5.21) a) $x[n] = u[n-2] - u[n-6]$

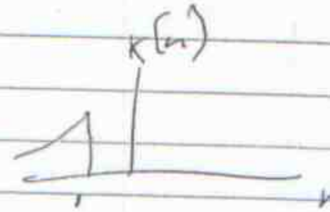
$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=2}^{5} e^{-j\omega n}$$

$$= e^{-j\omega 2} \sum_{n=0}^{3} e^{-j\omega n}$$

$$= e^{-2j\omega} \frac{1 - e^{-4j\omega}}{1 - e^{-j\omega}}$$

geom!
factor $e^{-j\omega}$
finite series
1st term



b) $x[n] = \left(\frac{1}{2}\right)^{-n} u[-n-1]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{-1} \left(\frac{1}{2}\right)^{-n} e^{-j\omega n}$$

Let $m = -n$.

$$= \sum_{m=1}^{\infty} \left(\frac{1}{2}\right)^{m} e^{j\omega m}$$

$$= \frac{1}{2} e^{j\omega} \sum_{m=0}^{\infty} \left(\frac{1}{2}\right)^{m} e^{j\omega m}$$

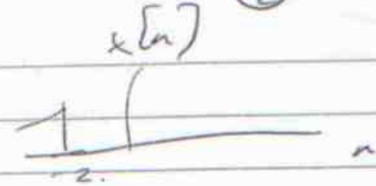
$$= \frac{1}{2} e^{j\omega} \frac{1}{1 - \frac{1}{2} e^{j\omega}}$$

geom!
factor $\frac{1}{2}e^{j\omega}$
$|\frac{1}{2} e^{j\omega}| < 1$
1st term

Hilroy

5.21 ②

c) $x[n] = \left(\frac{1}{3}\right)^{|n|} u[-n-2]$.



$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{-2} \left(\frac{1}{3}\right)^{|n|} e^{-j\omega n}$$

Let $m = -n$

$$= \sum_{m=2}^{\infty} \left(\frac{1}{3}\right)^{|-m|} e^{j\omega m}$$

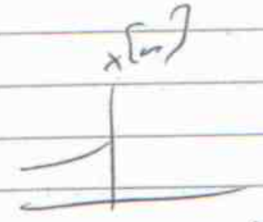
$$= \sum_{m=2}^{\infty} \left(\frac{1}{3}\right)^{m} e^{j\omega m}$$

$$= \frac{1}{3^2} e^{+2j\omega} \sum_{m=0}^{\infty} \left(\frac{1}{3}\right)^{m} e^{j\omega m}$$

geom Series $\left|\frac{1}{3}e^{j\omega}\right| < 1$ 1st term 0

$$= \frac{1}{9} e^{2j\omega} \frac{1}{1 - \frac{1}{3}e^{j\omega}}$$

d) $x[n] = 2^n \sin\left(\frac{\pi}{4} n\right) u[-n]$



$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{0} 2^n \sin \frac{\pi}{4} n \; e^{-j\omega n}$$

Let $n = -m$.

$$= -\sum_{m=0}^{\infty} 2^{-m} \sin \frac{\pi}{4} m \; e^{j\omega m}$$

Hilroy

5.21

③

$$= -\sum_{m=0}^{\infty} (0.5)^m \frac{e^{j\pi/4 m} - e^{-j\frac{\pi}{4}m}}{2j} e^{j\omega m}$$

$$= \frac{j}{2} \sum_{m=0}^{\infty} (0.5)^m \left(e^{jm(\omega+\frac{\pi}{4})} - e^{jm(\omega-\pi/4)} \right)$$

$$= \frac{j}{2} \left[\frac{1}{1 - 0.5 e^{j\pi/4} e^{j\omega m}} - \frac{1}{1 - 0.5 e^{-j\pi/4} e^{j\omega m}} \right]$$

