \frac{1}{2j}\left(\frac{1}{(-2+4j-j\omega)^2} - \frac{1}{(-2-4j-j\omega)^2}\right)$$

$$= \frac{j}{2}\left(\frac{1}{(2+4j+j\omega)^2} - \frac{1}{(2-4j+j\omega)^2}\right)$$

1

4.22 a) $X(j\omega) = \dfrac{2\sin 3(\omega - 2\pi)}{\omega - 2\pi}$

↗ a sinc function shifted by 2π

$T_1 = 3$

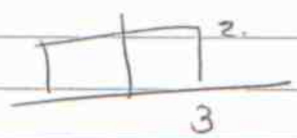
$\therefore$ $X(\omega j) = X_1\left(j(\omega - 2\pi)\right)$

$X_1(j\omega) = \dfrac{2\sin 3\omega}{\omega}$

See Table p. 329

$\therefore T_1 = 3$

$x_1(t)$



$\omega_0 = 2\pi$

$x(t) = e^{+j\omega_0 t} X_1(t)$

$= e^{+j2\pi t}\, x_1(t)$

$$\therefore x(t) = \begin{cases} 2e^{+j2\pi t} & |t| < 3 \\ 0 & \text{o.w.} \end{cases}$$

b) $X(j\omega) = \cos\left(4\omega + \pi/3\right) = \cos\left[4\left(\omega + \frac{\pi}{12}\right)\right]$

$= X_1\left(\omega + \pi/12\right)$

where $X_1(j\omega) = \cos 4\omega = \frac{1}{2}\left(e^{j4\omega} + e^{-j4\omega}\right)$

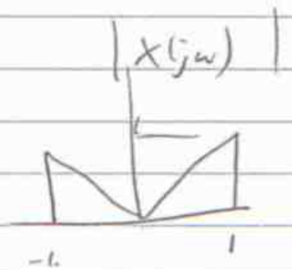
$$x_1(t) = \frac{1}{2}\left(\delta(t+4) + \delta(t-4)\right)$$

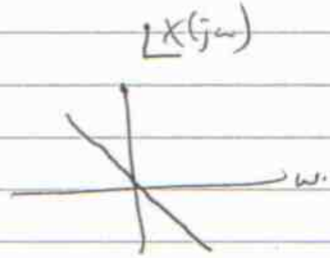
$$\therefore x(t) = e^{j\frac{\pi}{12}t}\, x(t)$$

$$= \frac{e^{-j\frac{\pi}{12}t}}{2}\left(\delta(t+4) + \delta(t-4)\right)$$

$$= \frac{e^{+j\frac{\pi}{3}}}{2}\delta(t+4) + \frac{e^{-j\pi/3}}{2}\delta(t-4)$$

c)





Note mag spectrum even
phase " odd

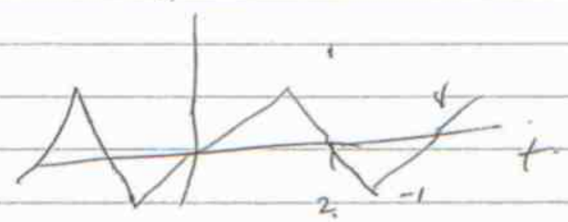
$\Rightarrow$ time signal real

$$X(j\omega) = \begin{cases} |\omega| e^{-j\omega 3} & |\omega| \le 1 \\ 0 & \text{o.w.} \end{cases}$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\left[\int_0^{1} \omega e^{-j\omega 3} e^{j\omega t}\, d\omega + \int_{-1}^{0} -\omega\, e^{-j\omega 3} e^{j\omega t}\, d\omega\right]$$

4.24 a)



odd signal
$\therefore$ F.T. purely imag

① $\therefore \mathrm{Re}(X(j\omega)) = 0$
true

② $\mathrm{Im}\{X(j\omega)\} \neq 0.$

③ $\exists\, \alpha$ s.t. $e^{j\alpha\omega} X(j\omega)$ is real ?

$X(j\omega)$ is real if $x(t)$ even.
mult by $e^{\alpha j\omega}$ is a shift in time domain.

So this question is asking can I shift $x(t)$ so that the shifted signal is even?

