Spatioterminological Reasoning: Subsumption Based on Geometrical Inferences
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Abstract: This paper presents a theoretical basis for terminological reasoning about objects and their qualitative spatial relationships. In contrast to existing work, which mainly focuses on reasoning about qualitative spatial relations alone, we integrate quantitative and qualitative information with terminological reasoning. This theory is motivated as basis for knowledge representation and query processing for instance in the domain of deductive geographic information systems.

Keywords: Qualitative spatial reasoning, terminological reasoning.

1 Introduction
The combination of formal conceptual and spatial reasoning serves as a theoretical basis for knowledge representation in domains such as geographical information systems (GIS) and can be used to solve important application problems. For instance, spatioterminological inferences can be applied to interpretation of map databases [6] and to spatial query processing [9].

Our treatment of spatial reasoning is based on Egenhofer’s set of topological relations [3] while the terminological reasoning part is based on description logic (DL) theory. In contrast to our earlier work presented in [7], [5] and [8] where topological relations are used as primitives in the sense of logic, we extend the treatment of topological relations with respect to conceptual reasoning by interpreting their semantic definition and by demonstrating their influence on automatic concept classification.

2 Integrating Spatial and Terminological Reasoning
This section introduces a space box (SBox) reasoner which implements inference services over spatial regions and concept terms. The SBox reasoner complements the usual DL reasoning facilities concerning the TBox and ABox. We analyze current possibilities to integrate the SBox into the CLASSIC system. The integration is based on a recent proposal [2] that extended the theory behind the CLASSIC DL for coping with external domains.

The fundamental idea of our SBox extension is the treatment of spatial regions as subsets of \( \mathbb{R}^2 \) represented by polygons and to define so-called spatial subsumption between polygons with respect to the relation \texttt{g.contains} (or \texttt{g.inside}, see Figure 1). Basically, spatial subsumption can be reduced to the polygon inclusion and intersection problem. The restriction to polygons is motivated by computational geometry offering efficient algorithms for polygon inclusion and intersection. As we will see, additional spatial inferences must be supported (reasoning with spatial relations) because not all polygons must necessarily be given as constants.

2.1 Spatial Relations
In a similar way as [4] we define 13 binary topological relations that are organized in a subsumption hierarchy (see Figure 1). The leaves of this hierarchy represent eight mutually exclusive relations (elementary relations) that are equivalent to the set of relations defined by Egenhofer [3]. The non-elementary relations are defined by a disjunction of relations represented as direct descendants of the corresponding nodes. Figure 2 illustrates five elementary relations (the inverses and the relation ‘equal’ have been omitted). Due to lack of space we refer to [6] for a formal definition of these relations.

2.2 New Language Constructs as External Concept Expressions
In order to support spatial inferences, we introduce new concept constructors based on these spatial relations. Our semantics assumes that each domain object is associated with its spatial representation (i.e. a polygon) via a predefined attribute \texttt{has_area} (see Figure 3).

Figure 1: Subsumption hierarchy of spatial relations.

Figure 2: Elementary spatial relations between A and B.
we define concepts for the federal states Hamburg and Schleswig-Holstein.

**federal_state_hh** ≡ (\forall \text{has_area } \text{equal}_{p_2})

**federal_state_sh** ≡ (\forall \text{has_area } \text{equal}_{p_2})

For instance, **federal_state_hh** is subsumed by **northern_german_region** since \(\xi[\text{equal}_{p_2}] \subseteq \xi[\text{inside}_{p_2}]\).

We like to emphasize that \(\text{equal}_{p_2}\) cannot subsume other spatial concepts. Algorithms for deciding subsumption between \(\text{sr}_p\) concepts are explained in Section 2.3.

In many cases, restrictions about spatial relations will have to be combined with additional restrictions. For example, how can we define a concept that describes a district of Hamburg that touches the federal state Hamburg from the inside? This requires some kind of qualified existential quantification. Thus, we propose the concept-forming operator \((\circ \text{sr} \ c)\) with the following semantics (let \(\text{sr}\) denote a spatial relation and \(c\) a concept term):

\[
\xi[(\circ \text{sr} \ c)] = \{x| \exists y_1, y_2, z : (x, y_1) \in \xi[\text{has_area}],
\]

\[
(z, y_2) \in \xi[\text{has_area}],
\]

\[
(y_1, y_2) \in \xi[\text{sr}], x \neq z, z \in [c]\}
\]

With this new operator we define the following two concepts. It can be proven that hh\_border\_district\_to\_sh is subsumed by hh\_border\_district.