5.21

4

e)

$$x[n]=\left(\frac{1}{2}\right)^{|n|}\cos\left(\frac{\pi}{8}(n-1)\right)$$

$$X\left(e^{j\omega}\right)=\sum_{n=-\infty}^{\infty}x[n]e^{-j\omega n}=\sum_{n=-\infty}^{\infty}\left(\frac{1}{2}\right)^{|n|}\cos\left(\frac{\pi}{8}(n-1)\right)e^{-j\omega n}$$

$$=\sum_{n=-\infty}^{-1}\left(\frac{1}{2}\right)^{-n}\cos\left(\frac{\pi}{8}(n-1)\right)e^{-j\omega n}+\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n}\cos\left(\frac{\pi}{8}(n-1)\right)e^{-j\omega n}$$

Substitute $m=-n$ in the first summation:

$$X\left(e^{j\omega}\right)=\sum_{m=1}^{\infty}\left(\frac{1}{2}\right)^{m}\cos\left(\frac{\pi}{8}(-m-1)\right)e^{j\omega m}+\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n}\cos\left(\frac{\pi}{8}(n-1)\right)e^{-j\omega n}$$

$$=\sum_{m=1}^{\infty}\left(\frac{1}{2}\right)^{m}\frac{e^{j\frac{\pi}{8}(-m-1)}+e^{-j\frac{\pi}{8}(-m-1)}}{2}e^{j\omega m}+\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n}\frac{e^{j\frac{\pi}{8}(n-1)}+e^{-j\frac{\pi}{8}(n-1)}}{2}e^{-j\omega n}$$

$$=\frac{1}{2}\sum_{m=1}^{\infty}\left(\frac{1}{2}\right)^{m}\left(e^{-j\frac{\pi}{8}}e^{-j\frac{\pi}{8}m}+e^{j\frac{\pi}{8}}e^{j\frac{\pi}{8}m}\right)e^{j\omega m}+\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n}\left(e^{-j\frac{\pi}{8}}e^{j\frac{\pi}{8}n}+e^{j\frac{\pi}{8}}e^{-j\frac{\pi}{8}n}\right)e^{-j\omega n}$$

$$=\left\{\frac{1}{2}e^{-j\frac{\pi}{8}}\sum_{m=1}^{\infty}\left(\frac{1}{2}e^{j\left(\omega-\frac{\pi}{8}\right)}\right)^{m}+\frac{1}{2}e^{j\frac{\pi}{8}}\sum_{m=1}^{\infty}\left(\frac{1}{2}e^{j\left(\omega+\frac{\pi}{8}\right)}\right)^{m}\right\}+$$

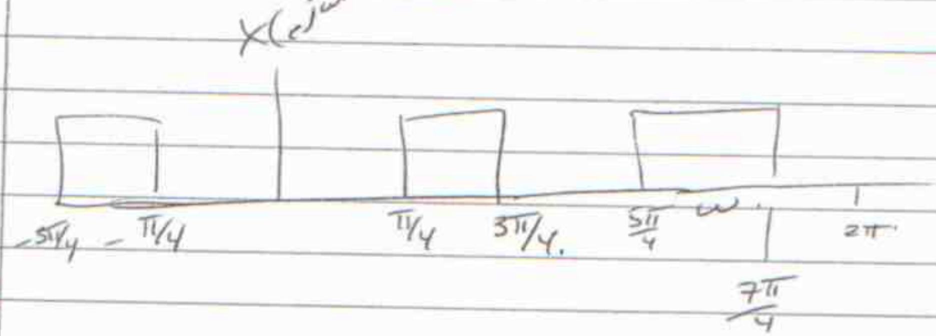
$$\left\{\frac{1}{2}e^{-j\frac{\pi}{8}}\sum_{n=0}^{\infty}\left(\frac{1}{2}e^{-j\left(\omega-\frac{\pi}{8}\right)}\right)^{n}+\frac{1}{2}e^{j\frac{\pi}{8}}\sum_{n=0}^{\infty}\left(\frac{1}{2}e^{-j\left(\omega+\frac{\pi}{8}\right)}\right)^{n}\right\}$$