Yes delay by 1

④ $\int_{-\infty}^{\infty} X(j\omega)\, d\omega = 0$? $\ast$

Note ~~x(0)~~ $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$

evaluate at $t = 0$

$$x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, d\omega$$

$$\int_{-\infty}^{\infty} X(j\omega)\, d\omega = 2\pi\, x(0)$$

Since for this signal $x(0) = 0$ then yes this one $\ast$ is true

(5) $\int_{-\infty}^{\infty} \omega X(j\omega)\, d\omega = 0$?

Recall $\frac{d}{dt} x(t) \overset{\mathcal{F}}{\longleftrightarrow} j\omega X(j\omega)$

$\therefore \frac{d}{dt} x(t) \Big|_{t=0} = \frac{1}{2\pi} \int_{-\infty}^{\infty} j\omega X(j\omega) e^{j\omega t} d\omega \Big|_{t=0}$

$\frac{2\pi}{j} \frac{d}{dt} x(t) \Big|_{t=0} = \cancel{\frac{1}{2\pi}} \int_{-\infty}^{\infty} \omega X(j\omega)\, d\omega$

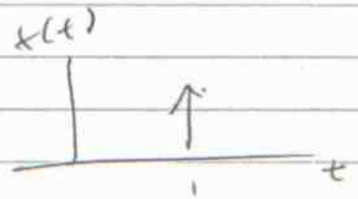
this one is asking if deriv. of $x(t)$ is zero at $t=0$

Here NO.

(6) $X(j\omega)$ periodic ?

periodic in freq domain $\Leftrightarrow$ discrete in time domain

Not true here

b) 

(1) not odd $\therefore Re\{X(j\omega)\} \neq 0$

(2) not even $\therefore Im\{X(j\omega)\} \neq 0$

(3) if we advance by 1 signal is even $\therefore$ yes to this one.

4.24

③

④ since $x(t)|_{t=0} = 0$ yes $\int_{-\infty}^{\infty} X(j\omega)\,d\omega = 0$

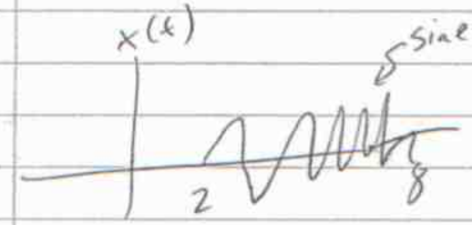
⑤ since $x'(0) = 0$ yes $\int_{-\infty}^{\infty} \omega X(j\omega)\,d\omega = 0$

⑥ since $x(t)$ only occurs at a finite # of pts

then yes $X(j\omega)$ is periodic.

(more generally countable # of times that are evenly spaced)

c) ~~① x(t) not~~



① $x(t)$ not odd $\therefore$ $Re\{X(j\omega)\} \neq 0$

② $x(t)$ not even $\therefore$ $Im\{X(j\omega)\} \neq 0$

③ Note: end pt at 2, & 8

$\therefore$ can't make even with a delay.

④ since $x(0) = 0$ then yes $\int_{-\infty}^{\infty} X(j\omega)\,d\omega = 0$

⑤ since $x'(0) = 0$ " yes $\int_{-\infty}^{\infty} \omega X(j\omega)\,d\omega = 0$

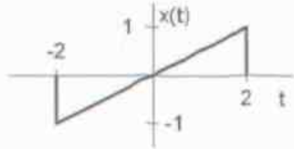
⑥ not discrete in time $\therefore$ $X(j\omega)$ not periodic

4. 24.

④

4.24 (Continue)

d)



(1) Since $x(t)$ is odd, $\mathrm{Re}\{X(j\omega)\}=0$

(2) Since $x(t)$ is odd, $\mathrm{Im}\{X(j\omega)\}\neq 0$

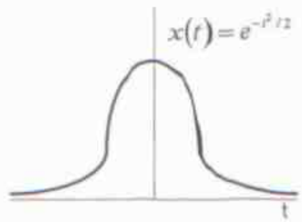
(3) Since $x(t)$ exists only between -2 and 2 and is odd, it's not possible to make it even by shifting $x(t)$.