**hh\_border\_district** ≡

\[
\text{district\_of\_hh} \cap (\circ \text{t\_inside federal_state_hh})
\]

**hh\_border\_district\_to\_sh** ≡

\[
\text{district\_of\_hh} \cap (\circ \text{touching federal_state_sh}) \cap
\]

\[
(\circ \text{spatially\_related federal_state_hh})
\]

In the next section we discuss how inferences about the new concept-forming terms can be realized with the CLASSIC extension interface.

### 2.3 Extending the CLASSIC Description Logic

Borgida et al. [2] defined the following set of functions for integrating a new concept-forming operator \(K\) into the CLASSIC description logic system. These functions are declared to the CLASSIC inference engine and are automatically called during subsumption proofs when required.

#### Normalization

In addition to syntax checking a normalization function for each term constructor \(K\) is required (in the following, the constructor pattern \(K\) is written in square brackets). As part of the normalization phase, all defined concepts are replaced by their definition.

- NormalizeTerm[\(\text{sr}_p\)](\(\text{sr}_p\)) = \(\text{sr}_p\text{NormalizePolygon}(p)\)
- NormalizeTerm[\((\circ \text{sr} \ c)\)]((\(\circ \text{sr} \ c\)) = (\(\circ \text{sr} \ \text{NormalizeTerm}(c)\))
Subsumption

Structural subsumption has to deal with terms that either contain external predicate terms (see [2]) or are equal to a predicate term. In our case, an external predicate term (used as a concept) refers to a polygon $p$ explicitly given in $sr_p$. In CLASSIC’s terminology, an external predicate term $sr_p$ is called a host concept. Host concepts may not be combined with abstract concepts (e.g. in conjunctions).

- **StructuralSubsumes**?[$sr_p$]($sr_p^1$,$sr_p^2$) returns true iff
  $\forall x \in P : sr_p^1(x) \iff sr_p^2(x)$. In other words:
  $\exists x \in P : \neg sr_p^1(x) \land sr_p^2(x, q)$ must be inconsistent.

Thus, in order to check whether $g_{inside_{ps}}$ subsumes $equal_{ps}$ (see above) the constraint system presented in Figure 5 must be solved. Before well-known algorithms for solving spatial constraint systems (based on Egenhofer’s composition table [3]) can be applied, restrictions concerning “concrete” polygon constants must be computed. In the example shown in Figure 5, $p_2$ (Hamburg) is known to be $s_{inside_{ps}}$ (Northern Germany). The constraint system is obviously inconsistent because $equal$ composed with $s_{inside}$ is defined to be $s_{inside}$. However, $\neg g_{inside}$ does not contain $s_{inside}$ (see also Figure 1). Thus, $equal_{ps}$ is subsumed by $g_{inside_{ps}}$. Grigni et al. have emphasized [4] that constraint systems that are (relationally) consistent must not necessarily lead to situations that are realizable in the plane. Thus, an additional planarity test must be added (see also [11]). For other concept-forming operators similar techniques can be applied.

- **StructuralSubsumes**?[(\(\bigcup sr\))\(\bigcup c1\)].(\(\bigcup sr^2 c2\)) returns true iff
  - $c2 \subseteq c1$ and
  - $\exists x, y, z \in P : \neg sr^1(x, y) \land sr^2(x, z)$ is inconsistent.

In order to compute whether a concept term based on the constructor $K$ subsumes a general concept term which is constructed with other concept constructors, we have to check whether the general concept implies the concept based on $K$.

Figure 5: Constraint network for computing the subsumption relation between two concepts. The constraint system is inconsistent.

\[
\begin{align*}
\text{StructuralSubsumes?} & \quad [sr_p] \quad (sr_p^1, sr_p^2) \quad \text{returns true iff} \\
\quad \forall x \in P & \quad : sr_p^1(x) \iff sr_p^2(x). \quad \text{In other words:} \\
\quad \exists x \in P & \quad : \neg sr^1(x, p) \land sr^2(x, q) \quad \text{must be inconsistent.}
\end{align*}
\]

\[
\begin{align*}
\text{In order to check whether } g_{\text{inside}_{ps}} & \text{ subsumes } equal_{ps} \quad \text{(see above) the constraint system presented in Figure 5 must be solved. Before well-known algorithms for solving spatial constraint systems (based on Egenhofer’s composition table [3]) can be applied, restrictions concerning “concrete” polygon constants must be computed. In the example shown in Figure 5, } p_2 \quad (\text{Hamburg}) \quad \text{is known to be } s_{\text{inside}_{ps}} \quad (\text{Northern Germany}). \quad \text{The constraint system is obviously inconsistent because } equal \quad \text{composed with } s_{\text{inside}} \quad \text{is defined to be } s_{\text{inside}}. \quad \text{However, } g_{\text{inside}} \quad \text{does not contain } s_{\text{inside}} \quad \text{(see also Figure 1).} \quad \text{Thus, } equal_{ps} \quad \text{is subsumed by } g_{\text{inside}_{ps}}. \quad \text{Grigni et al. have emphasized [4] that constraint systems that are (relationally) consistent must not necessarily lead to situations that are realizable in the plane. Thus, an additional planarity test must be added (see also [11]). For other concept-forming operators similar techniques can be applied.}
\end{align*}
\]