$$=\left\{\frac{1}{2}e^{-j\frac{\pi}{8}}\frac{\frac{1}{2}e^{j\left(\omega-\frac{\pi}{8}\right)}}{1-\frac{1}{2}e^{j\left(\omega-\frac{\pi}{8}\right)}}+\frac{1}{2}e^{j\frac{\pi}{8}}\frac{\frac{1}{2}e^{j\left(\omega+\frac{\pi}{8}\right)}}{1-\frac{1}{2}e^{j\left(\omega+\frac{\pi}{8}\right)}}\right\}+\left\{\frac{1}{2}e^{-j\frac{\pi}{8}}\frac{1}{1-\frac{1}{2}e^{-j\left(\omega-\frac{\pi}{8}\right)}}+\frac{1}{2}e^{j\frac{\pi}{8}}\frac{1}{1-\frac{1}{2}e^{-j\left(\omega+\frac{\pi}{8}\right)}}\right\}$$

$$=\frac{1}{2}e^{-j\frac{\pi}{8}}\left[\frac{\frac{1}{2}e^{j\left(\omega-\frac{\pi}{8}\right)}}{1-\frac{1}{2}e^{j\left(\omega-\frac{\pi}{8}\right)}}+\frac{1}{1-\frac{1}{2}e^{-j\left(\omega-\frac{\pi}{8}\right)}}\right]+\frac{1}{2}e^{j\frac{\pi}{8}}\left[\frac{\frac{1}{2}e^{j\left(\omega+\frac{\pi}{8}\right)}}{1-\frac{1}{2}e^{j\left(\omega+\frac{\pi}{8}\right)}}+\frac{1}{1-\frac{1}{2}e^{-j\left(\omega+\frac{\pi}{8}\right)}}\right]$$

$$=\frac{1}{2}e^{-j\frac{\pi}{8}}\frac{3}{5-4\cos\left(\omega-\frac{\pi}{8}\right)}+\frac{1}{2}e^{j\frac{\pi}{8}}\frac{3}{5-4\cos\left(\omega+\frac{\pi}{8}\right)}$$

5.22a) $X(e^{j\omega}) = \begin{cases} 1 & \pi/4 \le |\omega| \le \tfrac{3}{4}\pi \\ 0 & \text{o.w.} \end{cases}$



consider

$x_1(n) = \dfrac{\sin \frac{\pi}{4} n}{\pi n}$ Table 5.2

Now $X_2(e^{j\omega}) = X_1(e^{j(\omega - \pi/2)}) + X_1(e^{j(\omega+\pi/2)})$

$\therefore \quad x[n] = e^{j\pi/2 n} x_1[n] + e^{-j\pi/2 n} x_1[n]$

$= 2\cos\frac{\pi}{2} n \; x_1[n]$

$= 2 \dfrac{\cos\frac{\pi}{2} n \sin \frac{\pi}{4} n}{\pi n}$

b) $X(e^{j\omega}) = 1 + 3e^{-j\omega} + 2e^{-j2\omega} - 4e^{-3j\omega} + e^{-j10\omega}$

Compare to def^n $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$\therefore x[n] = \delta[n] + 3\delta[n-1] + 2\delta[n-2] - 4\delta[n-3] + \delta[n-10]$

Hilroy

5.22

(2)

c) $X(e^{j\omega}) = e^{-j\omega/2} \qquad -\pi \leq \omega \leq \pi$

inv. F.T.

