1. Show that $x^3 + log^5(x) = O(x^3)$.

   Solution: First, we want to show that $log^5(x) < Cx^3$ for some posive constant C and $x > 1$. We know $log(x) < x$, so $log(x^{\frac{3}{5}}) < x^{\frac{3}{5}}$ It is equivalent to $\frac{3}{5}log(x) < x^{\frac{3}{5}}$. Multiply five times, we get $(\frac{3}{5}log(x))^5 < (x^{\frac{3}{5}})^5$. Divide a constant, $log^5(x) < \frac{5^5}{3^5}x^3$. So $C = \frac{5^5}{3^5}$. Therefore $x^3 + log^5(x) < (1 + C)x^3$.

2. Show that $1 = O(x^3)$.

   Solution: when $x > 1$ we have $1 < 1 \times x^3$. From the Big-Oh notation definition we know $1 = O(x^3)$. Of course it is also true that $1 = O(1)$.

3. How many zeros are there at the end of 1200! ?.

   Solution: We know for every '0' we have a factor of $10 = 2 \times 5$. Because in 1200! there are many 2's, so we need to count the number of 5's. The count of numbers that can represented as a multiple of 5 is $\lfloor 1200/5 \rfloor$. The count of numbers that can be represented as a multiple of 5×5 is $\lfloor 1200/(5^2) \rfloor$. The count of numbers that can represented as a multiple of $5 \times 5 \times 5$ is $\lfloor 1200/(5^3) \rfloor$. Continue in this way until until $log_5(1200)$ steps. The solution is the sum of these results and it is 298.

4. Determine the last two digits of
   $S = 2^{5^1} + 2^{5^2} + \cdots + 2^{5^{1991}}$.

   Solution: We need to find $S(mod\ 100)$. Let's examine every term in the sum S.

   $2^{5^1}(mod\ 100) = 32$.

   $2^{5^2}(mod\ 100) = 32^5(mod\ 100) = 32^2(mod\ 100) \times 32^2(mod\ 100) \times 32(mod\ 100) = (24 \times 24 \times 32)(mod\ 100) = 32$.

   Continue in this way you will get $2^{5^k}(mod\ 100) = 32$. You need to prove it by Mathematical Induction. So
   $S(mod\ 100) = 32 \times 1991(mod\ 100) = 12$.