(4) Since $x(0)$=0, $\int_{-\infty}^{\infty} X(j\omega)d\omega=0$

(5) Since $x'(0)=1/2\neq 0$, $\int_{-\infty}^{\infty}\omega X(j\omega)d\omega\neq 0$

(6) Since $x(t)$ is not discrete, $X(j\omega)$ is *not* periodic.

e)



(1) Since $x(t)$ is even, $\mathrm{Re}\{X(j\omega)\}\neq 0$

(2) Since $x(t)$ is even, $\mathrm{Im}\{X(j\omega)\}=0$

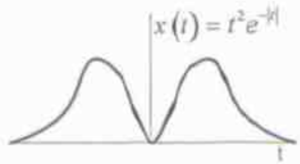
(3) Since $x(t)$ is already even, no need to shift ($\alpha=0$).

(4) Since $x(0)\neq 0$, $\int_{-\infty}^{\infty} X(j\omega)d\omega\neq 0$

(5) Since $\left.\frac{d}{dt}x(t)\right|_{t=0}=\left.\frac{d}{dt}e^{-t^2/2}\right|_{t=0}=\left.-te^{-t^2/2}\right|_{t=0}=0$, $\int_{-\infty}^{\infty}\omega X(j\omega)d\omega=0$

(6) Since $x(t)$ is not discrete, $X(j\omega)$ is *not* periodic.

f)



(1) Since $x(t)$ is even, $\mathrm{Re}\{X(j\omega)\}\neq 0$

(2) Since $x(t)$ is even, $\mathrm{Im}\{X(j\omega)\}=0$

(3) Since $x(t)$ is already even, no need to shift ($\alpha=0$).

(4) Since $x(0)$=0, $\int_{-\infty}^{\infty} X(j\omega)d\omega=0$

(5) For $t\geq 0$, $\left.\frac{d}{dt}x(t)\right|_{t=0}=\left.\frac{d}{dt}t^2e^{-t}\right|_{t=0}=\left.\left(2te^{-t}-t^2e^{-t}\right)\right|_{t=0}=0$

For $t\leq 0$, $\left.\frac{d}{dt}x(t)\right|_{t=0}=\left.\frac{d}{dt}t^2e^{t}\right|_{t=0}=\left.\left(2te^{t}+t^2e^{t}\right)\right|_{t=0}=0$

Therfore, $\left.\frac{d}{dt}x(t)\right|_{t=0}=0$, and $\int_{-\infty}^{\infty}\omega X(j\omega)d\omega=0$

(6) Since $x(t)$ is not discrete, $X(j\omega)$ is *not* periodic.

2/2

p339 b) want signal ① $Re\{X(j\omega)\} = 0$ — odd

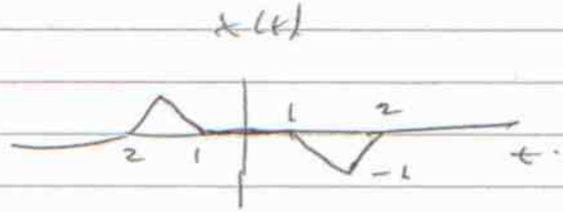
④ $\int_{-\infty}^{\infty} X(j\omega)\,d\omega = 0$ — $x(0)=0$ ~ generally needed for odd

⑤ $\int_{-\infty}^{\infty} \omega X(j\omega)\,d\omega = 0$ — slope is zero at $t=0$

but not ② – even easy

③ – can't make even by shift

⑥ – not desc.



Hilroy

4.28. a) $x(t) \xleftrightarrow{\mathcal{F}} X(j\omega)$

$p(t)$ periodic fundamental ω_0

$\therefore\ p(t) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t}$

F. T. of $y(t) = x(t)\,p(t)$

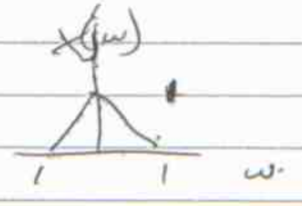
Note $P(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k\, \delta(\omega - k\omega_0)$

Now $Y(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$

$= \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} 2\pi a_k\, X(j(\omega - k\omega_0))$

$$Y(j\omega) = \sum_{k=-\infty}^{\infty} a_k\, X(j(\omega - k\omega_0))$$

b)