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega(n-\frac{1}{2})}\,d\omega$$

$$= \frac{1}{2\pi j(n-1/2)}\left[e^{j\omega(n-1/2)}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi j(n-1/2)}\left[e^{j\pi(n-1/2)} - e^{-j\pi(n-1/2)}\right]$$

$$= \frac{1}{2\pi j(n-1/2)}\left[-j e^{j\pi n} - j e^{-j\pi n}\right]$$

$$= \frac{(-1)^{n+1}}{\pi(n-1/2)}$$

Note 3rd entry from bottom in table 5.2 doesn't apply because constant n_0 would be 1/2. (can't have delta's at $n=1/2$.

if $e^{-j\omega n_0}$ $\quad n_0 \in \mathbb{Z}$

then we would get delta function not answer in form above

Hilroy

5.22

③

d) $X(e^{j\omega}) = \cos^2\omega + \sin^2 3\omega.$

recall

$$\cos 2\omega = 2\cos^2\omega - 1.$$

$$\cos^2\omega = \frac{\cos 2\omega + 1}{2}.$$

similarly

$$\sin^2 3\omega = \frac{1 - \cos 6\omega}{2}.$$

$$\therefore X(e^{j\omega}) = \frac{\cos 2\omega}{2} + \frac{1}{2} + \frac{1}{2} - \frac{\cos 6\omega}{2}$$

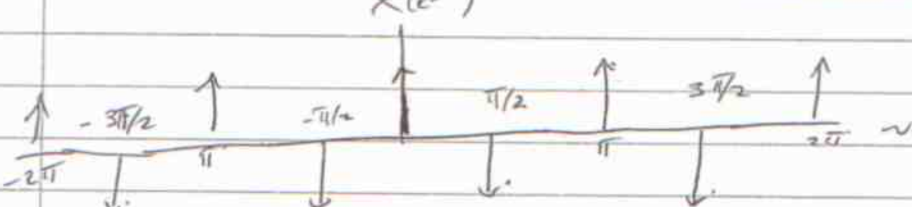
$$= \frac{e^{2j\omega} + e^{-2j\omega}}{4} + 1 - \frac{e^{6j\omega} + e^{-6j\omega}}{4}.$$

using table

$$x[n] = \tfrac{1}{4}\delta[n+2] + \tfrac{1}{4}\delta[n-2] + \delta[n]$$

$$- \tfrac{1}{4}\delta[n+6] - \tfrac{1}{4}\delta[n-6]$$

e) $X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(\omega - \frac{\pi}{2}k)$



missing 2π, π factor
conf -1 →
table 5.2

Clearly cos's.

$$x[n] = \frac{1}{2\pi} - \frac{1}{\pi}\cos\frac{\pi}{2}n + \frac{1}{\pi}\cos\pi n$$

$$- \frac{1}{\pi}\cos\frac{3\pi}{2}n.$$

2+4

Hilroy

manipulate

$$x[n] = \frac{1}{2\pi} - \frac{1}{\pi}\cos\frac{\pi}{2}n + \frac{1}{\pi}(-1)^n$$

$$-\frac{1}{\pi}\cos\left(\frac{3\pi}{2}-2\pi\right)n.$$

discrete cosine same if change ~~phase~~ freq by 2π

$$= \frac{1}{2\pi} - \frac{1}{\pi}\left[\cos\frac{\pi}{2}n + \cos -\frac{\pi}{2}n\right]$$