\[
\begin{align*}
\text{StructuralSubsumes?} & \quad [(\bigcup sr) \bigcup c1].(\bigcup sr^2 c2) \quad \text{returns true iff} \\
\quad c2 & \subseteq c1 \quad \text{and} \\
\quad \exists x, y, z \in P & \quad : \neg sr^1(x, y) \land sr^2(x, z) \quad \text{is inconsistent.}
\end{align*}
\]

In order to compute whether a concept term based on the constructor $K$ subsumes a general concept term which is constructed with other concept constructors, we have to check whether the general concept implies the concept based on $K$.

Figure 6: Example for a term that is implied by a conjunction of spatial host concepts.

\[
\begin{align*}
\text{StructuralSubsumes?} & \quad [sr_p] \quad (sr_p^1, sr_p^2) \quad \text{returns true iff} \\
\quad \text{the conjunction } & \quad \text{normalizedHostConcept} \land \neg sr_p \quad \text{is inconsistent. From normalizedHostConcept} \quad \text{we only consider the predicate terms } sr_p. \quad \text{This is a generalization of StructuralSubsumes?}. \quad \text{For instance, } s_{\text{inside}_{ps}} \quad \text{is subsumed by the conjunction } \text{spatially related}_{ps} \land g_{\text{inside}_{ps}} \quad (\text{see Figure 6}). \quad \text{Due to the constraint propagation process, } \text{spatially related} \quad \text{is restricted to } s_{\text{inside}} \quad \text{because, according to Figure 4, } p_2 \quad \text{is } s_{\text{inside}} {p_1}. \quad \text{If we claim that } \neg s_{\text{inside}_{ps}} \quad \text{holds, the constraint system becomes inconsistent.}
\end{align*}
\]

In order to check whether $hh$ border district is implied by $hh$ border district to sh it must be shown that the conjunction $\text{district of } hh \land (\text{touching federal state } sh) \land \text{spatially related}_{fs}$ (or its normalized form) implies $(\bigcup t_{\text{inside}} \text{ federal state } sh)$. To be able to prove this implication, a decision procedure for the pattern $K = (\bigcup sr \bigcup c)$ must be declared with CLASSIC’s extension interface.

- **StructuralSubsumes?$$[(\bigcup sr) \bigcup c1].(\bigcup sr^2 c2)$$ returns true iff $$[(\bigcup sr) \bigcup c1].(\bigcup sr^2 c2)$$ is implied by normalizedConcept. We have to extract from normalizedConcept every term of the form $[(\bigcup sr) \bigcup c1]$. and to combine them as a conjunction SC and check whether $\exists x : SC(x) \land \neg (\bigcup sr \bigcup c1)(x)$ is inconsistent.

The decision procedure will be explained with the example from above. We start with $SC = (\forall has_{area} g_{inside_{ps}}) \land (\bigcup touching federal state sh) \land (\bigcup spatially related federal state sh)$ and want to derive that $((\bigcup t_{\text{inside}} \text{ federal state } sh) \land \bigcup spatially related federal state sh)$ is implied. From the concept terms given with $SC$ we construct a graph. In Figure 7 an individual $x$ has been generated. For this individual $x$ all role fillers of has_{area} are $g_{inside}$. Because district of $hh$ holds. Since has_{area} is an attribute, a single filler can be generated as a representative ($q_2$, see Figure 7). The constraint $g_{inside}(q_2, p_2)$ is added. The other two terms are treated as follows. Due to the exists semantics of the circle operator, two additional individuals $y$ and $z$ are generated, together
with their associated geometrical representations $q_3$ and $q_4$, respectively. From the circle terms we know the constraints $\text{spatially\_related}(q_2, q_1)$ and $\text{touching}(q_2, q_3)$. Since $z$ is subsumed by $\text{federal\_state\_hh}$, $\text{equal}(q_1, p_2)$ also holds (see the structure created in Figure 7). Furthermore, $\text{equal}(q_3, p_4)$ holds, because $y$ is subsumed by $\text{federal\_state\_sh}$.