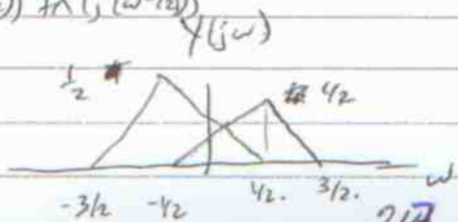
$y(t) = x(t)p(t)$

calc. $Y(j\omega)$

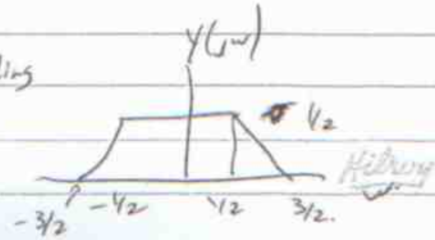
i) $p(t) = \cos t/2$.

$P(j\omega) = \left(\delta(\omega + 1/2) + \delta(\omega - \tfrac{1}{2})\right)\pi =$

$Y(j\omega) = \frac{1}{2}\left(X(j(\omega+1/2)) + X(j(\omega - 1/2))\right)$



$\longrightarrow$ adding

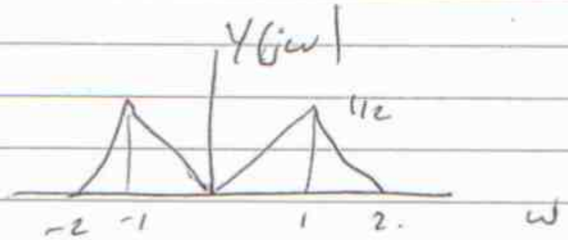


4.28 ②

ii) $p(t) = \cos t$

$$P(j\omega) = \pi \left(\delta(\omega-1) + \delta(\omega+1) \right)$$

$$Y(j\omega) = \frac{1}{2} \left(X(j(\omega-1)) + X(j(\omega+1)) \right)$$

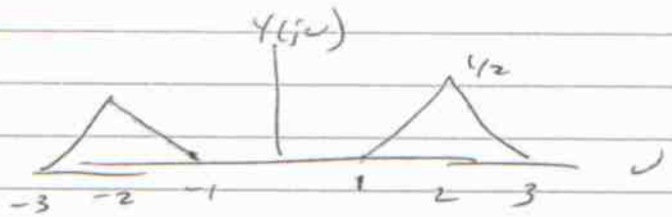


iii) $p(t) = \cos 2t$

$$P(j\omega) = \pi \left(\delta(\omega-2) + \delta(\omega+2) \right)$$

$$Y(j\omega) = \frac{1}{2\pi} \left(P(j\omega) * X(j\omega) \right)$$

$$= \frac{1}{2} \left[X(j(\omega-2)) + X(j(\omega+2)) \right]$$



iv) $p(t) = (\sin t)(\sin 2t)$

two ways

1st. $\sin A \sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$

$$\therefore p(t) = \frac{1}{2}\left[\cos(-t) - \cos 3t\right]$$

$$= \frac{1}{2}\cos t - \frac{1}{2}\cos 3t$$

4.28 (3)

$$\therefore \quad P(j\omega) = \frac{1}{2}\pi\left(\delta(\omega-1) + \delta(\omega+1)\right)$$
$$- \frac{1}{2}\pi\left(\delta(\omega-3) + \delta(\omega+3)\right) \quad (**)$$

other way

$$p_1(t) = \sin t$$
$$p_2(t) = \sin 2t.$$

$$P_1(j\omega) = \frac{\pi}{j}\left[\delta(\omega-1) - \delta(\omega+1)\right]$$

$$P_2(j\omega) = \frac{\pi}{j}\left[\delta(\omega-2) - \delta(\omega+2)\right]$$

Since $p(t) = p_1(t)\, p_2(t)$

$$P(j\omega) = \frac{1}{2\pi} P_1(j\omega) * P_2(j\omega)$$

$$= \frac{1}{2\pi}\frac{\pi}{j}\left[P_2(\omega-1) - P_2(\omega+1)\right]$$

$$= \frac{1}{2j}\left[\frac{\pi}{j}\left\{\delta(\omega-1-2) - \delta(\omega-1+2) - \left(\delta(\omega+1-2) - \delta(\omega+1+2)\right)\right\}\right]$$

$$= -\frac{\pi}{2}\left\{\delta(\omega-3) - \delta(\omega+1) - \delta(\omega-1) + \delta(\omega+3)\right\}$$