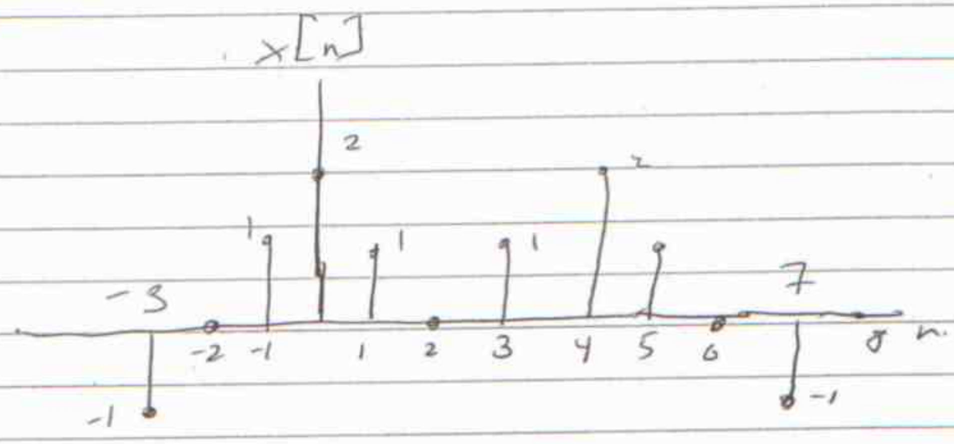
$$+\frac{1}{\pi}(-1)^n$$

cos even.

$$= \frac{1}{2\pi} - \frac{2}{\pi}\cos\frac{\pi}{2}n + \frac{1}{\pi}(-1)^n$$

Hilroy

5.23



$X(e^{j\omega})$ is F.T.

a) $X(e^{j\omega})\Big|_{\omega=0}$ D.C. value

(by adding

add up. $X(e^{j\omega})\Big|_{\omega=0} = 6.$

$$X(e^{j\omega})\Big|_{\omega=0} = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}\Big|_{\omega=0}$$

$$= \sum_{n=-\infty}^{\infty} x[n]$$

b) $\measuredangle X(e^{j\omega})$

Key observation this signal is symmetric about $n=2$.

if we define

$$g[n] = x[n+2]$$

y is even.

$\therefore Y(e^{j\omega})$ is real

$$X(e^{j\omega}) = Y(e^{j\omega})\, e^{-j\omega 2}.$$

$$\angle X(e^{j\omega}) = \angle Y(e^{j\omega}) - 2\omega.$$

$Y(e^{j\omega})$ ~~is~~ real implies this is 0 or π. ignore shifts by π

5.23

(2)

the $\underline{\angle X(e^{j\omega})} = -2\omega.$

c) $\int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega$?

Note $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\,d\omega$

let $n=0$

$$2\pi\, x[0] = \int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega.$$

$$\cancel{2\pi(2)} = 2(2\pi)$$

$$= \underline{\underline{4\pi}}$$

d) $X(e^{j\pi})$

$$X(e^{j\omega})\Big|_{\omega=\pi} = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}\Big|_{\omega=\pi}$$

$$= \sum_{n=-\infty}^{\infty} x[n]\,(-1)^{n}.$$

add up subtracting every other one. $= 2$

e) ~~F.T.~~ of $Re\{X(e^{j\omega})\}$ sketch. signal whose F.T. is

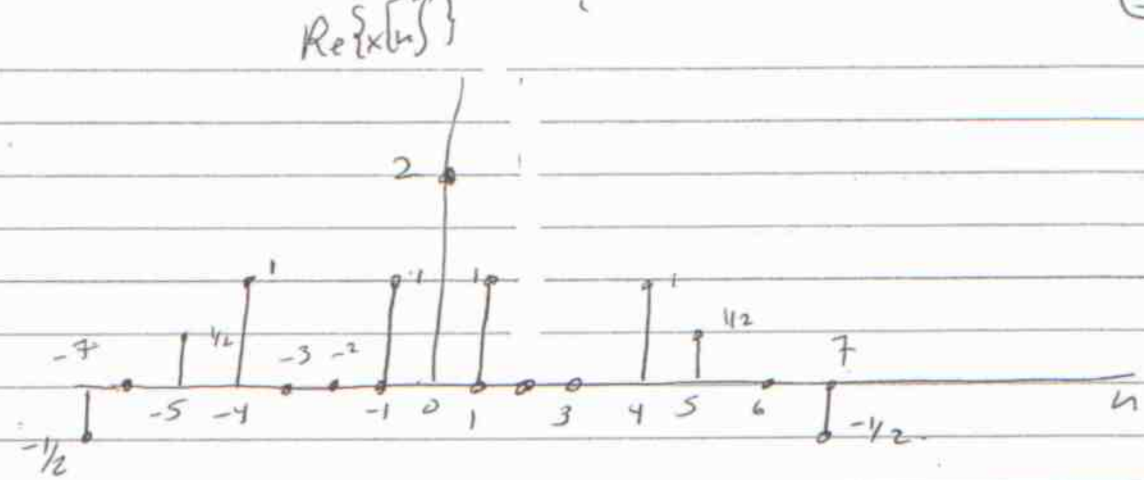
Recall that the real part of the F.T. correspond to the even part of the signal