In Figure 8, implicit relations between spatial objects have been added and the constraints have been solved. Obviously, because $q_1$ is equal to $p_2$, $\text{g\_inside}(q_2, q_1)$ also holds. Since $p_2$ is touching $p_4$ (see Figure 4), the relation between $q_2$ and $q_1$ is further restricted to $t\_inside$. Now, in order to check whether $((\bigcirc t\_inside \text{federal\_state\_hh})$ is subsumed, the resulting graph structure is traversed (starting from $x$ and following $\text{has\_area}$), i.e. direct paths to the generated objects are examined. In our example structure, there are two (direct) paths to new individuals ($z$ is reached via $t\_inside$ and $y$ is reached via $\text{touching}$). The concepts $c_i$ of the individuals at the end of each of these paths are “matched” against the concept term $c$ of the $((\bigcirc sr \ c)$ term in question. If there exists a path with relation $r$ to an individual whose $c_i$ is subsumed by $c$ with $r$ being equal to or a subrelation of $sr$, then the $((\bigcirc sr \ c)$ term is implied by $SC$. This is indeed the case for $((\bigcirc t\_inside \text{federal\_state\_hh})$.

In a similar way as for $sr_p$ we declare a subsumption checker for $K = (\forall \text{has\_area} \ sr_p)$.

- Subsumes?$(\forall r c)((\forall \text{has\_area} \ sr_p), \text{normalizedConcept})$ returns true iff $\text{normalizedConcept}$ contains $((\bigcirc sr_p \ c_0)$, $c_0$ implies $(\forall \text{has\_area} \ sr_p^2)$, and $\exists x, y : \neg sr^1(y, p) \land sr^2(y, x) \land sr^3(x, p)$ is inconsistent. $(\forall \text{has\_area} \ equal_p)$ is also implied by a $(\text{fills has\_area} \ p_2)$ term because $\text{has\_area}$ is an attribute. Note that although CLASSIC adopts a non-standard semantics for $\text{fills}$, this is not relevant for host concepts since properties of host individuals cannot be changed by concept terms.

### Conjoining Concept Terms

The functions for conjoining concept terms and consistency checking are similar to the subsumption functions given above. Implied terms (see above) must also be considered. In some cases, conjunctions can be simplified. For brevity, we do not discuss conjoin functions in detail in this paper.

### 3 Related Work

Concerning description logic theory, another general technique for integrating external domains into DLs is the ‘concrete domain’ approach [1; 10]. For instance, 

**Figure 7:** Initial structure used for deriving the subsumption relation between $\text{hh\_border\_district}$ and $\text{hh\_border\_district\_to\_sh}$. For symmetric relations the arrows point in both directions.

**Figure 8:** Structure from Figure 7 with implicit relations added and constraints propagated. Irrelevant relations have been omitted for clarity.

$\text{ALC}(D)$ provides a simple interface for external domains basically requiring that the satisfiability of finite conjunctions of concrete predicates be decidable. However, this approach can only define concepts with concrete predicates that depend on information available from attribute chains starting with this concept. Spatial relations cannot be adequately defined with the operators and primitive roles offered by $\text{ALC}(D)$. Another solution might be the new role-forming operator of $\text{ALCRP}(D)$ as proposed in [11]. Then, the term $((\bigcirc sr \ c)$ could be replaced by $(\exists sr(\text{has\_area})(\text{has\_area} \ c)$. However, the satisfiability problem for $\text{ALCRP}(D)$ has shown to be undecidable (see [11]).

Grigni et al. [4] study the computational problems in developing an inference system for checking the satisfiability of (conjunctive) combinations of spatial relations. They point out that in topological inferencing the aspects of relational consistency and planarity interact in
rather complex ways. They showed that besides the relational consistency problem a planarity problem has to be solved when areas are assumed to be disjoint. With this additional restriction, in many cases the complexity of the satisfiability problem becomes NP-hard.

4 Conclusions

In this paper, we have developed a DL formalization of space with two separate domains: the abstract and the space domain. The abstract domain is used to represent terminological knowledge about spatial domains on an abstract logical level. The space domain extends the abstract domain and allows access to efficient reasoning algorithms (e.g. computational geometry, spatial indexing) for concrete spatial regions (e.g. polygons in map databases). We have demonstrated that, on the one hand, topological relations directly influence the kind of conceptual or terminological knowledge that can (and must) be derived by a formal inference engine. On the other hand, assertions about concepts restrict the set of possible spatial relations between different individuals.

Due to CLASSIC’s complex extension scheme for external domains, the integration of our proposed operators into CLASSIC appears to be less elegant than, for instance, the $\mathcal{ALC}(D)$ approach. The high complexity is caused by delegating to the user the full responsibility for capturing all (hidden) inferences associated with an external domain. However, the spatial inference rules presented in this paper indicate that CLASSIC’s DL extended by our operators still remains decidable. We do not support spatial relations in $\forall$-terms and only a limited form of exists-in restrictions for spatial relations can be defined.

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References