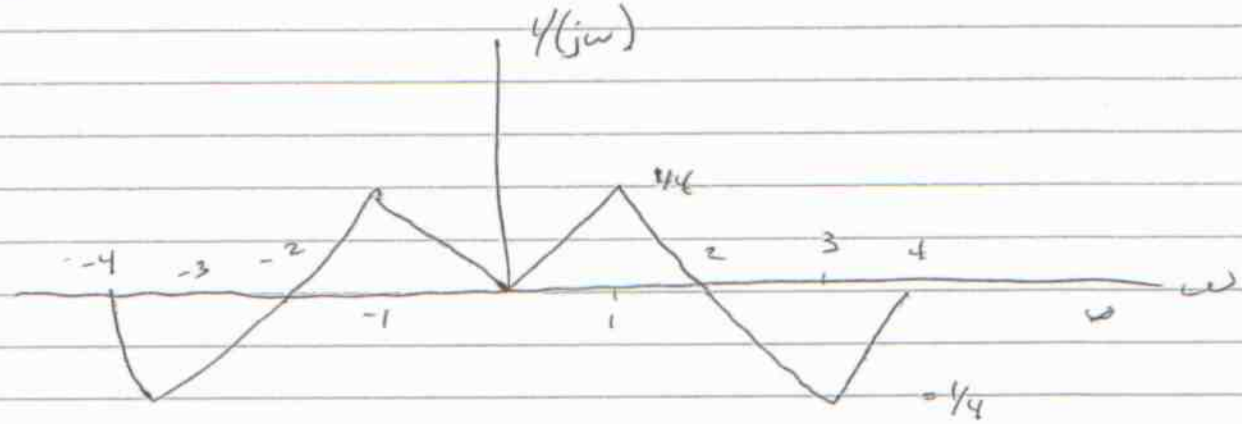
which of course is same as $**$

2/5

Hilroy

Now $Y(j\omega) = \frac{1}{2\pi}\left(X(j\omega) * P(j\omega) \right)$

$$= \frac{1}{4}\Big[X(j(\omega-1)) + X(j(\omega+1)) - X(j(\omega+3)) - X(j(\omega+3)) \Big]$$

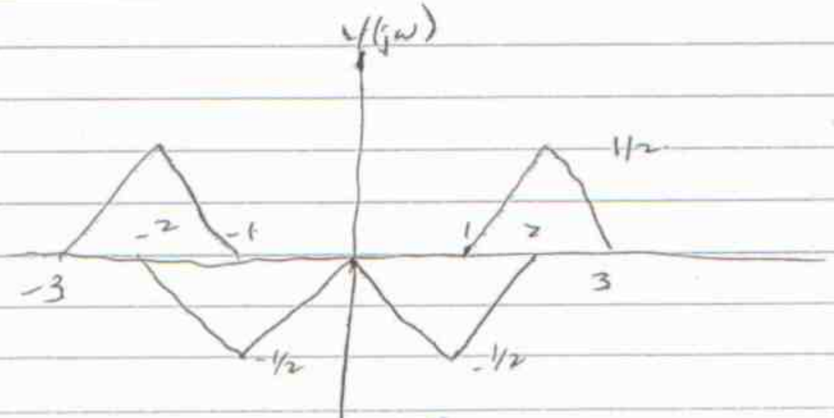


v) $p(t) = \cos 2t - \cos t$

$$P(j\omega) = \pi\left(\delta(\omega-2) + \delta(\omega+2)\right) - \pi\left(\delta(\omega+1) + \delta(\omega-1)\right)$$