∴ they are asking for:

$$Ev\{x[n]\} = \tfrac{1}{2}(x[n] + x[-n])$$

Hilroy

5.23



f) i) $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$ like Parseval

Recal $\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$

$\therefore \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 2\pi(14) = \underline{\underline{28\pi}}$

ii) $\int_{-\pi}^{\pi} \left| \frac{dX(e^{j\omega})}{d\omega} \right|^2 d\omega$

$j\frac{dX(e^{j\omega})}{d\omega} \overset{\mathcal{F}}{\longleftrightarrow} n x[n]$

energy of this signal is above

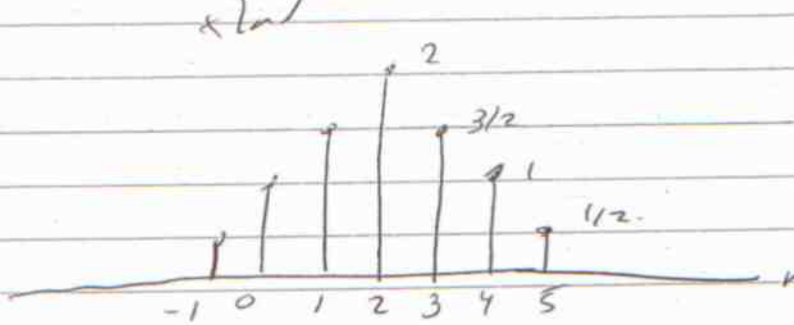
$\therefore \int_{-\pi}^{\pi} \left| \frac{dX(e^{j\omega})}{d\omega} \right|^2 d\omega = 2\pi \sum_{n=-\infty}^{\infty} |n x[n]|^2$

$= 2\pi(3^2 + 1^2 + 0^2 + 1^2 + 3^2 + 8^2 + 5^2 + 7^2)$

248 Hilbrwy

$= 2\pi(158) = 316\pi$ //

5.24a



i) $Re\{X(e^{j\omega})\} = 0$? only if odd signal
not odd ∴ not true.

ii) $Im\{X(e^{j\omega})\} = 0$ only if even signal
not even ∴ not true.

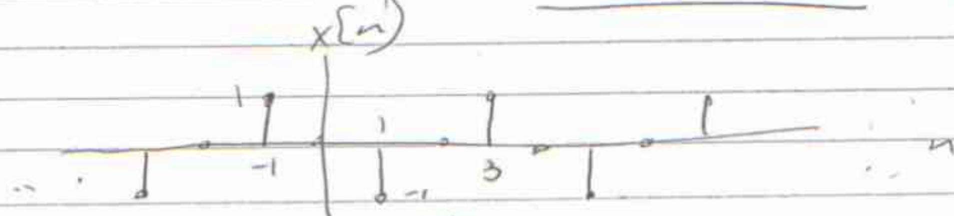
iii) $\exists\ \alpha$ s.t. $e^{j\alpha\omega}X(e^{j\omega})$ is real
i.e. by delay x[n] by any amount can we make signal even ?
yes. advance by 2.
∴ true.

iv) $\int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega = 0$ only if $x[0] = 0$
not true here.

v) $X(e^{j\omega})$ periodic. all discrete time signals ~~have~~ have spectrums periodic in 2π
∴ true.

vi) $X(e^{j\omega})\big|_{\omega=0} = 0$. DC value 0
not true here

b)



5.24.

