1. Recursively define the set of bit strings that have more 0s than 1s.

   Solution: Let S be the set of such strings and $0 \in S$. If $x \in S$ and $y \in S$, then $xy \in S$, $1xy \in S$, $x1y \in S$, $xy1 \in S$.

2. Show that $(w^R)^i = (w^i)^R$ whenever w is a string and i is a nonnegative integer.

   Solution: We use Mathematical Induction to prove it.

   BASIS STEP: when $i = 1$ we have $(w^R)^1 = (w^1)^R$.

   INDUCTIVE STEP: Assume $(w^R)^n = (w^n)^R$ is true for integer n. Then $(w^R)^{n+1} = (w^R)^n w^R = (w^n)^R w^R$. Since $(w_1 w_2)^R = w_2^R w_1^R$, we have $(w^n)^R w^R = (ww^n)^R = (w^{n+1})^R$. Hence, $(w^R)^{n+1} = (w^{n+1})^R$ is true.

   This complete the proof.

3. Let f_n be the nth Fibonacci number, prove that $f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}$ whenever n is a positive number.

   Solution: We use Mathematical Induction to prove it.

BASIS STEP: When $i = 1$ we have $f_1 = 1$ and $f_2 = 1$. So $f_1^2 = f_1 f_2$.

INDUCTIVE STEP: Assume it is true for integer n. We want to prove it is true for integer $n+1$.

$f_1^2 + f_2^2 + \cdots + f_n^2 + f_{n+1}^2 = f_n f_{n+1} + f_{n+1}^2 = f_{n+1}(f_n + f_{n+1}) = f_{n+1} f_{n+2}$.

This complete the proof.

4. Give a recursive definition of the sequence $\{a_n\}$ if $a_n = n^2$.

   Solution: $a_0 = 0$ and $a_{n+1} = a_n + (2n + 1)$. This is because $(n+1)^2 = n^2 + (2n+1)$.

5. Give a recursive definition of the set of positive integers not divisible by 5.

   Solution:

   $1, 2, 3, 4 \in S$, $x + 5 \in S$ if $x \in S$.

6. Give a recursive definition of the set of integers that are congruent to either 2 or 3 modulo 6.

   Solution: Let S be the set of integers that are congruent to either 2 or 3 modolo 6. A recursive definition is:

$2 \in S$, $3 \in S$, and
if $x \in S$ then $x + 6 \in S$ and $x - 6 \in S$.

7. Give a recursive definition of the set of bit strings that contain consecutive zeros.

   Solution: Let S be the set of bit strings that contain consecutive zeros. A recursive definition is:

   $00 \in S$, and
   if $x \in S$ then $0x \in S$, $1x \in S$, $x0 \in S$ and $x1 \in S$.

8. Give a recursive definition of the set of bit strings that have even length.

   Solution: Let S be the set of bit strings that have even length. A recursive definition is:

   $\lambda \in S$, and
   if $x \in S$ then $x01 \in S$, $x10 \in S$, $x00 \in S$ and $x11 \in S$.

9. Give a recursive definition of the set of bit

strings that are palindromes (i.e. bit strings that read the same forward as backwards).

Solution: Let S be the set of bit strings that are palindromes. A recursive definition is:

$\lambda, 0, 1 \in S$, and
if $x \in S$ then $0x0 \in S$ and $1x1 \in S$.

10. Devise a recursive algorithm to find a^{2^n} where a is a real number and n is a positive number.

    Solution:
    procedure iterative fun(a:real number,n positive integer)
    if $n = 1$ then $fun(a, n) = a^2$
    else $fun(a, n) = fun(a, n-1) * fun(a, n-1)$.

11. Give a recursive for finding the reversal of a bit string.

    Solution:
    procedure $reverse$(w:bit string)
    $n = length(w)$;
    if $n \leq 1$ then reverse(w) = w
    else $reverse(w) =$

$substr(w,n,n)reverse(substr(w,1,n{-}1))$.
where

$substr(w,a,b)$ is the substring of w consisting of the symbols in the ath through bth position.