$$Y(j\omega) = \frac{1}{2\pi} P(j\omega) * X(j\omega)$$

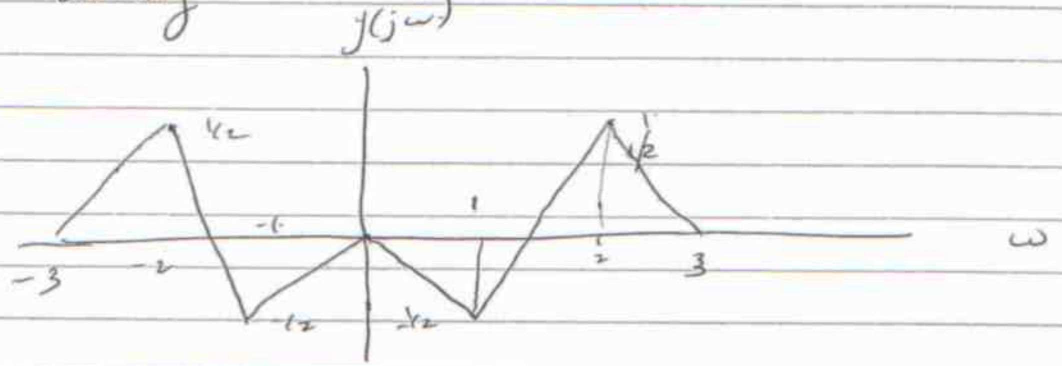
$$= \frac{1}{2}\Big[X(j(\omega-2)) + X(j(\omega+2)) - X(j(\omega+1)) - X(j(\omega-1)) \Big]$$



4.28

(5)

adding



vi) $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - \pi n)$ $\quad T = \pi$

periodic. $\quad \omega_0 = \frac{2\pi}{\pi} = 2.$

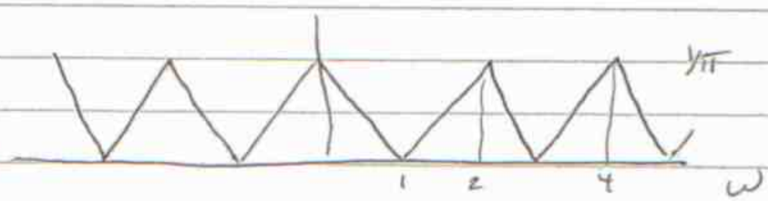
$$a_k = \frac{1}{\pi}$$

$$\therefore \quad P(j\omega) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{\pi} \delta(\omega - 2k) = 2\sum_{k=-\infty}^{\infty} \delta(\omega - 2k)$$

$$\therefore \; Y(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$= \frac{1}{\pi} \sum_{k=-\infty}^{\infty} X(j(\omega - 2k))$$

Y(jω).



4.28

(iii) $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - 2\pi n)$

$T = 2\pi$

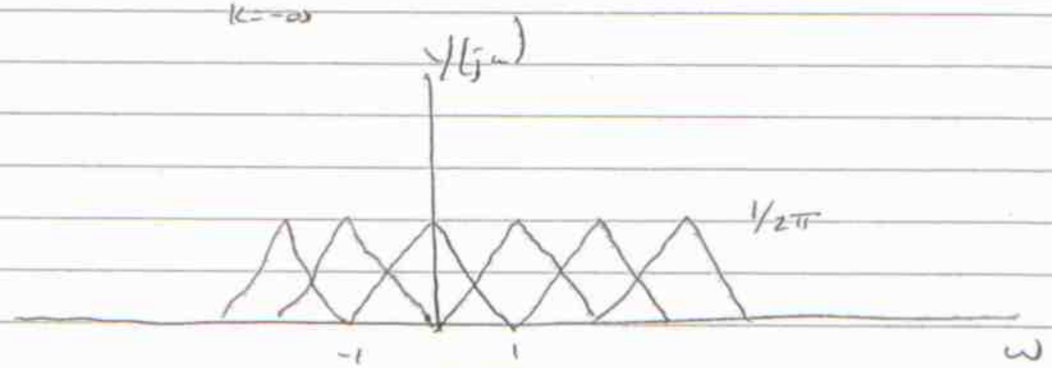
$\therefore \omega_0 = \frac{2\pi}{T} = 1$