2

i). this is odd signal $\therefore$ $Re\{X(e^{j\omega})\} = 0$

ii). not an even signal $\therefore$ not true

$$Im\{X(e^{j\omega})\} \neq 0.$$

iii). ~~can't~~ delay signal by 1 and it becomes even. i.e. $\alpha = -1$ and $e^{-j\omega} X(e^{j\omega}) \in \mathbb{R}.$

iv) $x[n] = 0$

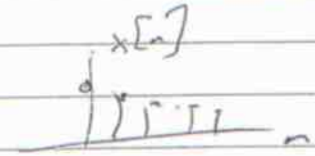
$$\therefore \int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega = 0$$

v). discrete time signal $\therefore$ $X(e^{j\omega})$ is periodic

vi) DC value is 0

$$\therefore X(e^{j\omega})\Big|_{\omega=0} = 0$$

c) $x[n] = \left(\frac{1}{2}\right)^n u[n].$



i). not odd $\therefore$ $Re\{X(e^{j\omega})\} \neq 0$

ii) not even $\therefore$ $Im\{X(e^{j\omega})\} \neq 0$

iii) signal is right sided $\therefore$ no delay will make it even. $\therefore$ $\nexists\, \alpha$ s.t $e^{j\alpha\omega} X(e^{j\omega}) \in \mathbb{R}$

iv) $x[0] = 1 \neq 0$

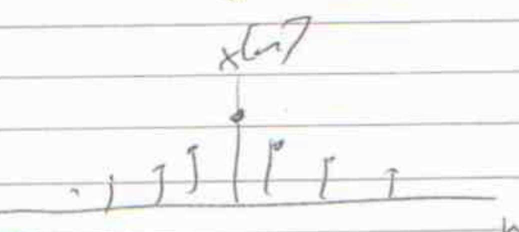
$$\therefore \int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega \neq 0$$

v) discrete time signal $\therefore$ $X(e^{j\omega})$ periodic

vi) Signal is 0 or pos. clearly DC value is not 0.

$$\therefore X(e^{j\omega})\Big|_{\omega=0} \neq 0$$

d) $x[n] = \left(\frac{1}{2}\right)^{|n|}$

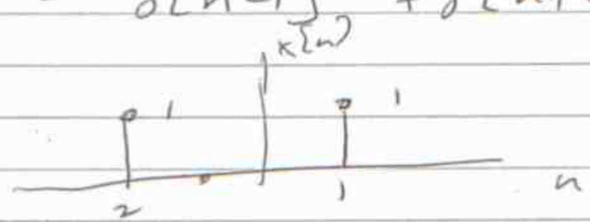


5.24

i) not odd $\therefore$ $Re\{X(e^{j\omega})\} \neq 0$
ii) even $\therefore$ $Im\{X(e^{j\omega})\} = 0$
iii) since its already even let $\alpha = 0$ and we satisfy condition.
iv) $x[0] = 1 \neq 0$
$$\therefore \int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega \neq 0$$

v) signal discrete $\therefore$ $X(e^{j\omega})$ periodic.
vi) DC value clearly not 0
$$\therefore X(e^{j\omega})\Big|_{\omega=0} \neq 0.$$

e) $x[n] = \delta[n-1] + \delta[n+2]$



i) not odd $\therefore$ $Re\{X(e^{j\omega})\} \neq 0$
ii) not even $\therefore$ $Im\{X(e^{j\omega})\} \neq 0$
iii) if signal were delayed by 1/2 then it would be even. (note we can't do this in time domain but $\alpha = 1/2$ no problem in freq
$\therefore$ $\alpha = 1/2$ satisfies condition.
iv) $x[0] = 1 \neq 0$
$$\therefore \int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega \neq 0$$

v) $x[n]$ discrete $\therefore$ $X(e^{j\omega})$ periodic
vi) $X(e^{j\omega})|_{\omega=0} \neq 0$ since DC value not 0

Hilroy