$a_k = \frac{1}{2\pi}$

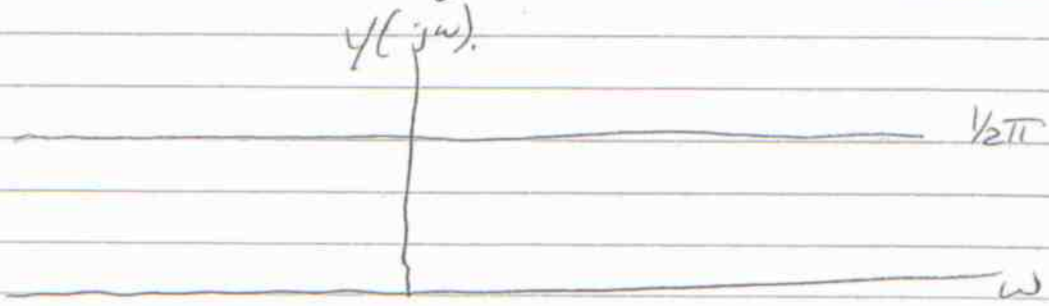
$P(j\omega) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{2\pi} \delta(\omega - k) = \sum_{k=-\infty}^{\infty} \delta(\omega - k)$

$\therefore Y(j\omega) = \frac{1}{2\pi} P(j\omega) * X(j\omega)$

$= \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} X(j(\omega - k))$



adding



4.28

(7)

viii) $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - 4\pi n)$

$T = 4\pi$

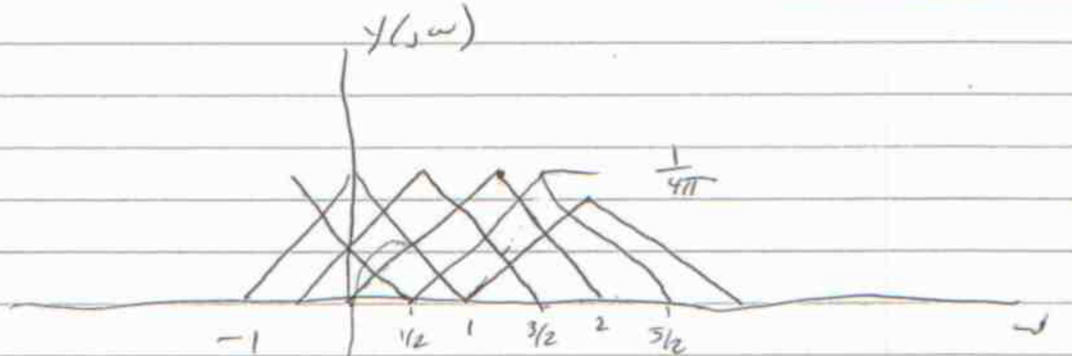
$\therefore \omega_0 = \frac{2\pi}{T} = \frac{1}{2}$

$a_k = \frac{1}{4\pi}$

$P(j\omega) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{4\pi} \delta(\omega - \frac{1}{2}k) = \frac{1}{2} \sum_{k=-\infty}^{\infty} \delta(\omega - \frac{k}{2})$

$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$

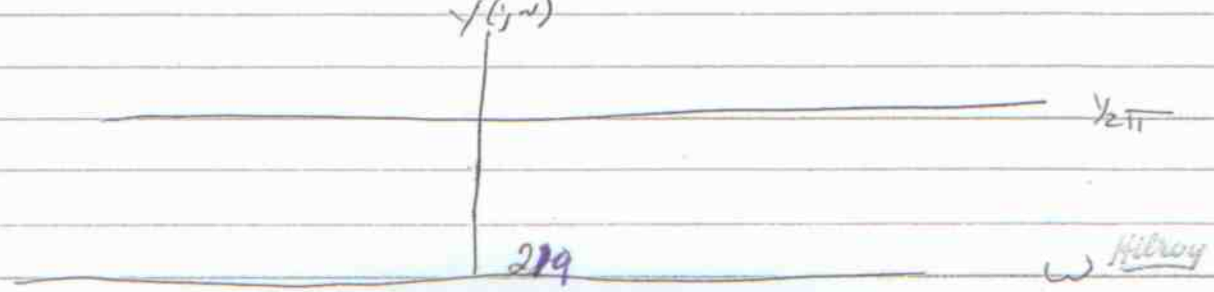
$= \frac{1}{4\pi} \sum_{k=-\infty}^{\infty} X(j(\omega - \frac{k}{2}))$



$\frac{1}{8\pi}$ at $\omega = 1/2$

we have $\frac{1}{4\pi} + \frac{1}{8\pi} + \frac{1}{8\pi} = \frac{1}{2\pi}$

constant with height $\frac{1}{2\pi}$



4.28 (8)

ix) $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - 2\pi n) - \frac{1}{2} \sum_{n=-\infty}^{\infty} \delta(t - \pi n)$

call p_7 like part vii

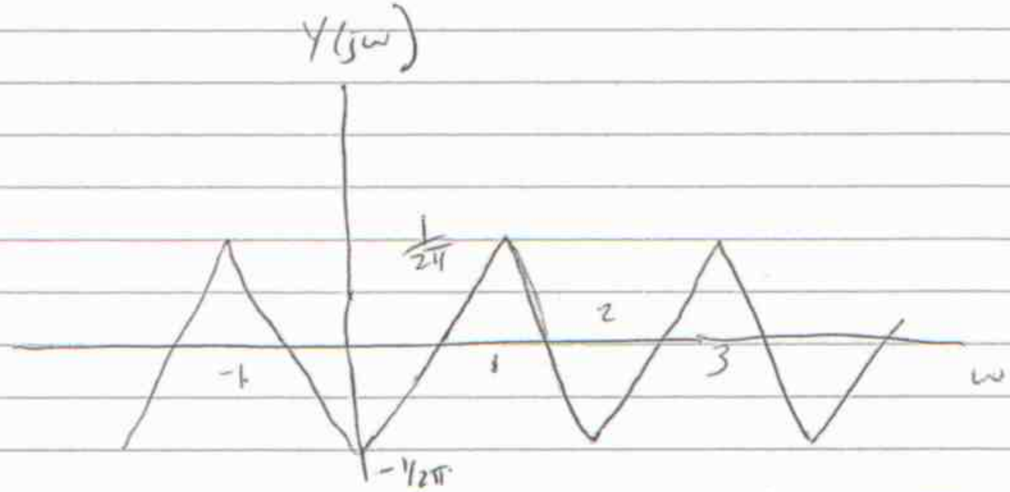
call p_6 like part 6

$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$

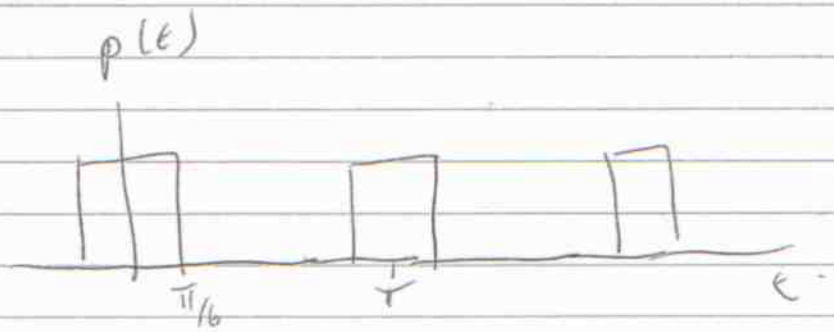
$= \frac{1}{2\pi} X(j\omega) * \left(P_7(j\omega) - \frac{1}{2} P_6(j\omega) \right)$

$= X_7(j\omega) - \frac{1}{2} X_6(j\omega)$

answers from part vii & vi



x)



like Ex 35

4.28

(9)

$\omega_0 = \frac{2\pi}{\pi} = 2$

p $\quad P(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi \frac{\sin \omega_0 k T_1}{\pi k} \delta(\omega - 2k)$

$T_1 = \frac{\pi}{6}$

$$= \sum_{k=-\infty}^{\infty} \frac{2 \sin \frac{\pi}{3} k}{k} \delta(\omega - 2k)$$

$$\therefore \; Y(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$= \sum_{k=-\infty}^{\infty} \frac{\sin \frac{\pi}{3} k}{\pi k} X(j(\omega - 2k))$$